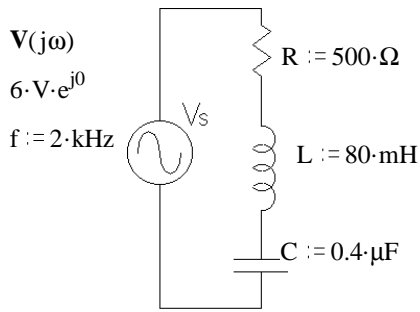


ECE 3600 Phasor Examples

Ex. 1 Find V_R , V_L , and V_C in polar phasor form. $f := 2 \cdot \text{kHz}$



$$\omega := 2 \cdot \pi \cdot f \quad \omega = 12566 \cdot \frac{\text{rad}}{\text{sec}}$$

$$Z_L := j \cdot \omega \cdot L \quad Z_L = 1.005j \cdot \text{k}\Omega$$

$$Z_C := \frac{1}{j \cdot \omega \cdot C} \quad Z_C = -0.199j \cdot \text{k}\Omega$$

$$Z_{eq} := R + j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C} \quad Z_{eq} = 500 + 806.366j \cdot \Omega$$

$$\sqrt{500^2 + 806^2} = 948.491 \quad \text{atan}\left(\frac{806}{500}\right) = 58.187 \cdot \text{deg} \quad Z_{eq} = 948.5 \Omega / 58.2^\circ$$

find the current: $I := \frac{6 \cdot V \cdot e^{j \cdot 0}}{Z_{eq}}$ magnitude: $\frac{6 \cdot V}{948.5 \Omega} = 6.326 \cdot \text{mA}$ angle: $0 \cdot \text{deg} - 58.2 \cdot \text{deg} = -58.2 \cdot \text{deg}$
 $I = 6.326 \text{mA} / -58.2^\circ$

find the magnitude

find the angle

$$V_R := I \cdot R \quad 6.326 \cdot \text{mA} \cdot 500 \cdot \Omega = 3.163 \cdot V \quad -58.2 \cdot \text{deg} + 0 \cdot \text{deg} = -58.2 \cdot \text{deg} \quad V_R = 3.163V / -58.2^\circ$$

$$V_L := I \cdot Z_L \quad 6.326 \cdot \text{mA} \cdot 1005 \cdot \Omega = 6.358 \cdot V \quad -58.2 \cdot \text{deg} + 90 \cdot \text{deg} = 31.8 \cdot \text{deg} \quad V_L = 6.358V / 31.8^\circ$$

$$V_C := I \cdot Z_C \quad 6.326 \cdot \text{mA} \cdot (-199) \cdot \Omega = -1.259 \cdot V \quad -58.2 \cdot \text{deg} + (-90) \cdot \text{deg} = -148.2 \cdot \text{deg} \quad V_C = -1.259V / 31.8^\circ$$

$$\text{OR: } 6.326 \cdot \text{mA} \cdot (199) \cdot \Omega = 1.259 \cdot V \quad -58.2 \cdot \text{deg} + (-90) \cdot \text{deg} = -148.2 \cdot \text{deg} \quad V_C = 1.259V / -148.2^\circ$$

OR, you can also find these voltages directly, using a voltage divider. I.E. to find V_C directly:

$$V_C := \frac{\frac{1}{j \cdot \omega \cdot C}}{R + j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C}} \cdot 6 \cdot V = \frac{1}{R \cdot (j \cdot \omega \cdot C) + j \cdot \omega \cdot L \cdot (j \cdot \omega \cdot C) + 1} \cdot 6 \cdot V = \frac{1}{R \cdot (j \cdot \omega \cdot C) - \omega^2 \cdot L \cdot C + 1} \cdot 6 \cdot V$$

$$= \frac{1}{(1 - \omega^2 \cdot L \cdot C) + j \cdot \omega \cdot R \cdot C} \cdot 6 \cdot V \quad (1 - \omega^2 \cdot L \cdot C) = -4.053 \quad j \cdot \omega \cdot R \cdot C = 2.513j$$

$$= \frac{6 \cdot V}{-4.053 + 2.513j} \cdot \frac{(-4.053 - 2.513j)}{(-4.053 - 2.513j)} = \frac{6 \cdot V \cdot (-4.053 - 2.513j)}{(-4.053)^2 + 2.513^2}$$

$$6 \cdot V \cdot (-4.053 - 2.513j) = -24.318 - 15.078j \cdot V$$

$$(-4.053)^2 + 2.513^2 = 22.742$$

$$= \left(\frac{-24.318}{22.742} - \frac{15.078j}{22.742} \right) \cdot V = -1.069 - 0.663j \cdot V$$

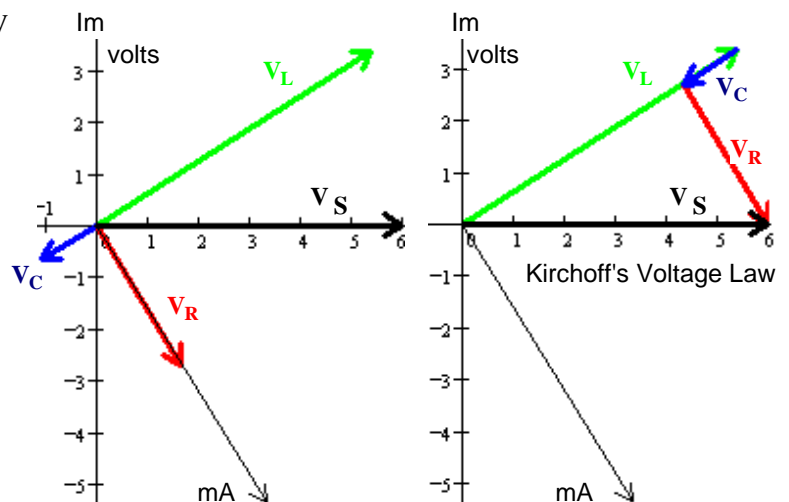
$$\text{magnitude: } \sqrt{1.069^2 + 0.663^2} = 1.258$$

$$\text{angle: } \text{atan}\left(\frac{-0.663}{-1.069}\right) = 31.81 \cdot \text{deg}$$

but this is actually in the third quadrant, so modify your calculator's results:

