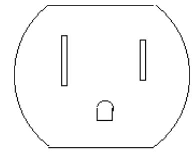
ECE 3600   Transmission Line notes

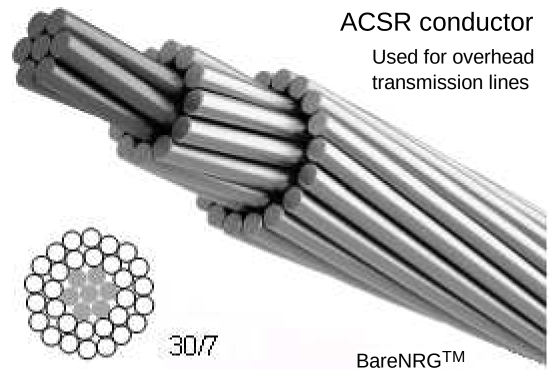


The beauty of electric power is that we have a ready source of zero-entropy energy available at any outlet. That energy can be made to do all kinds of things for us-- everything from washing our clothes to entertaining our children. But even as useful as electric power is, most of us don't want a power plant in our neighborhood. Power plants are best located close to energy sources and far from population centers. And that's the other great beauty of electric power (at least the AC version), it can be generated far from where it is used, transformed to very high voltages and moved efficiently over high-voltage transmission lines.

Watch the in-class slideshow of transmission line pictures

Pay attention to:

- Tower designs and sizes, and special designs at corners
- Multiple sets of 3-phase lines on a single set of towers
- Multiple sets of towers in the same corridor
- The number of insulator discs, which increase with voltage
- The wide variety of configurations
- Shield wire(s)
- Bundling & spacers
- Capacitor banks



Shield Wire

The very highest wire is nearly always a **shield wire**, a grounded wire placed above the rest for lightning protection. May be simple steel cable or aluminum with steel reinforcement, often with a fiber optic data line at the center

BTW, Don't try this at home

Common Voltages

$7.2\cdot kV\cdot\sqrt{3} = 12.47\cdot kV$ Local distribution, reduced to 240/120V at a transformer near you.

$46\cdot kV$ $69\cdot kV$ Distribution within a city or county, between substations

$115\cdot kV$ $138\cdot kV$ Short, light-use, rural, or older transmission lines or newer distribution lines

$161\cdot kV$ $230\cdot kV$ Common transmission lines

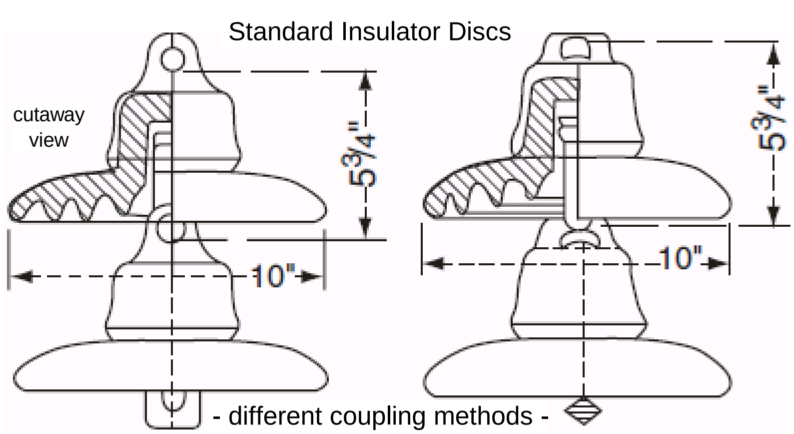
$345\cdot kV$ $500\cdot kV$ $765\cdot kV$ Long-distance lines

Power handling capabilities increase roughly proportional to the square of the voltage, and decrease with line length, see curve later in the notes.

	50 mile	300 mile
$230\cdot kV$	$420\cdot MW$	$140\cdot MW$
$345\cdot kV$	$1230\cdot MW$	$410\cdot MW$
$500\cdot kV$	$3000\cdot MW$	$1000\cdot MW$
$765\cdot kV$	$6800\cdot MW$	$2300\cdot MW$

Insulators

These standard-sized discs are made from porcelain or glass and coupled together to form strings. They come in different tensile force ratings (15 to 50,000 lbs) and can handle over $20kV$ each. They also come in special styles for fog or contamination.



cap & pin   ball & socket

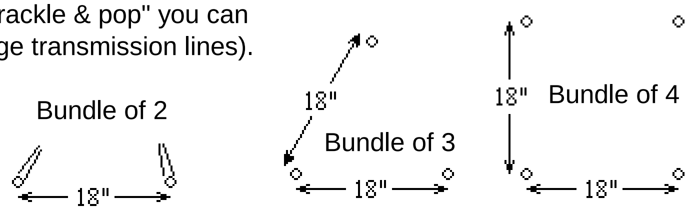
Number of standard insulator discs used in a string.

$46\cdot kV$	3-4
$69\cdot kV$	4-6
$115\cdot kV$	7-9
$138\cdot kV$	7-10
$161\cdot kV$	10-13
$230\cdot kV$	11-17
$345\cdot kV$	16-21
$500\cdot kV$	24-27
$765\cdot kV$	30-35

The conductors themselves are not insulated. Electrical insulation would also be **thermal insulation**, and that would **not** be **good**. Because of the high currents these lines carry, they heat up. Hanging out in the air helps keep them from overheating. Overhead lines are electrically insulated from ground and one another only by air and distance.

Bundling & $ Costs

As you saw in the pictures of transmission lines, it is not uncommon to use multiple conductors per phase. This is called bundling. Some 230kV lines and all lines above 230kV use bundling to reduce electric field strength and corona discharge (the source of the "crackle & pop" you can hear near high-voltage transmission lines).



	Bundling Conductors / phase	Million$ /mi
< 230·kV	1	
230·kV	1-2	1 - 2
345·kV	2-3	2 - 3
500·kV	3-4	2 - 3.2
765·kV	4	2.5 - 4

Bundling reduces the electric field around the lines. Multiple small-radius lines look like a single line of much greater radius, consequentially:

Corona discharge ↓ line inductance ↓ line capacitance ↑ , all good things

Line Parameters

R = resistance = $r \cdot len$ upper case for the whole line, lower case for resistance per unit length, len for length.

L = inductance = $l \cdot len$ X = reactance = $x \cdot len = \omega \cdot l \cdot len$

C = capacitance = $c \cdot len$ Y = admittance = $y \cdot len = j \cdot \omega \cdot c \cdot len$

G = conductance to ground = $g \cdot len$ caused primarily by corona discharge, usually neglected.

