ECE 3600 Lumped-Parameter Transmission Line Models b

Long-length Lines: over 240 km (150 miles) (over 200 mi in some texts)

Need: Units

line length: len , d — m or km — stick to the same unit length for all parameters; miles may also be used

Resistance per unit length: r — $\frac{\Omega}{m}$ or $\frac{\Omega}{km}$

Need		Units				Units
Inductance per unit length:	l	$\frac{H}{m}$ or $\frac{H}{km}$	OR	Inductive reactance per unit length:	x	$\frac{\Omega}{m}$ or $\frac{\Omega}{km}$
Capacitance per unit length:	c	$\frac{F}{m}$ or $\frac{F}{km}$	OR	Capacitance admittance per unit length:	y	$\frac{S}{m}$ or $\frac{S}{km}$
Conductance to ground:	g	$\frac{S}{m}$ or $\frac{S}{km}$		Common assumption: $g := 0\cdot\frac{S}{km}$		$S :=$ siemens

Find: Units

Surge impedance: $\mathbf{Z}_c = \sqrt{\dfrac{j\cdot x + r}{j\cdot y + g}}$ — Ω

Propagation constant: $\gamma = \sqrt{(j\cdot x + r)\cdot(j\cdot y + g)}$ — $\frac{1}{m}$ or $\frac{1}{km}$

If your calculator doesn't have hyperbolic trig functions

Series impedance $\mathbf{Z}_{series} = \mathbf{Z}_c\cdot\sinh(\gamma\cdot len) = \mathbf{Z}_c\cdot\dfrac{e^{\gamma\cdot len} - e^{-\gamma\cdot len}}{2}$ — Ω

Shunt admittance: $\dfrac{\mathbf{Y}_{shunt}}{2} = \dfrac{1}{\mathbf{Z}_c}\cdot\tanh\left(\gamma\cdot\dfrac{len}{2}\right) = \dfrac{1}{\mathbf{Z}_c}\cdot\dfrac{e^{\gamma\cdot\frac{len}{2}} - e^{-\gamma\cdot\frac{len}{2}}}{e^{\gamma\cdot\frac{len}{2}} + e^{-\gamma\cdot\frac{len}{2}}} = \dfrac{1}{\mathbf{Z}_c}\cdot\dfrac{\sqrt{e^{\gamma\cdot len}} - \sqrt{e^{-(\gamma\cdot len)}}}{\sqrt{e^{\gamma\cdot len}} + \sqrt{e^{-(\gamma\cdot len)}}}$ — Ω

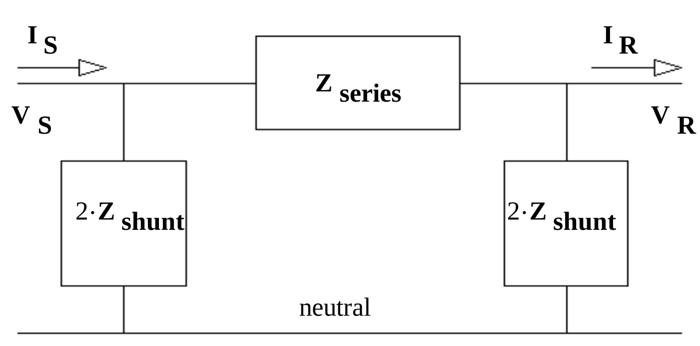
OR

Shunt impedance: $2\cdot\mathbf{Z}_{shunt} = \dfrac{\mathbf{Z}_c}{\tanh\left(\gamma\cdot\dfrac{len}{2}\right)}$ — S or $\frac{1}{\Omega}$

If your calculator can't handle complex exponents

$e^{(a+b\cdot j)} = e^{a}\cdot e^{b\cdot j} = e^{a}$ /b (in radians)

Model:



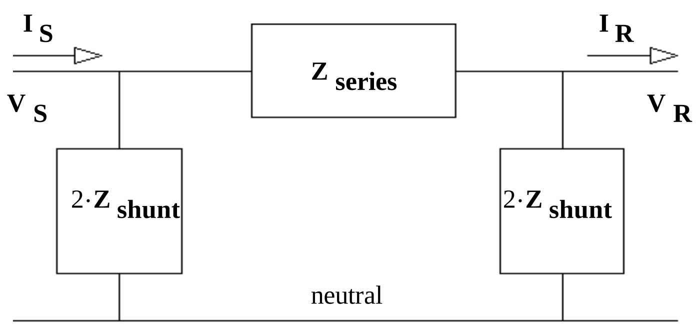
Medium-length Lines: 80 - 240 km (50 to 150 miles) (100 - 200 mi in some texts)