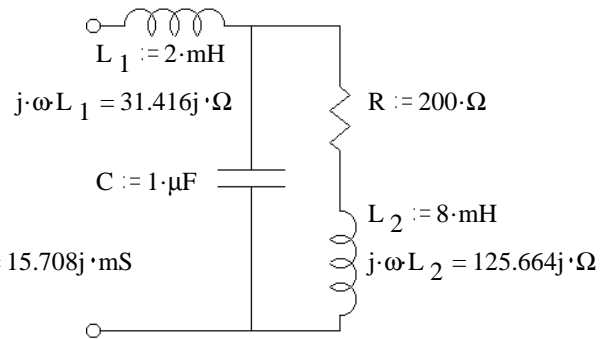
$$31.81 \cdot \text{deg} - 180 \cdot \text{deg} = -148.19 \cdot \text{deg}$$

$$= 1.258V / -148.2^\circ$$



ECE 3600 Phasor Examples p2

Ex. 2 a) Find Z_{eq} . $f := 2.5 \cdot \text{kHz}$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 15708 \cdot \frac{\text{rad}}{\text{sec}}$



$$Z_{eq} = j \cdot \omega L_1 + \frac{1}{\frac{1}{R + j \cdot \omega L_2} + \frac{1}{j \cdot \omega C}} = j \cdot \omega L_1 + \frac{1}{\frac{1}{R + j \cdot \omega L_2} + j \cdot \omega C}$$

$$Z_{eq} := j \cdot \omega L_1 + \frac{1}{\frac{1}{R + j \cdot \omega L_2} + j \cdot \omega C} = 31.416 \cdot j \cdot \Omega + \frac{1}{\frac{1}{(200 + 125.664 \cdot j) \cdot \Omega} + 15.708 \cdot j \cdot \text{mS}}$$

$$= 31.416 \cdot j \cdot \Omega + \frac{1}{(3.585 - 2.252 \cdot j + 15.708 \cdot j) \cdot \text{mS}} = 31.416 \cdot j \cdot \Omega + (18.487 - 69.391 \cdot j) \cdot \Omega = 18.487 - 37.975 \cdot j \cdot \Omega$$

$$|Z_{eq}| = 42.238 \cdot \Omega \quad \arg(Z_{eq}) = -64.043 \cdot \text{deg}$$

b) $V_{in} := 12 \cdot \text{V} \cdot e^{j \cdot 20 \cdot \text{deg}}$ Find I_{L1} , V_C $I_{L1} := \frac{V_{in}}{Z_{eq}} = \frac{12 \cdot \text{V}}{42.24 \cdot \Omega} = 284.1 \cdot \text{mA}$ $20 \cdot \text{deg} - (-64.04) \cdot \text{deg} = 84.04 \cdot \text{deg}$

$$I_{L1} = 284.1 \text{mA} / 84.04^\circ = 284.1 \cdot \text{mA} \cdot e^{j \cdot 84.04 \cdot \text{deg}} \quad I_{L1} = 29.485 + 282.569 \cdot j \cdot \text{mA}$$

$$V_C := I_{L1} \cdot (18.486 - 69.384 \cdot j) \cdot \Omega = 284.1 \cdot \text{mA} \cdot \sqrt{18.486^2 + 69.384^2} \cdot \Omega = 20.4 \cdot \text{V} \quad 84.04 \cdot \text{deg} + \text{atan}\left(\frac{-69.384}{18.486}\right) = 8.959 \cdot \text{deg}$$

To find V_C directly: $V_C := \frac{\frac{1}{\frac{1}{R + j \cdot \omega L_2} + j \cdot \omega C}}{j \cdot \omega L_1 + \frac{1}{\frac{1}{R + j \cdot \omega L_2} + j \cdot \omega C}} \cdot V_{in} = \frac{1}{j \cdot \omega L_1 \cdot \left(\frac{1}{R + j \cdot \omega L_2} + j \cdot \omega C\right) + 1} \cdot V_{in}$ $V_C = 20.153 + 3.178 \cdot j \cdot \text{V}$

You could then use another voltage divider to find V_R or V_{L2} .

c) Find I_{L2} $I_{L2} := \frac{V_C}{R + j \cdot \omega L_2} = \frac{20.4 \cdot \text{V} \cdot e^{j \cdot 8.96 \cdot \text{deg}}}{236.202 \cdot \Omega \cdot e^{j \cdot 32.142 \cdot \text{deg}}} = \frac{20.4 \cdot \text{V}}{236.202 \cdot \Omega} / 8.96 - 32.142^\circ = 86.4 \text{mA} / -23.18$