Resistance, R or *r*

$$R = \frac{\rho \cdot len}{A}$$

len = length of line
A = cross-sectional area
ρ = resistivity
increases with temperature

$$\rho_{T2} = \frac{M + T_2}{M + T_1} \cdot \rho_{T1} = \frac{M + T_2}{M + 20} \cdot \rho$$

Material	% Conductivity	ρ at 20 °C Resistivity $10^{-8} \cdot \Omega \cdot m$	M Temperature Constant
Copper			
Annealed	100·%	1.72	234.5 °C
Hard-drawn	97.3·%	1.77	241.5 °C
Aluminum			
Hard-drawn	61·%	2.83	228.1 °C
Silver	108·%	1.59	243 °C
Steel	2 - 14·%	12 - 88	180 - 980 °C

Resistance increases with:
- Temperature + 20% or more
- Frequency ("skin effect") + ~3% for 60 Hz
- Spiraling The aluminum conductors in the cables are longer because of the twisting + 1 to 2%

The large currents handled by transmission lines can cause significant heating of the lines, which causes the resistance to increase, making the problem even worse. Additionally, this heating causes the metal of the lines to expand and **sag** lower toward the ground, which can be a problem.

r = series resistance per unit length of the line = $\frac{\rho}{A} \left(\frac{\Omega}{m}\right)$ OR $\frac{1000 \cdot \rho}{A} \left(\frac{\Omega}{km}\right)$ The units will be important

Inductance, L or *l*

Your textbook goes through 5 pages of work and explanation (p.450 - 455) to get to the following expression of inductance per unit length of a single-phase, two-wire transmission line. Despite that it will still yield some useful information.

$$l = \frac{\mu}{\pi} \cdot \left(\frac{1}{4} + \ln\left(\frac{D}{r}\right)\right) \quad \left(\frac{henry}{m}\right)$$

D = spacing between line (phases)

r = radius of the conductor

series reactance = per unit length $x = \omega \cdot \frac{\mu}{\pi} \cdot \left(\frac{1}{4} + \ln\left(\frac{D}{r}\right)\right) \cdot 1000 \quad \left(\frac{henry}{km}\right)$

$\mu_0 := 4 \cdot \pi \cdot 10^{-7} \cdot \frac{henry}{meter}$ = permeability of a vacuum
air is about the same

The useful information, if D ↑, then l ↑, x ↑ and, if r ↑, then l ↓, x ↓, also true for 3-phase lines
bad good

Line voltage, V ↑, D ↑, x ↑ If the voltage and power handling of a line increase, the **D** must also increase.

This is BAD, but can be effectively countered by bundling.

Capacitance, C or c

Another 5 pages yields: $c = \dfrac{\pi \cdot \varepsilon}{\ln\left(\dfrac{D}{r}\right)} \cdot 10^{6} \left(\dfrac{\mu F}{m}\right)$ $\varepsilon_0 := 8.554 \cdot 10^{-12} \cdot \dfrac{farad}{meter}$ = permittivity of a vacuum = $\varepsilon_0 = 8.554 \cdot 10^{-3} \cdot \dfrac{\mu F}{km}$

capacitive admittance to ground = $y = j \cdot \omega \cdot \dfrac{\pi \cdot \varepsilon}{\ln\left(\dfrac{D}{r}\right)} \cdot 10^{9} \left(\dfrac{\mu S}{km}\right)$ (sometimes we get sloppy about the j)

if $D \uparrow$, then $c \downarrow$, $y \downarrow$ and, if $r \uparrow$, then $c \uparrow$, $y \uparrow$, usually good

usually not good See more in books by Weeks & Glover/Sarma

Conductance to Ground or Other Phases, G or c

$G = g \cdot len$ caused by corona discharge and leakage across insulators, usually neglected.

Bundling



Bundling makes the multiple small-radius lines look like a single line of much greater radius, so:

Corona discharge $\downarrow$, $g \downarrow$, good

Line inductance, $l \downarrow$, $x \downarrow$, good

Capacitance, $c \uparrow$, $y \uparrow$, usually good

Line resistance would depend on the conductors used, but will usually decrease. $r \updownarrow$?

Underground Cables

Common for distribution in residential areas and downtown urban areas.
Very problematic for high voltages and long distances.

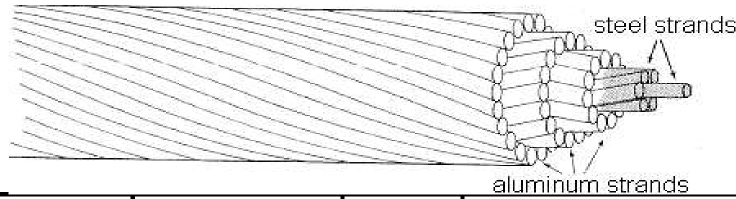
High Capacitance
By definition, these cables are always in close proximity to ground potential, plus they are usually made with a grounded outer conductive shield. This makes them big capacitors. While a bit of added capacitance in a neighborhood distribution system may be OK or even good, the amount you get in transmission systems is BAD. Using **H**igh-**V**oltage **DC** (HVDC) for underground and underwater transmission is a way to get around the problem of capacitive admittance, but has its own issues.

Heat problems
The thick electrical insulation also keeps the heat in, which may require forced liquid cooling systems and limits power carrying capability.

Very Expensive for transmission lines, esp. considering the reduced power rating.


Underground lines is a field unto itself and beyond the scope of this class.

Overhead Transmission Line Conductors



Overhead Transmission lines are usually **A**luminum **C**onductor **S**teel **R**einforced (ACSR) cables. This one is 54/7 "Cardinal".