Need:

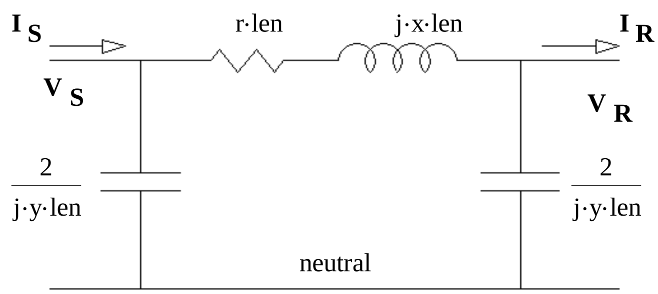
		Units				Units
line length:	len , d	m or km		stick to the same unit length for all parameters miles may also be used		
Resistance per unit length:	r	$\frac{\Omega}{m}$ or $\frac{\Omega}{km}$				
Inductance per unit length:	l	$\frac{H}{m}$ or $\frac{H}{km}$	OR	Inductive reactance per unit length:	x	$\frac{\Omega}{m}$ or $\frac{\Omega}{km}$
Capacitance per unit length:	c	$\frac{F}{m}$ or $\frac{F}{km}$	OR	Capacitance admittance per unit length:	y	$\frac{S}{m}$ or $\frac{S}{km}$
Conductance to ground:	g	$\frac{S}{m}$ or $\frac{S}{km}$		Common assumption: $g := 0 \cdot \frac{S}{km}$		

Find:

			Units
Surge Impedance:	$\mathbf{Z}_{\mathbf{c}} = \sqrt{\frac{x}{y}}$	Only needed if load is in terms of SIL	Ω
Series Resistance:	$R_{line} = r \cdot len$		Ω
Series impedance	$\mathbf{Z}_{\mathbf{series}} = (r + j \cdot x) \cdot len$		Ω
Shunt admittance:	$\frac{\mathbf{Y}_{\mathbf{shunt}}}{2} = j \cdot y \cdot \frac{len}{2}$		S or $\frac{1}{\Omega}$
OR			
Shunt impedance:	$2 \cdot \mathbf{Z}_{\mathbf{shunt}} = \frac{2}{j \cdot y \cdot len}$		Ω



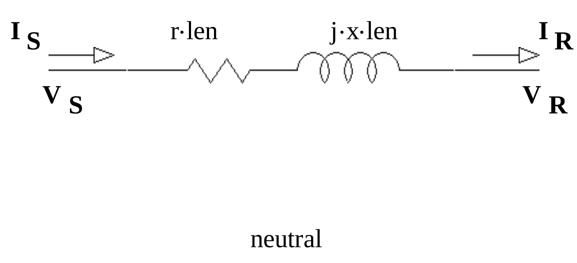
OR:



Short-length Lines: less than 80km (50 mi)

(less than 100 mi in some texts)

Same as above but without the capacitors



ECE 3600 Transmission Line Examples

b

Ex1. A 500 kV transmission line is 500 km long and has the line parameters shown below. Use the long-length model to find $\mathbf{V_S}$ and $\mathbf{I_S}$ if the line is loaded to 900 MVA and $|\mathbf{V_{RLL}}|$ is 490 kV. Assume the phase angle of $\mathbf{V_R}$ is 0° and assume load pf = 1.

$len := 500 \cdot km$ $\quad V_{RLL} := 490 \cdot kV$ $\quad \mathbf{V_R} := \frac{V_{RLL}}{\sqrt{3}}$ $\quad S_{1\phi} := \frac{900 \cdot MVA}{3}$

$r := 0.029 \cdot \frac{\Omega}{km}$ $\quad$ Assume: $g := 0 \cdot \frac{S}{km}$

$x := 0.326 \cdot \frac{\Omega}{km}$ $\quad y := 5.220 \cdot 10^{-6} \cdot \frac{S}{km}$

Note: These are typical values for a 500 kV transmission line

Long-length line model:

Surge Impedance: $\mathbf{Z_c} := \sqrt{\frac{j \cdot x + r}{j \cdot y + g}}$ $\qquad \mathbf{Z_c} = 250.151 - 11.104j \cdot \Omega$

Propagation constant: $\gamma := \sqrt{(j \cdot x + r) \cdot (j \cdot y + g)}$ $\qquad \gamma = 5.797 \cdot 10^{-5} + 1.306 \cdot 10^{-3} j \cdot \frac{1}{km}$