Or, directly by Current divider: $I_{L2} := \frac{\frac{1}{R + j \cdot \omega L_2}}{j \cdot \omega C + \frac{1}{R + j \cdot \omega L_2}} \cdot I_{L1} = \frac{1}{j \cdot \omega C \cdot (R + j \cdot \omega L_2) + 1} \cdot I_{L1} = 79.404 - 34.001 \cdot j \cdot \text{mA}$

d) How about I_C ? $I_C := \frac{V_C}{\left(\frac{1}{j \cdot \omega C}\right)} = V_C \cdot j \cdot \omega C = 20.4 \text{V} / 8.96^\circ \cdot 15.708 \text{mS} / 90^\circ = 320 \text{mA} / 98.96^\circ$

Or, directly by Current divider: $I_C := \frac{j \cdot \omega C}{j \cdot \omega C + \frac{1}{R + j \cdot \omega L_2}} \cdot I_{L1}$

This current is greater than the input current. What's going on?

The angle between I_C & I_{L2} is big enough that they somewhat cancel each other out, partial resonance.

Check Kirchoff's Current Law: $I_C + I_{L2} = 29.485 + 282.569 \cdot j \cdot \text{mA} = I_{L1} = 29.485 + 282.569 \cdot j \cdot \text{mA}$

ECE 3600 Phasor Examples p3

Ex. 3 a) Find Z_2 .

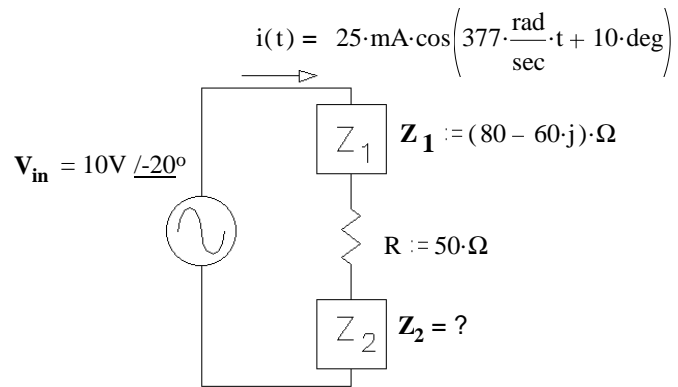
$$I := 25 \cdot \text{mA} \cdot e^{j \cdot 10 \cdot \text{deg}}$$

$$V_{in} := 10 \cdot \text{V} \cdot e^{-j \cdot 20 \cdot \text{deg}}$$

$$Z_T := \frac{V_{in}}{I} = \frac{10 \cdot \text{V}}{25 \cdot \text{mA}} \angle -20 - 10^\circ = 400 \Omega \angle -30^\circ$$

$$Z_T = 346.41 - 200j \cdot \Omega$$

$$Z_2 := Z_T - R - Z_1 = (346.41 - 200j) \cdot \Omega - 50 \cdot \Omega - (80 - 60j) \cdot \Omega = 216.41 - 140j \cdot \Omega$$



- b) Circle 1: i) The source current leads the source voltage <--- answer, because $10^\circ > -20^\circ$.
 ii) The source voltage leads the source current

Ex. 4 a) The impedance Z_1 (above) is made of two components in series. What are they and what are their values?

$$Z_1 = 80 - 60j \cdot \Omega \quad \omega := 377 \cdot \frac{\text{rad}}{\text{sec}}$$

Must have a resistor because there is a real part.

$$R := \text{Re}(Z_1)$$

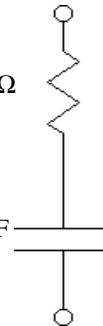
$$R = 80 \cdot \Omega$$

Must have a capacitor because the imaginary part is negative.

$$\text{Im}(Z_1) = -60 \cdot \Omega = \frac{-1}{\omega C}$$

$$C := \frac{-1}{\omega \text{Im}(Z_1)}$$

$$C = 44.21 \cdot \mu\text{F}$$



b) The impedance Z_1 is made of two components in parallel. What are they and what are their values?

$$Z_1 = 80 - 60j \cdot \Omega$$

Must have a resistor because there is a real part.

Must have a capacitor because the imaginary part is negative.

$$Z_1 = \frac{1}{\frac{1}{R} + j \cdot \omega C}$$

$$\frac{1}{Z_1} = \frac{1}{(80 - 60j) \cdot \Omega} \cdot \frac{(80 + 60j)}{(80 + 60j)} = \frac{80 + 60j}{80^2 + 60^2} = \frac{80 + 60j}{10,000} \cdot \frac{1}{\Omega}$$

$$80^2 + 60^2 = 10000$$

$$\frac{1}{Z_1} = 8 + 6j \cdot \text{mS} = 0.008 + 0.006j \cdot \frac{1}{\Omega} = \frac{1}{R} + j \cdot \omega C$$

$$\frac{1}{R} = 0.008 \cdot \frac{1}{\Omega}$$

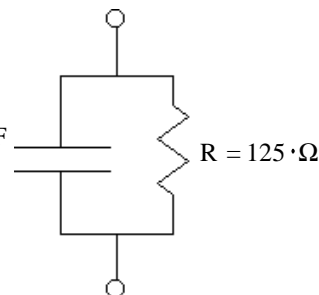
$$R := \frac{1}{0.008 \cdot \frac{1}{\Omega}}$$

$$R = 125 \cdot \Omega$$

$$\omega C = 0.006 \cdot \frac{1}{\Omega}$$

$$C := \frac{0.006 \cdot \frac{1}{\Omega}}{\omega}$$

$$C = 15.915 \cdot \mu\text{F}$$



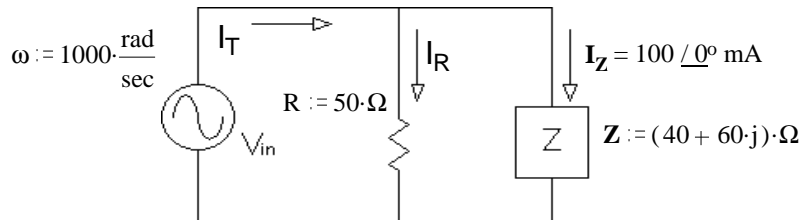
ECE 3600 Phasor Examples p4

Ex. 5 a) Find V_{in} in polar form.