Numbers below are not much use, b/c Based on a 1-foot spacing

ACSR	**Aluminum area**		Cable	**Resistance**				**Capacitve**	**Inductive Reactance**			**Ampacity**
Conductor Codeword	AWG or kcmil	mm^2	Strands AL/Steel	DC 20°C (Ω/km)	AC 25°C (Ω/km)	AC 50°C (Ω/km)	AC 75°C (Ω/km)	admittance 60Hz (µS/km)	25°C 60Hz (Ω/km)	50°C 60Hz (Ω/km)	75°C 60Hz (Ω/km)	(A)
Turkey	6	13.3	6/1	2.106	2.149	2.461	2.677	4.37	0.394	0.456	0.472	105
Swan	4	21.18	6/1	1.322	1.352	1.572	1.713	4.59	0.377	0.430	0.449	140
Swanate	4	21.12	7/1	1.309	1.335	1.519	1.693	4.62	0.371	0.407	0.427	140
Sparrow	2	33.59	6/1	0.830	0.850	1.010	1.102	4.84	0.361	0.404	0.420	185
Sparate	2	33.54	7/1	0.823	0.840	0.974	1.083	4.87	0.358	0.387	0.397	185
Robin	1	42.41	6/1	0.659	0.676	0.810	0.886	4.97	0.351	0.390	0.400	210
Raven	1/0	53.52	6/1	0.522	0.535	0.646	0.709	5.11	0.341	0.374	0.381	240
Quail	2/0	67.33	6/1	0.413	0.427	0.531	0.577	5.26	0.335	0.367	0.371	275
Pigeon	3/0	85.12	6/1	0.328	0.338	0.397	0.476	5.41	0.325	0.354	0.358	315
Penguin	4/0	107.2	6/1	0.261	0.270	0.351	0.381	5.50	0.316	0.344	0.344	365
Waxwing	266.8	135	18/1	0.211	0.216	0.237	0.259	5.70	0.296	0.296	0.296	445
Partridge	266.8	134.9	26/7	0.209	0.214	0.234	0.255	5.81	0.289	0.289	0.289	455
Merlin	336.4	170.2	18/1	0.167	0.172	0.188	0.205	5.86	0.271	0.271	0.271	515
Linnet	336.4	170.6	26/7	0.166	0.170	0.186	0.203	5.98	0.280	0.280	0.280	530
Oriole	336.4	170.5	30/7	0.165	0.168	0.185	0.201	6.03	0.277	0.277	0.277	530
Chickadee	397.5	200.9	18/1	0.142	0.145	0.160	0.173	6.03	0.281	0.281	0.281	575
Ibis	397.5	201.3	26/7	0.140	0.144	0.158	0.172	6.09	0.274	0.274	0.274	590
Pelican	477	242.3	18/1	0.118	0.121	0.133	0.145	6.21	0.274	0.274	0.274	640
Flicker	477	241.6	24/7	0.117	0.120	0.132	0.144	6.26	0.268	0.268	0.268	670
Hawk	477	241.6	26/7	0.117	0.120	0.132	0.144	6.29	0.267	0.267	0.267	660
Hen	477	241.3	30/7	0.116	0.119	0.131	0.142	6.35	0.263	0.263	0.263	660
Osprey	556.5	282.5	18/1	0.101	0.104	0.114	0.124	6.33	0.268	0.268	0.268	710
Parakeet	556.5	282.3	24/7	0.101	0.103	0.114	0.124	6.41	0.263	0.263	0.263	720
Dove	556.5	282.6	26/7	0.100	0.103	0.113	0.123	6.43	0.261	0.261	0.261	730
Rook	636	323.1	24/7	0.0879	0.0909	0.0994	0.1083	6.54	0.258	0.258	0.258	780
Grosbeak	636	321.8	26/7	0.0876	0.0902	0.0988	0.1076	6.57	0.256	0.256	0.256	790
Drake	795	402.6	26/7	0.0702	0.0728	0.0794	0.0863	6.81	0.248	0.248	0.248	910
Tern	795	403.8	45/7	0.0709	0.0738	0.0807	0.0876	6.72	0.252	0.252	0.252	890
Rail	954	483.8	45/7	0.0591	0.0617	0.0676	0.0732	6.92	0.245	0.245	0.245	970
Cardinal	954	484.5	54/7	0.0587	0.061	0.0673	0.0728	6.98	0.242	0.242	0.242	990
Curlew	1033.5	525.5	54/7	0.0541	0.0564	0.062	0.0673	7.07	0.239	0.239	0.239	1040
Bluejay	1113	565.5	45/7	0.0509	0.0535	0.0584	0.0633	7.12	0.240	0.240	0.240	1070
Bittern	1272	644.4	45/7	0.0443	0.0472	0.0515	0.0558	7.27	0.235	0.235	0.235	1160
Lapwing	1590	804.1	45/7	0.0354	0.0384	0.042	0.0453	7.56	0.226	0.226	0.226	1340
Falcon	1590	806.2	54/19	0.0354	0.0381	0.0423	0.0459	7.63	0.222	0.222	0.222	1360
Bluebird	2156	1092	84/19	0.0263	0.0296	0.0321	0.0344	8.02	0.214	0.214	0.214	1610
Kiwi	2167	1098	72/7	0.0262	0.0302	0.0317	0.0348	7.98	0.223	0.223	0.223	1607
Thrasher	2312	1172	76/19	0.0246	0.0282	0.0299	0.0328	8.10	0.213	0.213	0.213	1673
Joree	2515	1274	76/19	0.0226	0.0266	0.0279	0.0305	8.22	0.210	0.210	0.210	1751

Warning, the column for Capacitance is often given as reactance per lenght rather than admittance which means you have to ***divide*** by line length to get overall capacitance.

Types of conductors used for overhead lines:

- **A**luminum **C**onductor **S**teel **R**einforced (ACSR) conductors are the most common.
- **A**ll **A**luminum **C**onductor (AAC).
- **A**ll **A**luminum-**A**lloy **C**onductor (AAAC).
- **A**luminum **C**onductor **A**lloy-**R**einforced (ACAR).
- Alumoweld, an aluminum-clad steel conductor.
- Expanded ACSR, which includes filler material between the steel and aluminum to make the outer diameter bigger.

Surge Impedance, SIL & Characteristic Impedance

Take a representative km somewhere along the transmission line, where the voltage is the nominal voltage at 0º.
Over that km, the 1ϕ complex power due to the voltage would be:

$$V_{LN}\cdot\overline{\mathbf{I_L}} = V_{LN}\cdot\overline{\left[V_{LN}\cdot(g+j\cdot\omega\cdot c)\right]} = V_{LN}^2\cdot\overline{(g+j\cdot\omega\cdot c)} = V_{LN}^2\cdot(g-j\cdot\omega\cdot c)$$ assuming the voltage is constant

And the reactive power would be: $-V_{LN}^2\cdot(j\cdot\omega\cdot c)$

The 1ϕ complex power due to the current would be:

$$\Delta V_{LN}\cdot\overline{\mathbf{I_L}} = \left[\mathbf{I_L}\cdot(r+j\cdot\omega\cdot l)\right]\cdot\overline{\mathbf{I_L}} = I_L^2\cdot(r+j\cdot\omega\cdot l)$$ assuming the current is constant

And the reactive power would be: $I_L^2\cdot(j\cdot\omega\cdot l)$

If the two reactive powers were equal and opposite, then the Q of the line would be 0, IE:

$$I_L^2\cdot(j\cdot\omega\cdot l) = -\left[-V_{LN}^2\cdot(j\cdot\omega\cdot c)\right] \quad \text{and} \quad \frac{V_{LN}}{I_L} = \sqrt{\frac{j\cdot\omega\cdot l}{j\cdot\omega\cdot c}} = \sqrt{\frac{l}{c}} = \mathbf{Z_0}$$

Where $\mathbf{Z_0}$ is the magnitude of the impedance I should hook to the line here to get this to happen.

Surge Impedance = $\mathbf{Z_0} = \sqrt{\frac{l}{c}} = \sqrt{\frac{L}{C}} = \sqrt{\frac{x}{|y|}}$

SO, to get the line Q to be 0 (or pretty close), hook this impedance to the end of the line. If $\mathbf{Z_0}$ was purely resistive, and the line voltage at the receiving end were nominal, then the load power would be one "**S**urge **I**mpedance **L**oad", 1SIL.