Series impedance: $\mathbf{Z_{series}} := \mathbf{Z_c} \cdot \sinh(\gamma \cdot len)$ $\qquad \mathbf{Z_{series}} = 12.508 + 151.772j \cdot \Omega$

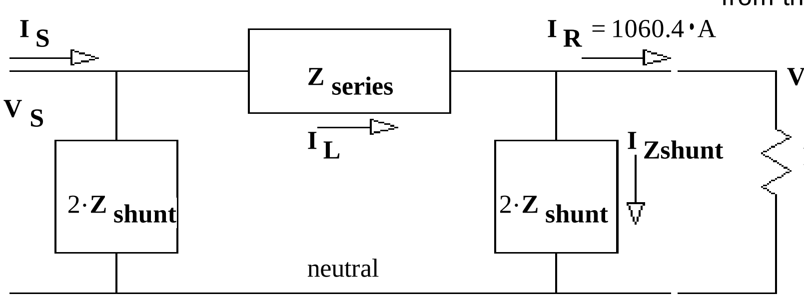
Shunt admittance: (Not used in my solution) $\mathbf{Y_{shunt}} := \frac{2}{\mathbf{Z_c}} \cdot \tanh\left(\gamma \cdot \frac{len}{2}\right)$ $\qquad \frac{\mathbf{Y_{shunt}}}{2} = 4.49 \cdot 10^{-6} + 1.353 \cdot 10^{-3} j \cdot S$

Shunt impedance: $\mathbf{Z_{shunt}} := \frac{\mathbf{Z_c}}{2 \cdot \tanh\left(\gamma \cdot \frac{len}{2}\right)}$ $\qquad 2 \cdot \mathbf{Z_{shunt}} = 2.451 - 738.924j \cdot \Omega$

Solve circuit:

$\mathbf{I_R} := \frac{S_{1\phi}}{|\mathbf{V_R}|}$ (Not complex in this case because pf = 1 otherwise include a phase angle calculated from the pf or load other information)

$\mathbf{I_R} = 1060.4 \cdot A$



$\mathbf{V_R} = 282.902 \cdot kV$

$\mathbf{I_{Zshunt}} := \frac{\mathbf{V_R}}{2 \cdot \mathbf{Z_{shunt}}}$

$\mathbf{I_{Zshunt}} = 1.27 + 382.852j \cdot A$

$\mathbf{I_L} := \mathbf{I_{Zshunt}} + \mathbf{I_R}$ $\qquad \mathbf{I_L} = 1.062 \cdot 10^3 + 382.852j \cdot A$

$\mathbf{V_S} := \mathbf{V_R} + \mathbf{I_L} \cdot \mathbf{Z_{series}}$ $\qquad \mathbf{V_S} = 2.381 \cdot 10^5 + 1.659 \cdot 10^5 j \cdot V$ $\qquad |\mathbf{V_S}| = 290.192 \cdot kV$ $\quad \arg(\mathbf{V_S}) = 34.874 \cdot deg$

$\mathbf{I_{ZshuntS}} := \frac{\mathbf{V_S}}{2 \cdot \mathbf{Z_{shunt}}}$ $\qquad \mathbf{I_{ZshuntS}} = -223.48 + 322.934j \cdot A$ $\qquad |\sqrt{3} \cdot \mathbf{V_S}| = 502.628 \cdot kV$

$\mathbf{I_S} := \mathbf{I_{ZshuntS}} + \mathbf{I_L}$ $\qquad \mathbf{I_S} = 838.23 + 705.786j \cdot A$ $\qquad |\mathbf{I_S}| = 1096 \cdot A$ $\quad \arg(\mathbf{I_S}) = 40.097 \cdot deg$

Ex 2. A 345 kV transmission line is 220 km long and has the line parameters shown below.
Find $\mathbf{V_S}$ and $\mathbf{I_S}$ if the line is loaded to 0.9 SIL with pf = 95% lagging. $|\mathbf{V_{RLL}}|$ is 510 kV. $pf := 0.95$

$len := 220 \cdot km$ $\quad V_{RLL} := 510 \cdot kV$ $\quad \mathbf{V_R} := \frac{V_{RLL}}{\sqrt{3}}$

Assume the phase angle of $\mathbf{V_R}$ is 0^o if $\mathbf{V_R}$ is given

$r := 0.037 \cdot \frac{\Omega}{km}$ Assume: $g := 0 \cdot \frac{S}{km}$

Note: These are typical values for a 345 kV transmission line

$x := 0.376 \cdot \frac{\Omega}{km}$ $\quad y := 4.518 \cdot 10^{-6} \cdot \frac{S}{km}$