$$I_Z := 100 \cdot \text{mA} \quad Z := (40 + 60j) \cdot \Omega$$

$$V_{in} := I_Z \cdot Z \quad V_{in} = 4 + 6j \cdot \text{V}$$

$$\sqrt{4^2 + 6^2} = 7.211 \quad \text{atan}\left(\frac{6}{4}\right) = 56.31 \cdot \text{deg} \quad V_{in} = 7.21 \text{V} \angle -56.3^\circ$$



b) Find I_T in polar form. $I_R := \frac{V_{in}}{R} = \frac{(4 + 6j) \cdot \text{V}}{50 \cdot \Omega} = \frac{4 \cdot \text{V}}{50 \cdot \Omega} + \frac{6j \cdot \text{V}}{50 \cdot \Omega} = 80 + 120j \cdot \text{mA}$

$$I_T := I_R + I_Z = (80 + 120j) \cdot \text{mA} + 100 \cdot \text{mA} = 180 + 120j \cdot \text{mA}$$

$$|I_T| = 216.3 \cdot \text{mA} \quad \arg(I_T) = 33.69 \cdot \text{deg} \quad I_T = 216.3 \text{mA} \angle 33.7^\circ$$

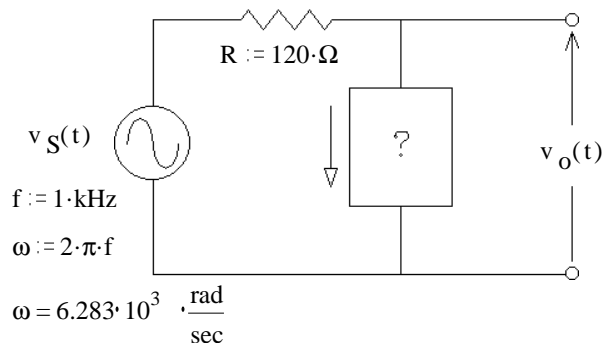
c) Circle 1: i) I_T leads V_{in} ii) V_{in} leads I_T answer ii), $56.3^\circ > 33.7^\circ$

Ex. 6 You need to design a circuit in which the the "output" voltage leads the input voltage ($v_S(t)$) by 30° of phase.

a) What should go in the box: R, L, C?

$$V_o = \frac{Z_{\text{box}}}{R + Z_{\text{box}}} \cdot V_S$$

$$\text{angle of } \frac{Z_{\text{box}}}{R + Z_{\text{box}}} \text{ is } 30^\circ.$$



This can only happen if the angle of Z_{box} is positive, so Z_{box} is an inductor

b) Find its value. $V_o = \frac{j \cdot \omega L}{R + j \cdot \omega L} \cdot V_S$ angle: $\frac{j \cdot \omega L}{R + j \cdot \omega L}$ is $90 - \text{atan}\left(\frac{\omega L}{R}\right) = 30^\circ$ so $\text{atan}\left(\frac{\omega L}{R}\right) = 60^\circ$.

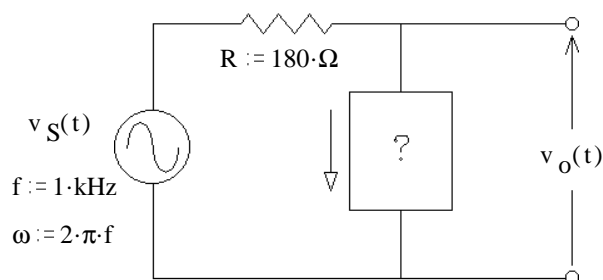
$$\frac{\omega L}{R} = \tan(60 \cdot \text{deg}) = 1.732 \quad L := \frac{R \cdot 1.732}{\omega} \quad L = 33.1 \cdot \text{mH}$$

Ex. 7 You need to design a circuit in which the the "output" voltage lags the input voltage ($v_S(t)$) by 40° of phase.

a) What should go in the box: R, L, C?

$$V_o = \frac{Z_{\text{box}}}{R + Z_{\text{box}}} \cdot V_S$$

$$\text{angle of } \frac{Z_{\text{box}}}{R + Z_{\text{box}}} \text{ is } -40^\circ.$$



This can only happen if the angle of Z_{box} is negative, so Z_{box} is a capacitor

b) Find its value. $V_o = \frac{1}{R + \frac{1}{j \cdot \omega C}} \cdot V_S$ angle: $\frac{1}{R + \frac{1}{j \cdot \omega C}}$ is $-90 - \text{atan}\left(\frac{1}{\omega C \cdot R}\right) = -90 - \text{atan}\left(-\frac{1}{\omega C \cdot R}\right)$ so $\text{atan}\left(-\frac{1}{\omega C \cdot R}\right) = -50^\circ$

$$-\frac{1}{\omega C \cdot R} = \tan(-50 \cdot \text{deg}) = -1.192 \quad C := \frac{1}{\omega R \cdot 1.192} \quad C = 0.742 \cdot \mu\text{F}$$

ECE 3600 Phasor Examples p5

Ex. 8 The magnitudes of \mathbf{I}_1 and \mathbf{I}_2 are 3A and 2A. They lag the supply voltage by 20° and 30° , respectively.

