Review of Phasors

ECE 3600

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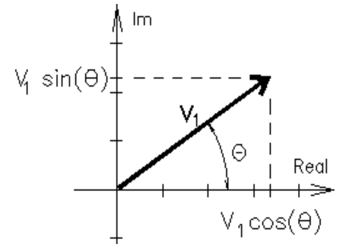
9/3/08
rev, 1/12/25

For steady-state sinusoidal response ONLY
(Which is pretty common in AC power)

Phasors



Time domain

$v(t) = V_1 \cdot \cos(377 \cdot t + \theta)$

Phasor, frequency domain

$\mathbf{V_1} = V_1 \cdot e^{j \cdot \theta} = V_1 \underline{/\theta} = V_1 \cdot \cos(\theta) + j \cdot V_1 \cdot \sin(\theta)$

Impedances, Reactances, Admittances, Susceptances

Inductor

$$v_L = L \cdot \frac{d}{dt} i_L = L \cdot \frac{d}{dt} I_L \cdot e^{j \cdot (\omega \cdot t + \theta)} = j \cdot \omega \cdot L \cdot \left[I_L \cdot e^{j \cdot (\omega \cdot t + \theta)} \right]$$

$$\mathbf{V_L}(\omega) = j \cdot \omega \cdot L \cdot \mathbf{I_L}(\omega)$$

AC impedance

$$\mathbf{Z_L} = j \cdot \omega \cdot L$$

Capacitor

$$i_C = C \cdot \frac{d}{dt} v_c = C \cdot \frac{d}{dt} V_c \cdot e^{j \cdot (\omega \cdot t + \theta)} = j \cdot \omega \cdot C \cdot \left[V_c \cdot e^{j \cdot (\omega \cdot t + \theta)} \right]$$

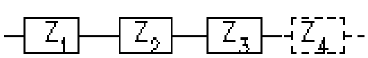
$$\mathbf{I_C}(\omega) = j \cdot \omega \cdot C \cdot \mathbf{V_c}(\omega) \qquad \mathbf{V_c}(\omega) = \frac{1}{j \cdot \omega \cdot C} \cdot \mathbf{I_C}(\omega) \qquad \mathbf{Z_C} = \frac{1}{j \cdot \omega \cdot C} = \frac{-j}{\omega \cdot C}$$

	Impedance	**Reactance**	**Admittance**	**Susceptance**
Resistor	$\mathbf{Z_R} = R$	$X_R = R = \mathbf{Z_R}$	$\mathbf{Y_R} = G_R = \frac{1}{R}$ (conductance)	$B_R = G_R = \frac{1}{R}$ (conductance)
Inductor	$\mathbf{Z_L} = j \cdot \omega \cdot L$	$X_L = \omega \cdot L = \lvert \mathbf{Z_L} \rvert$	$\mathbf{Y_L} = \frac{1}{j \cdot \omega \cdot L} = \frac{-j}{\omega \cdot L}$	$B_L = -\frac{1}{\omega \cdot L} = -\lvert \mathbf{Y_L} \rvert$
Capacitor	$\mathbf{Z_C} = \frac{1}{j \cdot \omega \cdot C} = \frac{-j}{\omega \cdot C}$	$X_C = -\frac{1}{\omega \cdot C} = -\lvert \mathbf{Z_C} \rvert$	$\mathbf{Y_C} = j \cdot \omega \cdot C$	$B_C = \omega \cdot C = \lvert \mathbf{Y_C} \rvert$

Units: $\frac{1}{\Omega} = \text{Siemen}$

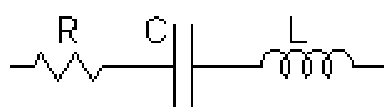
You can use impedances just like resistances as long as you deal with the complex arithmetic.
ALL the DC circuit analysis techniques will work with AC.

series:



$$\mathbf{Z_{eq}} = \mathbf{Z_1} + \mathbf{Z_2} + \mathbf{Z_3} + \ldots$$

Example: $f := 60 \cdot Hz \qquad \omega := 2 \cdot \pi \cdot f \qquad \omega = 377 \cdot \frac{rad}{sec}$