Medium-length line model:

Surge Impedance: $\mathbf{Z_c} := \sqrt{\frac{x}{y}}$ $\qquad \mathbf{Z_c} = 288.5 \cdot \Omega$

Series impedance: $\mathbf{Z_{series}} := (r + j \cdot x) \cdot len$ $\qquad \mathbf{Z_{series}} = 8.14 + 82.72j \cdot \Omega$

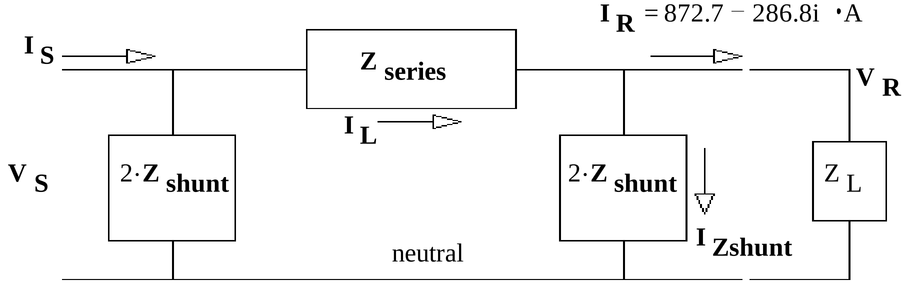
Shunt admittance: $\mathbf{Y_{shunt}} := j \cdot y \cdot len$ $\qquad \frac{\mathbf{Y_{shunt}}}{2} = 496.98j \cdot \mu S$
Not used in my solution

Shunt impedance: $\mathbf{Z_{shunt}} := \frac{1}{j \cdot y \cdot len}$ $\qquad 2 \cdot \mathbf{Z_{shunt}} = -2.012 \cdot 10^3 j \cdot \Omega$

Figure out 1 SIL: $SIL := 3 \cdot \frac{(|\mathbf{V_R}|)^2}{|\mathbf{Z_c}|}$ $\qquad SIL = 902 \cdot MVA$ $\qquad$ Actual: $S_{1\phi} := \frac{0.9 \cdot SIL}{3}$ (0.9 SIL loading)

Solve circuit: $\mathbf{I_R} := \frac{S_{1\phi}}{|\mathbf{V_R}|} \cdot e^{-j \cdot acos(pf)}$ (Negative phase angle because the pf is lagging)

$\mathbf{I_R} = 872.7 - 286.8i \cdot A$



$\mathbf{V_R} = 294.449 \cdot kV$

$\mathbf{I_{Zshunt}} := \frac{\mathbf{V_R}}{2 \cdot \mathbf{Z_{shunt}}}$ $\qquad \mathbf{I_{Zshunt}} = 146.335j \cdot A$

$\mathbf{I_L} := \mathbf{I_{Zshunt}} + \mathbf{I_R}$ $\qquad \mathbf{I_L} = 872.68 - 140.501j \cdot A$

$\mathbf{V_S} := \mathbf{V_R} + \mathbf{I_L} \cdot \mathbf{Z_{series}}$ $\qquad \mathbf{V_S} = 3.132 \cdot 10^5 + 7.104 \cdot 10^4 j \cdot V$ $\qquad |\mathbf{V_S}| = 321.132 \cdot kV$ $\qquad arg(\mathbf{V_S}) = 12.781 \cdot deg$

Line voltage: $|\sqrt{3} \cdot \mathbf{V_S}| = 556.216 \cdot kV$

$\mathbf{I_{ZshuntS}} := \frac{\mathbf{V_S}}{2 \cdot \mathbf{Z_{shunt}}}$ $\qquad \mathbf{I_{ZshuntS}} = -35.308 + 155.641j \cdot A$

$\mathbf{I_S} := \mathbf{I_{ZshuntS}} + \mathbf{I_L}$ $\qquad \mathbf{I_S} = 837.372 + 15.141j \cdot A$ $\qquad |\mathbf{I_S}| = 838 \cdot A$ $\qquad arg(\mathbf{I_S}) = 1.036 \cdot deg$

Ex3. A 230 kV transmission line has the following length and line parameters.