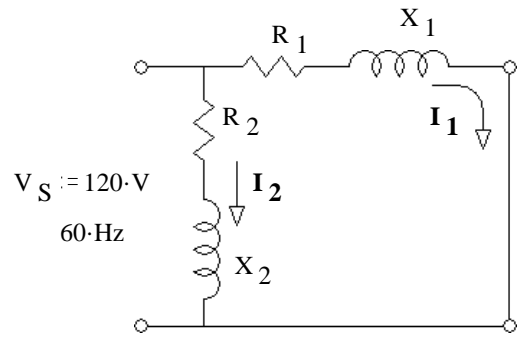
a) Find the values of R_1 , R_2 , X_1 and X_2 .

$$\mathbf{Z}_1 := \frac{120 \cdot \text{V}}{3 \cdot \text{A} \cdot e^{-j \cdot 20 \cdot \text{deg}}} \quad \mathbf{Z}_1 = 37.588 + 13.681j \cdot \Omega$$

$$R_1 := \text{Re}(\mathbf{Z}_1) \quad R_1 = 37.588 \cdot \Omega$$

$$X_1 := \text{Im}(\mathbf{Z}_1) \quad X_1 = 13.681 \cdot \Omega$$

$$\mathbf{Z}_2 := \frac{120 \cdot \text{V}}{2 \cdot \text{A} \cdot e^{-j \cdot 30 \cdot \text{deg}}} \quad \mathbf{Z}_2 = 51.962 + 30j \cdot \Omega \quad R_2 := \text{Re}(\mathbf{Z}_2) \quad R_2 = 51.962 \cdot \Omega \quad X_2 := \text{Im}(\mathbf{Z}_2) \quad X_2 = 30 \cdot \Omega$$



b) Add C to the circuit such that \mathbf{I}_{1C} leads \mathbf{I}_2 by 90° . Find the value of C.
 $\omega := 2 \cdot \pi \cdot 60 \cdot \text{Hz}$

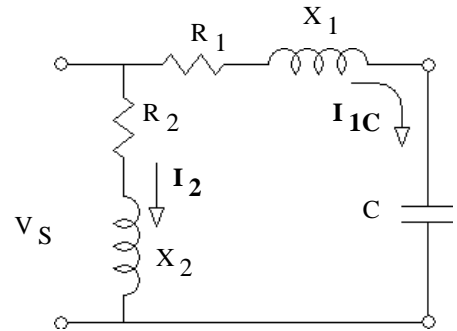
$$\mathbf{I}_{1C} = \frac{120 \cdot \text{V}}{R_1 + j \cdot X_1 + j \cdot X_C} \text{ needs to be at an angle of } +60^\circ$$

$$\text{So: } \text{atan}\left(\frac{X_1 + X_C}{R_1}\right) = -60 \cdot \text{deg}$$

$$\frac{X_1 + X_C}{R_1} = \tan(-60 \cdot \text{deg})$$

$$X_C := R_1 \cdot \tan(-60 \cdot \text{deg}) - X_1 \quad X_C = -78.785 \cdot \Omega = \frac{-1}{\omega C}$$

$$C := \frac{-1}{\omega X_C} \quad C = 33.7 \cdot \mu\text{F}$$



c) Change C so that the magnitudes of \mathbf{I}_{1C} and \mathbf{I}_2 are the same. Find the new C.

$$|\mathbf{I}_{1C}| = \left| \frac{120 \cdot \text{V}}{R_1 + j \cdot X_1 + j \cdot X_C} \right| \text{ needs to be } 2\text{A.} \quad \text{So: } |R_1 + j \cdot X_1 + j \cdot X_C| = 60 \cdot \Omega$$

$$\sqrt{R_1^2 + (X_1 + X_C)^2} = 60 \cdot \Omega$$

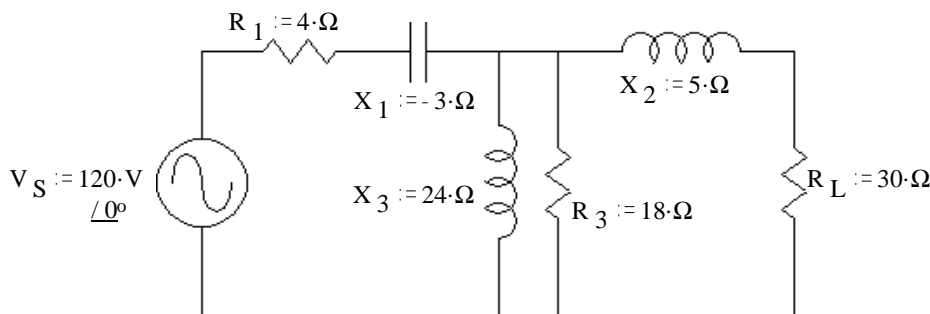
$$(X_1 + X_C) = \sqrt{(60 \cdot \Omega)^2 - R_1^2} = 46.767 \cdot \Omega$$