SIL: $\text{SIL} = 3\cdot\frac{V_R^2}{\mathbf{Z_0}} = \frac{V_{LL}^2}{\mathbf{Z_0}}$ Sometimes load powers or line power capabilities are expressed in terms of SIL.

Characteristic Impedance

The complex version of the surge impedance arises out of the full-fledged calculation of the distributed effect the transmission line parameters. It is known as the characteristic impedance and is EXACTLY the same as the characteristic impedance you found (or will find) for transmission lines in your Electromagnetics (EM) class.

$$\mathbf{Z_C} = \sqrt{\frac{r+j\cdot\omega\cdot l}{g+j\cdot\omega\cdot c}} = \sqrt{\frac{r+j\cdot x}{g+y}} = \sqrt{\frac{R+j\cdot X}{G+Y}}$$ We only use this in caculations for long-length lines

And **if** the line is **lossless**, then: $\mathbf{Z_C} = \mathbf{Z_0} = \sqrt{\frac{l}{c}} = \sqrt{\frac{L}{C}} = \sqrt{\frac{x}{|y|}}$ ONLY if $r = 0 = g$

Propagation Constant

Another number used in caculations for long-length lines is the propagation constant: $\gamma = \sqrt{(r+j\cdot x)\cdot(g+y)}$
Although power transmission lines share some characteristics with EM transmission lines, the wavelength (λ) for 60Hz is about 5000km (3000mi), so, no Smithcharts or stub tuning for 60Hz. However, 360º/5000km still works out to 1º/13.9km, so phase-angle changes may be important to consider.

Transients

Transients on the power lines can happen on much shorter time scales than the 60Hz waveform. Lightning strikes are assumed to produce peak currents of 10 to 20,000 amps in 1.2μs and then exponentially decay at a much slower rate. Switching lines on or off can result in impulses which peak in about 250μs and last longer than lightning impulses. These impulses produce traveling waves on the lines which can bounce back and forth along the line.

The first concern raised by these impulses is the insulation, especially in transformers, where insulation failure results in very-expensive, permanent damage. Studies are done of the **B**asic **I**nsulation **L**evel (BIL) for lightning impulses and **B**asic **S**witching Insulation **L**evel (BSL) for switching impulses.

The insulation discs used with transmission lines are rarely damaged permanently by over-voltages and flashovers.

Surge & Lightning Arresters are highly nonlinear devices which have a high resistance at normal voltages and low resistance at voltages over their threshold. They protect transformers and other devices from over-voltages.

Transient stability of transmission lines play only a part in the overall transient and dynamic stability of entire power systems. Stability and the control of voltage, frequency and generators are fields beyond the scope of this class.

HVDC Transmission Lines

High-**V**oltage **DC** (HVDC) is used for long-distance transmission of large amounts of power, and for some underground and most underwater transmission. HVDC is also used as a power link between two AC grids which are not in sync.

The insulation requirements of transmission lines are set by the peak voltage, but the power is determined by the RMS voltage. For sinusoidal waveforms the peak is 40% higher than the RMS. For DC they are both the same, so the RMS voltage can be 40% higher and the power can be twice as much for the same insulation. For each positive line there will also be a negative line with the same voltage magnitude so the neutral current can be zero.

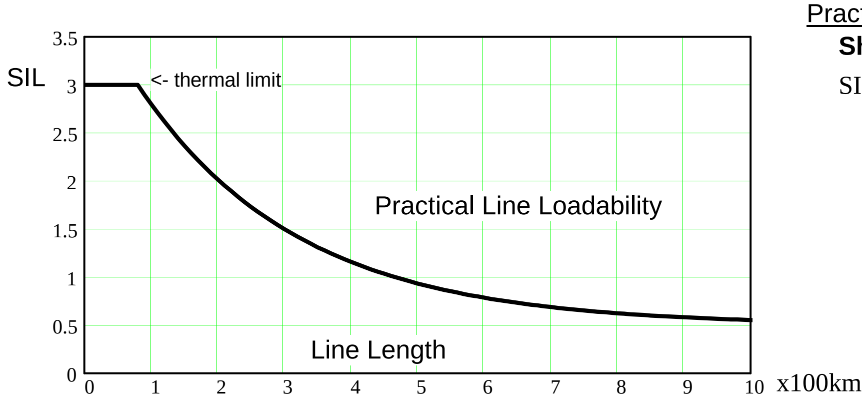
HVDC systems require rectifiers at the sending end to change the AC to DC and an inverter at the receiving end to return the power to AC. These require very high-voltage, very high-power, very expensive, semi-conductor parts. The sending end typically uses transformers with Y-Y, Y-Δ and Δ-Y windings arranged so the rectifiers see a peak voltage every 30º of phase angle (every 1.39ms). This minimizes the need for filtering.

HVDC lines and the associated power conversions are a field unto themselves and beyond the scope of this class.

Transmission Line Typical Values

Nominal Voltages	Bundling Conductors / phase	Line Parameters r Ω/km	ωl Ω/km	ωc μS/km	$\lvert Z_C \rvert$ Characteristic Impedance (Surge Impedance) Ω	SIL Surge Impedance Loading MW	Line Current at 1 SIL A	Line Current at 3 SIL (maximum) A
69·kV	1	0.47	0.47	3.3	383	12.4	104	312
138·kV	1	0.14	0.48	3.4	380	50	210	629
230·kV	1	0.055	0.489	3.373	380	140	350	1050
345·kV	2	0.037	0.376	4.518	290	410	687	2061
500·kV	3	0.029	0.326	5.220	250	1000	1155	3464
765·kV	4	0.013	0.339	4.988	260	2250	1700	5100

Power Handling Capability		Multiple of SIL		230kV MW	345kV MW	500kV MW	765kV MW	Reason For Limit
Short-length Lines:	< 80km (50 miles)	3	3·SIL=	420	1230	3000	6750	Overheating lines
Medium-length:	80 - 240 km (50 - 150 mi)	1.75 – 3	1.75·SIL=	245	718	1750	3938	Voltage Drop
Long-length:	> 240 km (150 mi)	1.0 – 1.75	1·SIL=	140	410	1000	2250	Transient Stability



Practical Limitations

Short Lines should be limited to 3 times the SIL in order to limit the I^2R heating of the line.

Mid-length lines are limited in order to limit the voltage drop across line less than 5%.
$$\frac{|V_R|}{|V_S|} \le 0.95$$

Long-length lines can become unstable, which limits loadability. The power angle should be limited: $\delta \le 30^o$

Sometimes series capacitors and/or shunt inductors are added to these lines to "compensate" for the line reactance.

