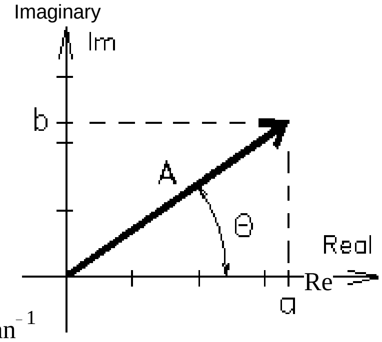
Complex Numbers



$j = \sqrt{-1}$ the imaginary number

Rectangular Form $\mathbf{A} = a + b\cdot j$

$\mathrm{Re}(\mathbf{A}) = a \qquad \mathrm{Im}(\mathbf{A}) = b$

Polar Form $\mathbf{A} = A\cdot e^{j\cdot\theta} = A\ \underline{/\ \theta}$

$\mathrm{Re}(\mathbf{A}) = A\cdot\cos(\theta) \qquad \mathrm{Im}(\mathbf{A}) = A\cdot\sin(\theta)$

Conversions $A = |\mathbf{A}| = \sqrt{a^2 + b^2} \qquad \theta = \arg(\mathbf{A}) = \mathrm{atan}\left(\frac{b}{a}\right)$ Note: $\mathrm{atan} = \tan^{-1}$

$a = A\cdot\cos(\theta) \qquad b = A\cdot\sin(\theta)$

$\mathbf{A} = A\cdot e^{j\cdot\theta} = A\ \underline{/\ \theta} = A\cdot\cos(\theta) + A\cdot\sin(\theta)\cdot j \qquad \mathbf{A} = a + b\cdot j = \left(\sqrt{a^2+b^2}\right)\cdot e^{j\cdot\mathrm{atan}\left(\frac{b}{a}\right)}$

Special Cases $j := \sqrt{-1} = e^{j\cdot 90\cdot deg} \qquad \frac{1}{j} = -j = e^{-j\cdot 90\cdot deg} \qquad e^{j\cdot 0\cdot deg} = 1 \qquad e^{j\cdot 180\cdot deg} = e^{-j\cdot 180\cdot deg} = -1$

$j\cdot e^{j\cdot\theta} = e^{j\cdot(\theta + 90\cdot deg)}$

Define a 2nd number: rect: $\mathbf{D} = c + d\cdot j$ polar: $\mathbf{D} = D\cdot e^{j\cdot\phi} = D\ \underline{/\ \theta}$

Equality $\mathbf{A} = \mathbf{D}$ if and only if $a = c$ and $b = d$ OR $A = D$ and $\theta = \phi$

Addition and Subtraction

$\mathbf{A} + \mathbf{D} = (a + b\cdot j) + (c + d\cdot j) = (a + c) + (b + d)\cdot j$

$\mathbf{A} - \mathbf{D} = (a + b\cdot j) - (c + d\cdot j) = (a - c) + (b - d)\cdot j$

Convert polars to rectangular form first

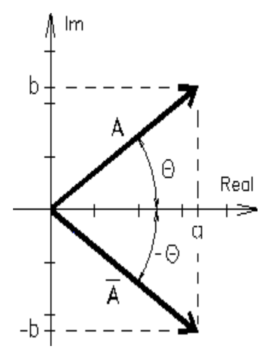
Multiplication and Division $\mathbf{A}\cdot\mathbf{D} = (a + b\cdot j)\cdot(c + d\cdot j) = (a\cdot c - b\cdot d) + (b\cdot c + a\cdot d)\cdot j$

Rectangular: $\frac{\mathbf{A}}{\mathbf{D}} = \frac{a + b\cdot j}{c + d\cdot j} = \frac{a + b\cdot j}{c + d\cdot j}\cdot\frac{c - d\cdot j}{c - d\cdot j} = \frac{a\cdot c + b\cdot d}{c^2 + d^2} + \frac{b\cdot c - a\cdot d}{c^2 + d^2}\cdot j$

Polar: $\mathbf{A}\cdot\mathbf{D} = A\cdot e^{j\cdot\theta}\cdot D\cdot e^{j\cdot\phi} = A\cdot D\cdot e^{j\cdot(\theta + \phi)} = AD\ \underline{/\ \theta + \phi}$

$\frac{\mathbf{A}}{\mathbf{D}} = \frac{A\cdot e^{j\cdot\theta}}{D\cdot e^{j\cdot\phi}} = \frac{A}{D}\cdot e^{j\cdot(\theta - \phi)} = \frac{A}{D}\ \underline{/\ \theta - \phi} = A/D\ \underline{/\ \theta - \phi}$

Powers $\mathbf{A}^n = A^n\cdot e^{j\cdot n\cdot\theta} = A^n\cdot\cos(n\cdot\theta) + A^n\cdot\sin(n\cdot\theta)\cdot j$ Convert rectangulars first, usually



Conjugates

complex number	Conjugate	
$\mathbf{A} = a + b\cdot j$	$\overline{\mathbf{A}} = a - b\cdot j$	$\overline{\overline{\mathbf{A}}} = \mathbf{A}$
$\mathbf{A} = A\cdot e^{j\cdot\theta}$	$\overline{\mathbf{A}} = A\cdot e^{-j\cdot\theta} = A\ \underline{/\ -\theta}$	
$\mathbf{F} = \frac{3 + 5\cdot j}{(2 - 6\cdot j)\cdot e^{j\cdot 40\cdot deg}}$	$\overline{\mathbf{F}} = \frac{3 - 5\cdot j}{(2 + 6\cdot j)\cdot e^{-j\cdot 40\cdot deg}}$	

Euler's equation $e^{j\cdot\alpha} = \cos(\alpha) + j\cdot\sin(\alpha)$ OR: $\cos(\alpha) = \frac{e^{j\cdot\alpha} + e^{-j\cdot\alpha}}{2} \qquad \sin(\alpha) = \frac{e^{j\cdot\alpha} - e^{-j\cdot\alpha}}{2\cdot j}$

$e^{j\cdot(\omega\cdot t + \theta)} = \cos(\omega\cdot t + \theta) + j\cdot\sin(\omega\cdot t + \theta)$

$\mathrm{Re}\left[e^{j\cdot(\omega\cdot t + \theta)}\right] = \cos(\omega\cdot t + \theta)$

If we freeze this at time t=0, then we can represent $\cos(\omega\cdot t + \theta)$ by $e^{j\cdot\theta}$

Calculus Remember, when we write $e^{j\cdot\theta}$, we really mean $e^{j\cdot(\omega\cdot t + \theta)}$

$$\frac{d}{dt}\mathbf{A} = \frac{d}{dt}\left(A\cdot e^{j\cdot\theta}\right) = j\cdot\omega\cdot A\cdot e^{j\cdot\theta} = \omega\cdot A\cdot e^{j\cdot(\theta + 90\cdot deg)}$$

$$\int \mathbf{A}\, dt = \int A\cdot e^{j\cdot\theta}\, dt = \frac{1}{j\cdot\omega}\cdot A\cdot e^{j\cdot\theta} = \frac{1}{\omega}\cdot A\cdot e^{j\cdot(\theta - 90\cdot deg)}$$

Complex Numbers Notes