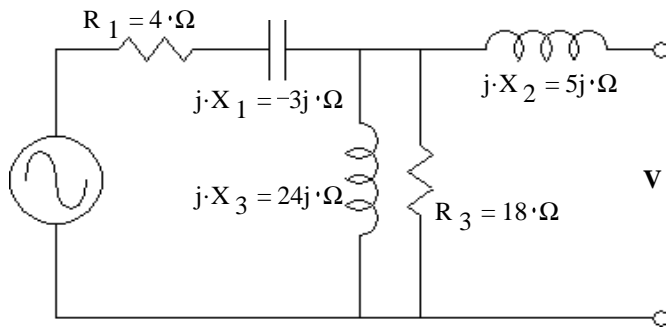
$$X_C := \sqrt{(60 \cdot \Omega)^2 - R_1^2} - X_1 \quad X_C = 33.086 \cdot \Omega = \frac{-1}{\omega C} \quad \text{NOT POSSIBLE}$$

$$\text{So: } (X_1 + X_C) = -46.767 \cdot \Omega$$

$$\text{And: } X_C := -\sqrt{(60 \cdot \Omega)^2 - R_1^2} - X_1 \quad X_C = -60.448 \cdot \Omega = \frac{-1}{\omega C} \quad C := \frac{-1}{\omega X_C} \quad C = 43.9 \cdot \mu\text{F}$$

Ex. 9 a) In the circuit below R_L is the load resistor. Find and draw the Thevenin equivalent of the rest of the circuit.



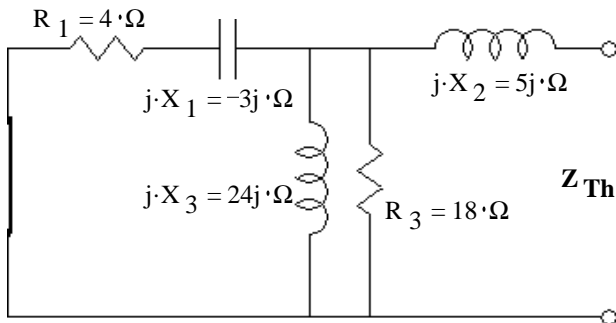


$$\mathbf{V_{Th}} := \mathbf{V_S} \cdot \frac{\left(\frac{1}{\frac{1}{j \cdot X_3} + \frac{1}{R_3}} \right)}{R_1 + j \cdot X_1 + \left(\frac{1}{\frac{1}{j \cdot X_3} + \frac{1}{R_3}} \right)}$$

$$|\mathbf{V_{Th}}| = 104.645 \cdot \text{V}$$

$$\arg(\mathbf{V_{Th}}) = 16.899 \cdot \text{deg}$$

$$\mathbf{V_{Th}} = 100.126 + 30.418j \cdot \text{V}$$



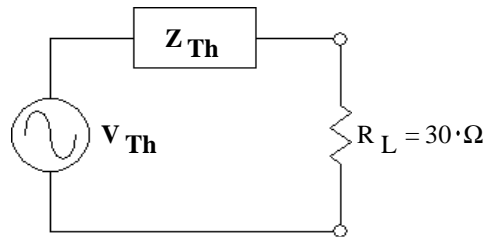
$$\mathbf{Z_{Th}} := \frac{1}{\frac{1}{R_1 + j \cdot X_1} + \frac{1}{j \cdot X_3} + \frac{1}{R_3}} + j \cdot X_2$$

$$|\mathbf{Z_{Th}}| = 5.3962 \cdot \Omega$$

$$\arg(\mathbf{Z_{Th}}) = 40.587 \cdot \text{deg}$$

$$\mathbf{Z_{Th}} = 4.098 + 3.511j \cdot \Omega$$

- b) Use the Thevenin equivalent to find the current through the load resistor and the voltage across the load resistor.



$$\mathbf{I_{RL}} := \frac{\mathbf{V_{Th}}}{\mathbf{Z_{Th}} + R_L}$$

$$\mathbf{I_{RL}} = 2.997 + 0.584j \cdot \text{A}$$

$$|\mathbf{I_{RL}}| = 3.053 \cdot \text{A}$$

$$\arg(\mathbf{I_{RL}}) = 11.02 \cdot \text{deg}$$

$$\mathbf{V_{RL}} := \mathbf{I_{RL}} \cdot R_L$$

$$\mathbf{V_{RL}} = 89.895 + 17.507j \cdot \text{V}$$

$$|\mathbf{V_{RL}}| = 91.584 \cdot \text{V}$$

$$\arg(\mathbf{V_{RL}}) = 11.02 \cdot \text{deg}$$

- c) Find a replacement for R_L in order to maximize the power delivered to R_L .

$$R_L := |\mathbf{Z_{Th}}|$$

$$R_L = 5.396 \cdot \Omega$$

- d) Find the new current and voltage for the load resistor.