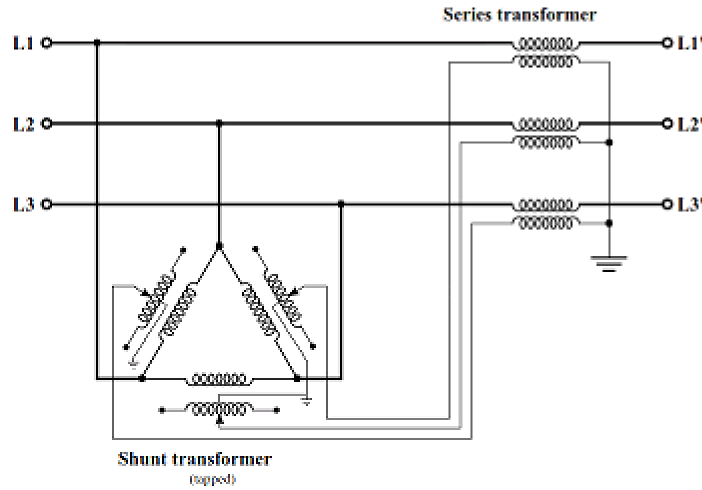
Power Flow (Driven by δ)

Power and current are pushed down the line by a phase angle difference (δ, the power angle), NOT a voltage difference.

if you neglect the line losses $P_{in} = P_{out} = 3 \cdot \frac{V_S \cdot V_R \cdot \sin(\delta)}{X_{line}}$

The power grid often has multiple paths for power to flow from one substation to another. Power will flow down the various paths depending only on the line impedances and lengths. Phase-Shifting transformers allow operators of the lines to take control over the power flow. This use is still relatively rare.

Phase-Shifting transformers are more commonly found where one control area connects to another within a power region (tie line). In this position, they can control power flow from area to area.

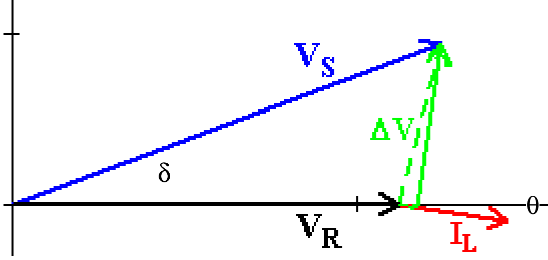


Combating Voltage Sag

The effect of Adding Capacitance at Receiving end of a Transmission Line

All drawing show the same V_S.

Watch the Animations Shown in class



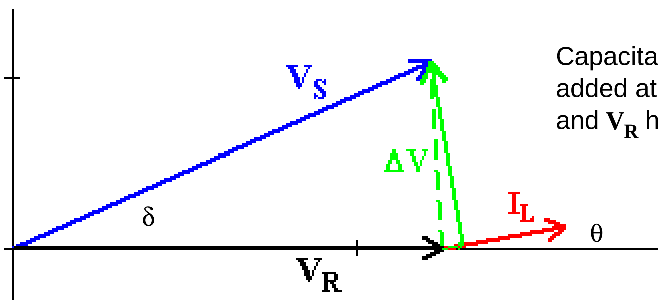
Receiving-end is inductive

$\mathbf{V_S}$ is the line-neutral voltage at the sending end.

$\mathbf{V_R}$ is the line-neutral voltage at the receiving end.

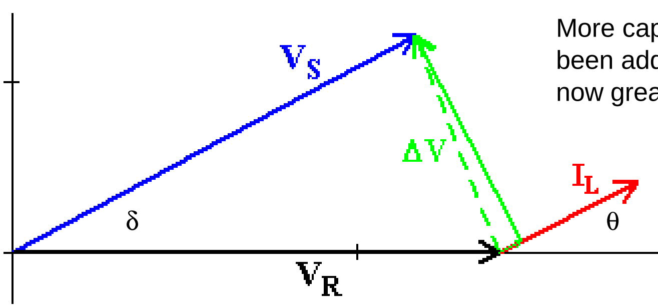
$\mathbf{\Delta V}$ is the voltage difference between the two ends.

$\mathbf{I_L}$ is the line current.



Capacitance has been added at the receiving end and $\mathbf{V_R}$ has increased.

You should notice a similarity between this phasor diagram and that of synchronous generators.



More capacitance has been added and $\mathbf{V_R}$ is now greater than $\mathbf{V_S}$.

Power companies add capacitance at the receiving end of transmission lines if the voltage is too low in order to raise the voltage. These drawing show how this works.

Sources:

Electric Machinery and Power System Fundamentals, Stephen J, Chapman
Power Systems Analysis and Design, Glover & Sarma
First Course on Power Systems, Ned Mohan
Transmission and Distribution of Electrical Energy, Walter L. Weeks
Standard Handbook for Electrical Engineers, Fink & Beaty
www.nexans.us

Long-length Lines: over 240 km (150 miles) (over 200 mi in some texts)

Need:

		Units				
line length:	len , d	m or km		stick to the same unit length for all parameters miles may also be used		
Resistance per unit length:	r	$\frac{\Omega}{m}$ or $\frac{\Omega}{km}$				Units
Inductance per unit length:	l	$\frac{H}{m}$ or $\frac{H}{km}$	OR	Inductive reactance per unit length:	x	$\frac{\Omega}{m}$ or $\frac{\Omega}{km}$
Capacitance per unit length:	c	$\frac{F}{m}$ or $\frac{F}{km}$	OR	Capacitance admittance per unit length:	y	$\frac{S}{m}$ or $\frac{S}{km}$
Conductance to ground:	g	$\frac{S}{m}$ or $\frac{S}{km}$		Common assumption: $g := 0 \cdot \frac{S}{km}$		S := siemens

Find: Units

Characteristic Impedance: $\mathbf{Z_c} = \sqrt{\dfrac{j \cdot x + r}{y + g}}$ Ω

Propagation constant: $\gamma = \sqrt{(j \cdot x + r) \cdot (y + g)}$ $\frac{1}{m}$ or $\frac{1}{km}$

If your calculator doesn't have hyperbolic trig functions

Series impedance $\mathbf{Z_{series}} = \mathbf{Z_c} \cdot \sinh(\gamma \cdot \mathrm{len}) = \mathbf{Z_c} \cdot \dfrac{e^{\gamma \cdot \mathrm{len}} - e^{-\gamma \cdot \mathrm{len}}}{2}$ Ω