$len := 150 \cdot km$ $\quad r := 0.06 \cdot \frac{\Omega}{km}$ $\quad x := 0.5 \cdot \frac{\Omega}{km}$ $\quad g := 0 \cdot \frac{S}{km}$ $\quad y := 4 \cdot 10^{-6} \cdot \frac{S}{km}$

Medium-length line model:

Series impedance: $\mathbf{Z}_{series} := (r + j \cdot x) \cdot len$ $\quad \mathbf{Z}_{series} = 9 + 75j \cdot \Omega$

Shunt admittance: $\mathbf{Y}_{shunt} := j \cdot y \cdot len$ $\quad \frac{\mathbf{Y}_{shunt}}{2} = 0.3j \cdot mS$

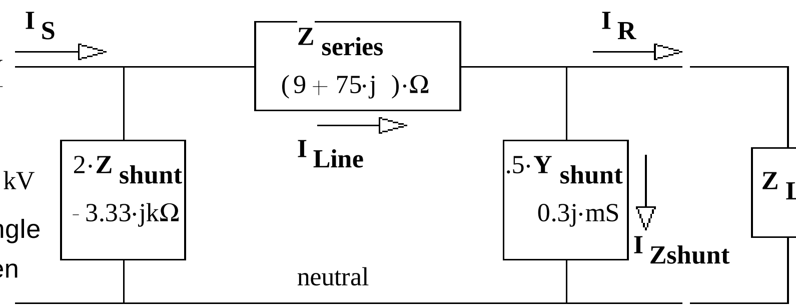
Shunt impedance: $\mathbf{Z}_{shunt} := \frac{1}{j \cdot y \cdot len}$ $\quad 2 \cdot \mathbf{Z}_{shunt} = -3.333j \cdot k\Omega$

a) The load is 250Ω with a power factor of 0.87, leading. Find the line current, $\mathbf{I}_{Line}$.

$\mathbf{V}_S := \frac{230 \cdot kV}{\sqrt{3}}$

$\mathbf{V}_S = 132.79 \cdot kV$

Assume the phase angle of $\mathbf{V}_S$ is 0° if $\mathbf{V}_S$ is given



$Z_L := 250 \cdot \Omega$

$pf := 0.87$

$\mathbf{Z}_L := Z_L \cdot e^{-j \cdot acos(pf)}$

$\mathbf{Z}_L = 217.5 - 123.263j \cdot \Omega$

$= 250\Omega \angle 29.54°$

$$\mathbf{Z} := \mathbf{Z}_{series} + \frac{1}{\frac{\mathbf{Y}_{shunt}}{2} + \frac{1}{\mathbf{Z}_L}}$$

$\mathbf{Z} = 210.467 - 56.544j \cdot \Omega \quad = \quad 219.7\Omega \angle -15.04°$

$\mathbf{I}_{Line} := \frac{\mathbf{V}_S}{\mathbf{Z}}$ $\quad \mathbf{I}_{Line} = 588.459 + 158.096j \cdot A \quad = \quad 609.3A \angle 15.04°$

b) Find the load line voltage. $\quad \mathbf{I}_{Line} \cdot \mathbf{Z}_{series} = -6.561 + 45.557j \cdot kV$

$\mathbf{V}_R := \mathbf{V}_S - \mathbf{I}_{Line} \cdot \mathbf{Z}_{series}$ $\quad \mathbf{V}_R = 139.352 - 45.557j \cdot kV \quad = \quad 146.6kV \angle -18.1°$

Receiving line voltage = $\left| \sqrt{3} \cdot \mathbf{V}_R \right| = 253.9 \cdot kV$

Notice that $|\mathbf{V}_R|$ is bigger than $|\mathbf{V}_S|$, this can happen when the receiving-end power factor is leading.

c) What is the "power angle" (δ)? $\quad \delta \;=\; -arg(\mathbf{V}_R) = 18.104 \cdot deg$

d) How much power is delivered to the load?

$I_R := \frac{|\mathbf{V}_R|}{|\mathbf{Z}_L|}$ $\quad P_L \;=\; 3 \cdot |\mathbf{V}_R| \cdot I_R \cdot pf = 224.4 \cdot MW$

Power estimate for the same $|\mathbf{V}_R|$ and $|\mathbf{V}_S|$, but neglecting the line resistance: $\quad \simeq 3 \cdot \frac{|\mathbf{V}_S| \cdot |\mathbf{V}_R| \cdot \sin(18.1 \cdot deg)}{|\mathbf{Z}_{series}|} = 240 \cdot MW$

e) Express this loading in terms of SIL

Surge Impedance: $\mathbf{Z}_c := \sqrt{\frac{x}{y}}$ $\quad \mathbf{Z}_c = 353.6 \cdot \Omega$ $\quad \frac{\mathbf{Z}_c}{Z_L} = 1.414$ SIL load