$$\mathbf{I_{RL}} := \frac{\mathbf{V_{Th}}}{\mathbf{Z_{Th}} + R_L}$$

$$\mathbf{I_{RL}} = 10.32 - 0.612j \cdot \text{A}$$

$$|\mathbf{I_{RL}}| = 10.338 \cdot \text{A}$$

$$\arg(\mathbf{I_{RL}}) = -3.395 \cdot \text{deg}$$

$$\mathbf{V_{RL}} := \mathbf{I_{RL}} \cdot R_L$$

$$\mathbf{V_{RL}} = 55.687 - 3.303j \cdot \text{V}$$

$$|\mathbf{V_{RL}}| = 55.785 \cdot \text{V}$$

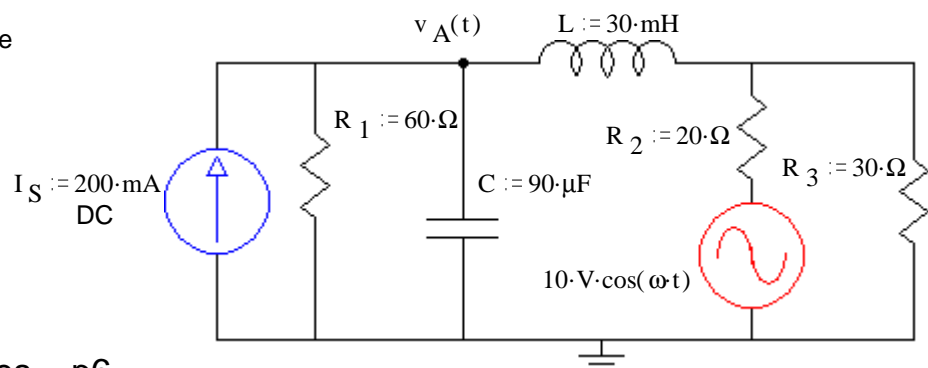
$$\arg(\mathbf{V_{RL}}) = -3.395 \cdot \text{deg}$$

Ex. 10 The circuit shown has two sources. The current source is DC and the voltage source is 60Hz.

Using superposition, find the nodal voltage $v_A(t)$. Be sure to redraw the circuit twice as part of your solution.

$$v_A(t) = ?$$

$$\omega := 2 \cdot \pi \cdot 60 \cdot \text{Hz}$$



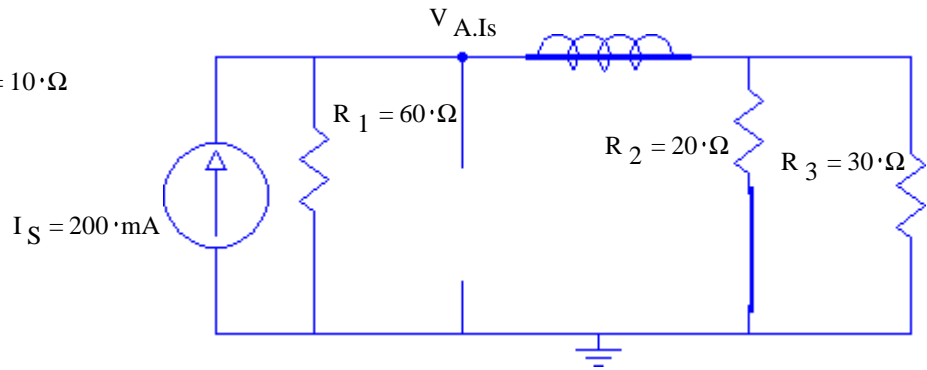
Eliminate voltage source

$$R_{eq} := \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$R_{eq} = 10 \cdot \Omega$$

$$V_{A,Is} := I_S \cdot R_{eq}$$

$$V_{A,Is} = 2 \cdot V$$



Eliminate current source

Let's use nodal analysis

node A

$$I_L = I_1 + I_C$$

$$\frac{V_B - V_A}{j \cdot \omega L} = \frac{V_A}{R_1} + V_A \cdot j \cdot \omega C$$

$$V_B - V_A = \left(\frac{V_A}{R_1} + V_A \cdot j \cdot \omega C \right) \cdot (j \cdot \omega L)$$

$$j \cdot \omega L = 11.31j \cdot \Omega$$

$$V_B = \left(\frac{V_A}{R_1} + V_A \cdot j \cdot \omega C \right) \cdot j \cdot \omega L + V_A$$

$$j \cdot \omega C = 33.929j \cdot \text{mS}$$

node B

$$I_2 = I_L + I_3$$

$$\frac{V_S - V_B}{R_2} = \frac{V_B - V_A}{j \cdot \omega L} + \frac{V_B}{R_3}$$

$$\frac{V_S}{R_2} + \frac{V_A}{j \cdot \omega L} = V_B \cdot \left(\frac{1}{j \cdot \omega L} + \frac{1}{R_3} + \frac{1}{R_2} \right) = V_B \cdot (83.333 - 88.419j) \cdot \text{mS} = V_B \cdot 121.5 \cdot \text{mS} \cdot e^{-46.696 \cdot \frac{\pi}{180} j}$$

$$V_B = \frac{V_S}{R_2 \cdot \left(\frac{1}{j \cdot \omega L} + \frac{1}{R_3} + \frac{1}{R_2} \right)} + \frac{V_A}{j \cdot \omega L \cdot \left(\frac{1}{j \cdot \omega L} + \frac{1}{R_3} + \frac{1}{R_2} \right)} = \left(\frac{V_A}{R_1} + V_A \cdot j \cdot \omega C \right) \cdot j \cdot \omega L + V_A$$