$R := 20 \cdot \Omega \qquad C := 60 \cdot \mu F \qquad L := 80 \cdot mH$

$j \cdot \omega \cdot L = 30.159j \cdot \Omega$

$\frac{1}{j \cdot \omega \cdot C} = -44.21j \cdot \Omega$

$$\mathbf{Z_{eq}} := R + \frac{1}{j \cdot \omega \cdot C} + j \cdot \omega \cdot L$$

$$= 20 \cdot \Omega - 44.21 \cdot j \cdot \Omega + 30.16 \cdot j \cdot \Omega = 20 - 14.05j \cdot \Omega$$

$$\sqrt{(20 \cdot \Omega)^2 + (14.05 \cdot \Omega)^2} = 24.44 \cdot \Omega \qquad \operatorname{atan}\left(\frac{-14.05 \cdot \Omega}{20 \cdot \Omega}\right) = -35.09 \cdot deg \qquad \mathbf{Z_{eq}} = 24.44\Omega \underline{/-35.1}^{o}$$

If: $\mathbf{V} := 120 \cdot V \cdot e^{j \cdot 0 \cdot deg} \qquad \mathbf{I} := \frac{\mathbf{V}}{\mathbf{Z_{eq}}} = \frac{120 \cdot V}{24.44 \cdot \Omega} = 4.91 \cdot A \quad \underline{/} \quad 0 - -35.1 = 35.1 \quad deg$

$4.91 \cdot \cos(35.1 \cdot deg) = 4.017 \qquad 4.91 \cdot \sin(35.1 \cdot deg) = 2.823$

slight roundoff error

$\mathbf{I} = 4.017 + 2.822j \cdot A$

Voltage divider:

$$\mathbf{V}_{Zn} = \mathbf{V}_{total}\cdot\frac{\mathbf{Z}_n}{\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\ldots}$$

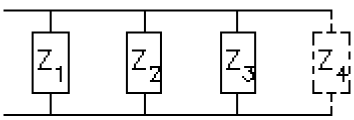
Eg: $\mathbf{V}_C := \mathbf{V}\cdot\frac{\frac{1}{j\cdot\omega\cdot C}}{\mathbf{Z}_{eq}} = 120\cdot V\cdot e^{j\cdot 0\cdot deg}\cdot\frac{44.21\cdot e^{-j\cdot 90\cdot deg}\cdot\Omega}{24.44\cdot e^{-j\cdot 35.1\cdot deg}\cdot\Omega}$

$120\cdot V\cdot\frac{44.21\cdot\Omega}{24.44\cdot\Omega} = 217.07\cdot V$ $\qquad \angle\ 0 + -90 - -35.1 = -54.9\ \text{deg}$

$\mathbf{V}_C = 217.1V\ \underline{/-54.9^o}$ $\qquad \mathbf{V}_C = 124.771 - 177.604j\ \cdot V$

$217.1\cdot\cos(-54.9\cdot deg) = 124.8$ $\qquad 217.1\cdot\sin(-54.9\cdot deg) = -177.6$

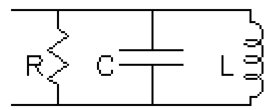
parallel:



$$\mathbf{Z}_{eq} = \frac{1}{\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3}+\ldots}$$

Example:

$f := 60\cdot Hz$ $\qquad \omega := 2\cdot\pi\cdot f$ $\qquad \omega = 377\cdot\frac{rad}{sec}$



$L := 80\cdot mH$

$j\cdot\omega\cdot L = 30.159j\cdot\Omega$

$R := 20\cdot\Omega$ $\qquad C := 60\cdot\mu F$ $\qquad \frac{1}{\omega\cdot L} = 3.316\cdot 10^{-2}\cdot\frac{1}{\Omega}$