Shunt admittance: $\dfrac{\mathbf{Y_{shunt}}}{2} = \dfrac{1}{\mathbf{Z_c}} \cdot \tanh\left(\gamma \cdot \dfrac{\mathrm{len}}{2}\right) = \dfrac{1}{\mathbf{Z_c}} \cdot \dfrac{e^{\gamma \cdot \frac{\mathrm{len}}{2}} - e^{-\gamma \cdot \frac{\mathrm{len}}{2}}}{e^{\gamma \cdot \frac{\mathrm{len}}{2}} + e^{-\gamma \cdot \frac{\mathrm{len}}{2}}} = \dfrac{1}{\mathbf{Z_c}} \cdot \dfrac{\sqrt{e^{\gamma \cdot \mathrm{len}}} - \sqrt{e^{-(\gamma \cdot \mathrm{len})}}}{\sqrt{e^{\gamma \cdot \mathrm{len}}} + \sqrt{e^{-(\gamma \cdot \mathrm{len})}}}$ Ω

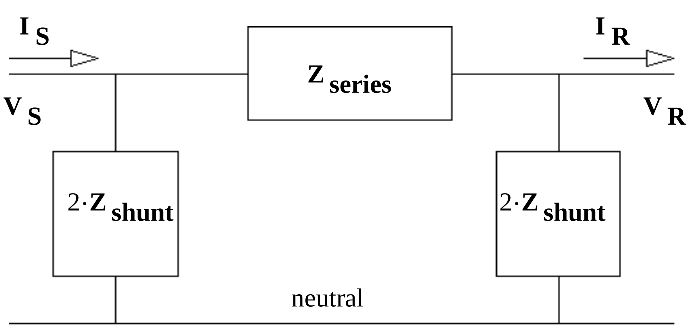
OR

Shunt impedance: $2 \cdot \mathbf{Z_{shunt}} = \dfrac{\mathbf{Z_c}}{\tanh\left(\gamma \cdot \dfrac{\mathrm{len}}{2}\right)}$ S or $\frac{1}{\Omega}$

If your calculator can't handle complex exponents

$e^{(a + b \cdot j)} = e^a \cdot e^{b \cdot j} = e^a \angle b$ (in radians)

Model:



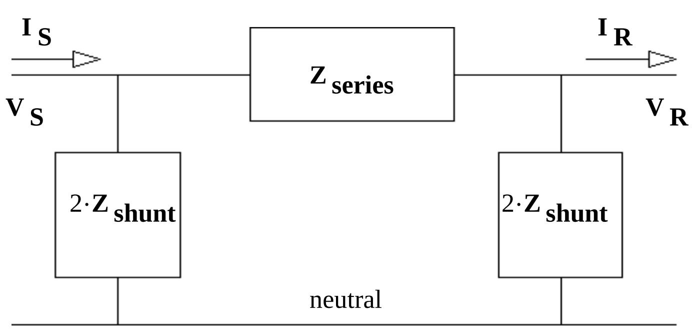
Medium-length Lines: 80 - 240 km (50 to 150 miles) (100 - 200 mi in some texts)

Need:

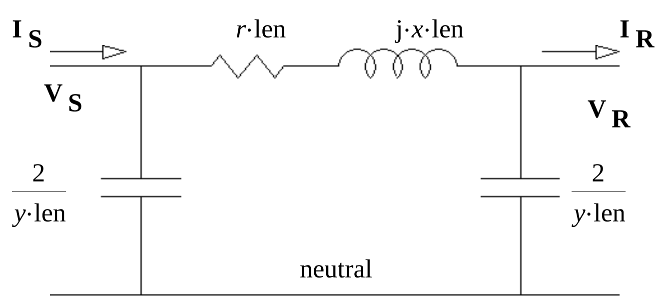
		Units				Units
line length:	len , d	m or km		stick to the same unit length for all parameters miles may also be used		
Resistance per unit length:	r	$\frac{\Omega}{\text{m}}$ or $\frac{\Omega}{\text{km}}$				
Inductance per unit length:	l	$\frac{\text{H}}{\text{m}}$ or $\frac{\text{H}}{\text{km}}$	OR	Inductive reactance per unit length:	x	$\frac{\Omega}{\text{m}}$ or $\frac{\Omega}{\text{km}}$
Capacitance per unit length:	c	$\frac{\text{F}}{\text{m}}$ or $\frac{\text{F}}{\text{km}}$	OR	Capacitance admittance per unit length:	y	$\frac{\text{S}}{\text{m}}$ or $\frac{\text{S}}{\text{km}}$
Conductance to ground:	g	$\frac{\text{S}}{\text{m}}$ or $\frac{\text{S}}{\text{km}}$		Common assumption: $g := 0\cdot\frac{\text{S}}{\text{km}}$		

Find:

			Units
Surge Impedance:	$\mathbf{Z_0} = \sqrt{\frac{x\cdot j}{y}}$	Only needed if load is in terms of SIL	Ω
Series Resistance:	$R_{line} = r\cdot\text{len}$		Ω
Series impedance	$\mathbf{Z_{series}} = (r + j\cdot x)\cdot\text{len}$		Ω
Shunt admittance:	$\frac{\mathbf{Y_{shunt}}}{2} = y\cdot\frac{\text{len}}{2}$		S or $\frac{1}{\Omega}$
OR			
Shunt impedance:	$2\cdot\mathbf{Z_{shunt}} = \frac{2}{y\cdot\text{len}}$		Ω



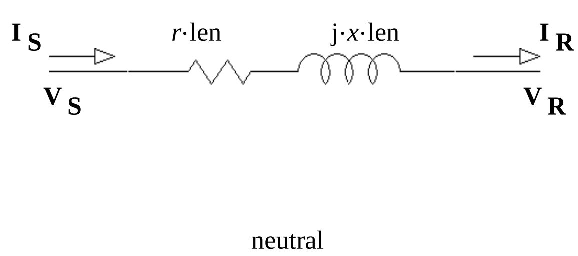
OR:



Short-length Lines: less than 80km (50 mi)

(less than 100 mi in some texts)

Same as above but without the capacitors



b

Ex1. A 500 kV transmission line is 500 km long and has the line parameters shown below. Use the long-length model to find $\mathbf{V_S}$ and $\mathbf{I_S}$ if the line is loaded to 900 MVA and $|\mathbf{V_{RLL}}|$ is 490 kV. Assume the phase angle of $\mathbf{V_R}$ is 0^o and assume load pf = 1.