Equate to node A equation:

$$\frac{V_S}{R_2 \cdot \left(\frac{1}{j \cdot \omega L} + \frac{1}{R_3} + \frac{1}{R_2} \right)} = \left(\frac{V_A}{R_1} + V_A \cdot j \cdot \omega C \right) \cdot j \cdot \omega L + V_A - \frac{V_A}{1 + j \cdot \omega L \cdot \left(\frac{1}{R_3} + \frac{1}{R_2} \right)}$$

$$= V_A \cdot \left[\left(\frac{1}{R_1} + j \cdot \omega C \right) \cdot j \cdot \omega L + 1 - \frac{1}{1 + j \cdot \omega L \cdot \left(\frac{1}{R_3} + \frac{1}{R_2} \right)} \right]$$

$$1 + j \cdot \omega L \cdot \left(\frac{1}{R_3} + \frac{1}{R_2} \right) = 1 + 0.942j$$

$$\left(\frac{1}{R_1} + j \cdot \omega C \right) \cdot j \cdot \omega L = -0.384 + 0.188j$$

$$V_A := \frac{V_S}{R_2 \cdot \left(\frac{1}{j \cdot \omega L} + \frac{1}{R_3} + \frac{1}{R_2} \right)} \cdot \frac{1}{\left[\left(\frac{1}{R_1} + j \cdot \omega C \right) \cdot j \cdot \omega L + 1 - \frac{1}{1 + j \cdot \omega L \cdot \left(\frac{1}{R_3} + \frac{1}{R_2} \right)} \right]}$$

$$= \frac{V_S}{R_2 \cdot 121.5 \cdot \text{mS} \cdot e^{-j \cdot 46.696 \cdot \text{deg}} \cdot \left[(-0.384 + 0.188j) + 1 - \frac{1}{1 + 0.942j} \right]}$$

$$V_A = 4.796 - 3.5j \cdot V$$

$$|V_A| = 5.938 \cdot V \quad \arg(V_A) = -36.12 \cdot \text{deg}$$

$$V_{A,V_S} = 5.938 \cdot V \cdot \cos(377 \cdot t - 36.1 \cdot \text{deg})$$

Add the results $v_A(t) = 2 \cdot V + 5.938 \cdot V \cdot \cos(377 \cdot t - 36.1 \cdot \text{deg})$

Ex. 11 The circuit shown has two sources. The frequency of the current source is the third harmonic of the voltage source.

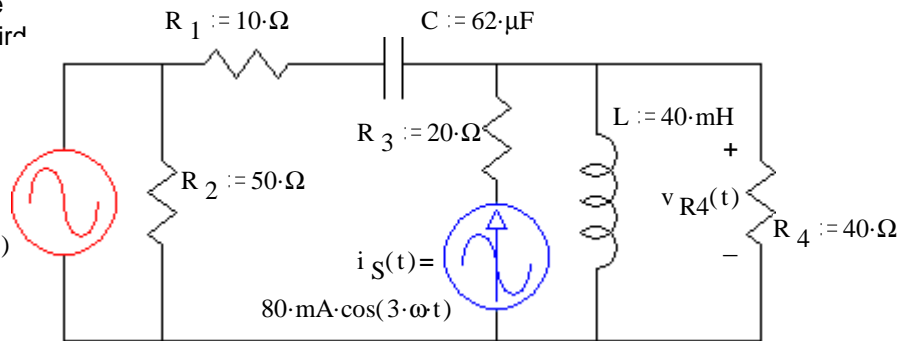
Using superposition, find the voltage across R_4 . Be sure to redraw the circuit twice as part of your solution.

$$v_{R4}(t) = ?$$

$$v_S(t) := 10 \cdot V \cdot \cos(\omega t)$$

$$f := 60 \cdot \text{Hz}$$

$$\omega := 2 \cdot \pi \cdot f$$



Eliminate current source

$$Z_C = \frac{1}{j \cdot \omega C} = -42.784j \cdot \Omega$$

$$Z_L = j \cdot \omega L = 15.08j \cdot \Omega$$

$$V_{R4.Vs} := V_S \cdot \frac{\frac{1}{\left(\frac{1}{j \cdot \omega L} + \frac{1}{R_4}\right)}}{R_1 + \frac{1}{j \cdot \omega C} + \frac{1}{\left(\frac{1}{j \cdot \omega L} + \frac{1}{R_4}\right)}}$$

$$V_{R4.Vs} = -2.875 + 3.138j \cdot V$$

$$|V_{R4.Vs}| = 4.256 \cdot V \quad \arg(V_{R4.Vs}) = 132.5 \cdot \text{deg}$$

$$v_{R4.Vs}(t) := 4.256 \cdot V \cdot \cos(\omega t + 132.5 \cdot \text{deg})$$

Eliminate voltage source

$$Z_C = \frac{1}{j \cdot 3 \cdot \omega C} = -14.261j \cdot \Omega$$

$$Z_L = j \cdot 3 \cdot \omega L = 45.239j \cdot \Omega$$

$$V_{R4.Is} := I_S \cdot \frac{1}{\left(\frac{1}{R_1 + \frac{1}{j \cdot 3 \cdot \omega C}} + \frac{1}{j \cdot 3 \cdot \omega L} + \frac{1}{R_4}\right)}$$

$$V_{R4.Is} = 1.165 - 0.501j \cdot V$$

$$|V_{R4.Is}| = 1.268 \cdot V \quad \arg(V_{R4.Is}) = -23.25 \cdot \text{deg}$$

$$v_{R4.Is}(t) := 1.268 \cdot V \cdot \cos(3 \cdot \omega t - 23.25 \cdot \text{deg})$$

Add the results

$$v_{R4}(t) := 4.256 \cdot V \cdot \cos(\omega t + 132.5 \cdot \text{deg}) + 1.268 \cdot V \cdot \cos(3 \cdot \omega t - 23.25 \cdot \text{deg})$$

$$t := 0, .2 \dots 30$$