$\frac{1}{j\cdot\omega\cdot C} = -44.21j\cdot\Omega$ $\qquad \omega\cdot C = 2.262\cdot 10^{-2}\cdot\frac{1}{\Omega}$

$$\mathbf{Z}_{eq} := \frac{1}{\frac{1}{R}+\frac{1}{\left(\frac{1}{j\cdot\omega\cdot C}\right)}+\frac{1}{j\cdot\omega\cdot L}} = \frac{1}{\frac{1}{R}+j\cdot\omega\cdot C-\frac{j}{\omega\cdot L}} = \frac{1}{\frac{1}{20\cdot\Omega}+2.262\cdot 10^{-2}\cdot j\cdot\frac{1}{\Omega}-3.316\cdot 10^{-2}\cdot j\cdot\frac{1}{\Omega}} = \frac{1}{\left(5\cdot 10^{-2}-1.054\cdot 10^{-2}\cdot j\right)\cdot\frac{1}{\Omega}}$$

$$= \frac{1}{\left(5\cdot 10^{-2}-1.054\cdot 10^{-2}\cdot j\right)\cdot\frac{1}{\Omega}}\cdot\frac{5\cdot 10^{-2}+1.054\cdot 10^{-2}\cdot j}{\left(5\cdot 10^{-2}+1.054\cdot 10^{-2}\cdot j\right)} = 19.149+4.037j\ \cdot\Omega$$

OR, If you want a polar result, it's actually easier to change the denominator to polar and then do polar division.

$\sqrt{\left(5\cdot 10^{-2}\cdot\frac{1}{\Omega}\right)^2+\left(1.054\cdot 10^{-2}\cdot\frac{1}{\Omega}\right)^2} = 5.11\cdot 10^{-2}\cdot\frac{1}{\Omega}$ $\qquad \operatorname{atan}\left(\frac{-1.054\cdot 10^{-2}\cdot\Omega}{5\cdot 10^{-2}\cdot\Omega}\right) = -11.9\cdot deg$

$\frac{1}{5.11\cdot 10^{-2}\cdot\frac{1}{\Omega}} = 19.569\cdot\Omega$ $\qquad \angle\ 0 - -11.9 = 11.9\ \text{deg}$ $\qquad \mathbf{Z}_{eq} = 19.57\Omega\ \underline{/11.9^o}$

If: $\mathbf{V} := 120\cdot V\cdot e^{j\cdot 0\cdot deg}$ $\qquad \mathbf{I} := \frac{\mathbf{V}}{\mathbf{Z}_{eq}} = \frac{120\cdot V}{19.57\cdot\Omega} = 6.132\cdot A$ $\qquad \angle\ 0 - 11.9 = -11.9\ \text{deg}$

$6.132\cdot\cos(-11.9\cdot deg) = 6$ $\qquad 6.132\cdot\sin(-11.9\cdot deg) = -1.264$ $\qquad \mathbf{I} = 6 - 1.265j\ \cdot A$

slight roundoff error

Current divider:

$$\mathbf{I}_{Zn} = \mathbf{I}_{total}\cdot\frac{\frac{1}{\mathbf{Z}_n}}{\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3}+\ldots}$$

Eg: $\mathbf{I}_L := \mathbf{I}\cdot\frac{\frac{1}{j\cdot\omega\cdot L}}{\frac{1}{R}+j\cdot\omega\cdot C+\frac{1}{j\cdot\omega\cdot L}} = \mathbf{I}\cdot\frac{\left(\frac{1}{j\cdot\omega\cdot L}\right)}{\left(\frac{1}{\mathbf{Z}_{eq}}\right)} = \mathbf{I}\cdot\frac{\mathbf{Z}_{eq}}{j\cdot\omega\cdot L}$

$= 6.132\cdot A\cdot e^{j\cdot -11.9\cdot deg}\cdot\frac{19.57\cdot e^{j\cdot 11.9\cdot deg}\cdot\Omega}{30.159\cdot e^{j\cdot 90\cdot deg}\cdot\Omega}$

$\mathbf{I}_L = 6.132\cdot A\cdot\frac{19.57\cdot\Omega}{30.159\cdot\Omega} = 3.979\cdot A$ $\qquad \angle -11.9 + 11.9 - 90 = -90\ \text{deg}$ $\qquad \mathbf{I}_L = -3.979j\cdot A$

$= \frac{\mathbf{V}}{j\cdot\omega\cdot L} = -3.979j\cdot A$ $\qquad$ Duh...