$len := 500\cdot km$ $V_{RLL} := 490\cdot kV$ $\mathbf{V_R} := \dfrac{V_{RLL}}{\sqrt{3}}$ $S_{1\phi} := \dfrac{900\cdot MVA}{3}$

$r := 0.029\cdot\dfrac{\Omega}{km}$ Assume: $g := 0\cdot\dfrac{S}{km}$

$x := 0.326\cdot\dfrac{\Omega}{km}$ $y := j\cdot\left(5.220\cdot10^{-6}\right)\cdot\dfrac{S}{km}$

Note: These are typical values for a 500 kV transmission line

Long-length line model:

Characteristic Impedance: $\mathbf{Z_c} := \sqrt{\dfrac{j\cdot x + r}{y + g}}$ $\mathbf{Z_c} = 250.151 - 11.104j \cdot\Omega$

Propagation constant: $\gamma := \sqrt{(j\cdot x + r)\cdot(y + g)}$ $\gamma = 5.797\cdot10^{-5} + 1.306\cdot10^{-3}j \cdot\dfrac{1}{km}$

Series impedance: $\mathbf{Z_{series}} := \mathbf{Z_c}\cdot\sinh(\gamma\cdot len)$ $\mathbf{Z_{series}} = 12.508 + 151.772j \cdot\Omega$

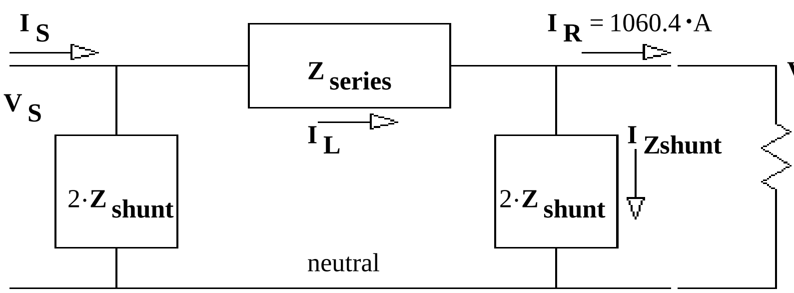
Shunt admittance: $\mathbf{Y_{shunt}} := \dfrac{2}{\mathbf{Z_c}}\cdot\tanh\left(\gamma\cdot\dfrac{len}{2}\right)$ $\dfrac{\mathbf{Y_{shunt}}}{2} = 4.49\cdot10^{-6} + 1.353\cdot10^{-3}j \cdot S$
(Not used in my solution)

Shunt impedance: $\mathbf{Z_{shunt}} := \dfrac{\mathbf{Z_c}}{2\cdot\tanh\left(\gamma\cdot\dfrac{len}{2}\right)}$ $2\cdot\mathbf{Z_{shunt}} = 2.451 - 738.924j \cdot\Omega$

Solve circuit:

$\mathbf{I_R} := \dfrac{S_{1\phi}}{|\mathbf{V_R}|}$ (Not complex in this case because pf = 1 otherwise include a phase angle calculated from the pf or load other information)

$\mathbf{I_R} = 1060.4\cdot A$



$\mathbf{V_R} = 282.902\cdot kV$

$\mathbf{I_{Zshunt}} := \dfrac{\mathbf{V_R}}{2\cdot\mathbf{Z_{shunt}}}$

$\mathbf{I_{Zshunt}} = 1.27 + 382.852j \cdot A$

$\mathbf{I_L} := \mathbf{I_{Zshunt}} + \mathbf{I_R}$ $\mathbf{I_L} = 1.062\cdot10^3 + 382.852j \cdot A$

$\mathbf{V_S} := \mathbf{V_R} + \mathbf{I_L}\cdot\mathbf{Z_{series}}$ $\mathbf{V_S} = 2.381\cdot10^5 + 1.659\cdot10^5 j \cdot V$ $|\mathbf{V_S}| = 290.192\cdot kV$ $\arg(\mathbf{V_S}) = 34.874\cdot deg$

$\mathbf{I_{ZshuntS}} := \dfrac{\mathbf{V_S}}{2\cdot\mathbf{Z_{shunt}}}$ $\mathbf{I_{ZshuntS}} = -223.48 + 322.934j \cdot A$ $\left|\sqrt{3}\cdot\mathbf{V_S}\right| = 502.628\cdot kV$

$\mathbf{I_S} := \mathbf{I_{ZshuntS}} + \mathbf{I_L}$ $\mathbf{I_S} = 838.23 + 705.786j \cdot A$ $|\mathbf{I_S}| = 1096\cdot A$ $\arg(\mathbf{I_S}) = 40.097\cdot deg$

Ex 2. A 345 kV transmission line is 220 km long and has the line parameters shown below.
Find $\mathbf{V_S}$ and $\mathbf{I_S}$ if the line is loaded to 800MVA with pf = 91% lagging. $|\mathbf{V_{RLL}}|$ is 350 kV. $pf := 0.91$

$len := 220 \cdot km$ $\quad V_{RLL} := 350 \cdot kV$ $\quad \mathbf{V_R} := \dfrac{V_{RLL}}{\sqrt{3}}$

Assume the phase angle of $\mathbf{V_R}$ is 0^o if $\mathbf{V_R}$ is given

$r := 0.037 \cdot \dfrac{\Omega}{km}$ $\quad$ Assume: $g := 0 \cdot \dfrac{S}{km}$

Note: These are typical values for a 345 kV transmission line

$x := 0.376 \cdot \dfrac{\Omega}{km}$ $\quad y := j \cdot \left(4.518 \cdot 10^{-6}\right) \cdot \dfrac{S}{km}$

Medium-length line model:

Series impedance: $\mathbf{Z_{series}} := (r + j \cdot x) \cdot len$ $\quad \mathbf{Z_{series}} = 8.14 + 82.72j \ \cdot \Omega$

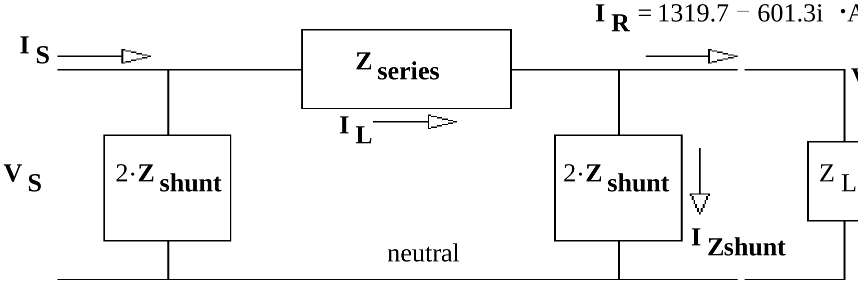
Shunt admittance: $\mathbf{Y_{shunt}} := y \cdot len$ $\quad \dfrac{\mathbf{Y_{shunt}}}{2} = 496.98j \cdot \mu S$
Not used in my solution

Shunt impedance: $\mathbf{Z_{shunt}} := \dfrac{1}{y \cdot len}$ $\quad 2 \cdot \mathbf{Z_{shunt}} = -2.012 \cdot 10^3 j \ \cdot \Omega$

Solve circuit:

$S_{1\phi} := \dfrac{800 \cdot MW}{3 \cdot pf}$ $\quad \mathbf{I_R} := \dfrac{S_{1\phi}}{|\mathbf{V_R}|} \cdot e^{-j \cdot acos(pf)}$ (Negative phase angle because the pf is lagging)

$\mathbf{I_R} = 1319.7 - 601.3i \ \cdot A$



$\mathbf{V_R} = 202.073 \cdot kV$ $\quad 0^o$ reference, only because this is a given number

$\mathbf{I_{Zshunt}} := \dfrac{\mathbf{V_R}}{2 \cdot \mathbf{Z_{shunt}}}$ $\quad \mathbf{I_{Zshunt}} = 100.426j \cdot A$

$\mathbf{I_L} := \mathbf{I_{Zshunt}} + \mathbf{I_R}$ $\quad \mathbf{I_L} = 1.32 \cdot 10^3 - 500.828j \ \cdot A$

$\mathbf{V_S} := \mathbf{V_R} + \mathbf{I_L} \cdot \mathbf{Z_{series}}$ $\quad \mathbf{V_S} = 2.542 \cdot 10^5 + 1.051 \cdot 10^5 j \ \cdot V$ $\quad |\mathbf{V_S}| = 275.104 \cdot kV$ $\quad arg(\mathbf{V_S}) = 22.457 \cdot deg$

Line voltage: $\left|\sqrt{3} \cdot \mathbf{V_S}\right| = 476.495 \cdot kV$

power angle = δ = $arg(\mathbf{V_S}) - arg(\mathbf{V_R}) = 22.457 \cdot deg$

$\mathbf{I_{ZshuntS}} := \dfrac{\mathbf{V_S}}{2 \cdot \mathbf{Z_{shunt}}}$ $\quad \mathbf{I_{ZshuntS}} = -52.225 + 126.354j \ \cdot A$

$\mathbf{I_S} := \mathbf{I_{ZshuntS}} + \mathbf{I_L}$ $\quad \mathbf{I_S} = 1.267 \cdot 10^3 - 374.474j \ \cdot A$ $\quad |\mathbf{I_S}| = 1322 \cdot A$ $\quad arg(\mathbf{I_S}) = -16.46 \cdot deg$

Ex3. A 230 kV transmission line has the following length and line parameters.

$len := 150 \cdot km \qquad r := 0.06 \cdot \frac{\Omega}{km} \qquad x := 0.5 \cdot \frac{\Omega}{km} \qquad g := 0 \cdot \frac{S}{km} \qquad y := j \cdot \left(4 \cdot 10^{-6}\right) \cdot \frac{S}{km}$

Medium-length line model:

Series impedance: $\mathbf{Z}_{series} := (r + j \cdot x) \cdot len \qquad \mathbf{Z}_{series} = 9 + 75j \cdot \Omega$

Shunt admittance: $\mathbf{Y}_{shunt} := y \cdot len \qquad \frac{\mathbf{Y}_{shunt}}{2} = 0.3j \cdot mS$

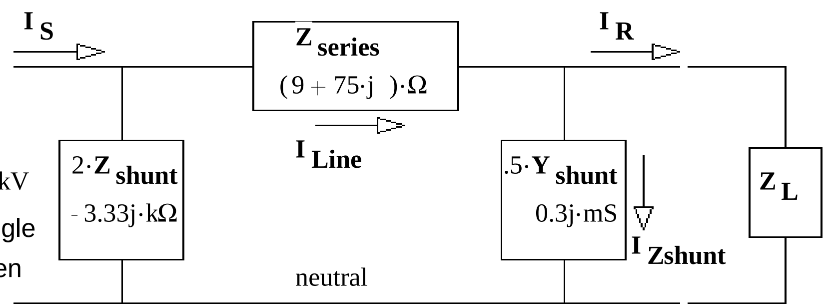
Shunt impedance: $\mathbf{Z}_{shunt} := \frac{1}{y \cdot len} \qquad 2 \cdot \mathbf{Z}_{shunt} = -3.333j \cdot k\Omega$

a) The load is 250Ω with a power factor of 0.87, leading. Find the line current, $\mathbf{I}_{Line}$.

$\mathbf{V}_S := \frac{230 \cdot kV}{\sqrt{3}}$

$\mathbf{V}_S = 132.79 \cdot kV$

Assume the phase angle of $\mathbf{V}_S$ is 0º if $\mathbf{V}_S$ is given



$Z_L := 250 \cdot \Omega$

$pf := 0.87$

$\mathbf{Z}_L := Z_L \cdot e^{-j \cdot acos(pf)}$

$\mathbf{Z}_L = 217.5 - 123.263j \cdot \Omega$

$= 250\Omega \, \underline{/\ 29.54º}$

$$\mathbf{Z} := \mathbf{Z}_{series} + \frac{1}{\frac{\mathbf{Y}_{shunt}}{2} + \frac{1}{\mathbf{Z}_L}} \qquad \mathbf{Z} = 210.467 - 56.544j \cdot \Omega \;=\; 217.9\Omega \, \underline{/\ -15.04º}$$

$$\mathbf{I}_{Line} := \frac{\mathbf{V}_S}{\mathbf{Z}} \qquad \mathbf{I}_{Line} = 588.459 + 158.096j \cdot A \;=\; 609.3A \, \underline{/\ 15.04º}$$

b) Find the load line voltage. $\qquad \mathbf{I}_{Line} \cdot \mathbf{Z}_{series} = -6.561 + 45.557j \cdot kV$

$$\mathbf{V}_R := \mathbf{V}_S - \mathbf{I}_{Line} \cdot \mathbf{Z}_{series} \qquad \mathbf{V}_R = 139.352 - 45.557j \cdot kV \;=\; 146.6kV \, \underline{/\ -18.1º}$$

Receiving line voltage = $\left|\sqrt{3} \cdot \mathbf{V}_R\right| = 253.9 \cdot kV$

Notice that $|\mathbf{V}_R|$ is bigger than $|\mathbf{V}_S|$, this can happen when the receiving-end power factor is leading.

c) What is the "power angle" (δ)? $\qquad \delta = -arg\left(\mathbf{V}_R\right) = 18.104 \cdot deg$

d) How much power is delivered to the load?

$$I_R := \frac{|\mathbf{V}_R|}{|\mathbf{Z}_L|} \qquad P_L = 3 \cdot |\mathbf{V}_R| \cdot I_R \cdot pf = 224.4 \cdot MW$$

Power estimate for the same $|\mathbf{V}_R|$ and $|\mathbf{V}_S|$, but neglecting the line resistance: $\qquad \simeq 3 \cdot \frac{|\mathbf{V}_S| \cdot |\mathbf{V}_R| \cdot \sin(18.1 \cdot deg)}{|\mathbf{Z}_{series}|} = 240 \cdot MW$

e) Express this loading in terms of SIL

Surge Impedance: $\qquad \mathbf{Z}_0 := \sqrt{\frac{j \cdot x}{y}} \qquad \mathbf{Z}_0 = 353.6 \cdot \Omega \qquad \frac{\mathbf{Z}_0}{Z_L} = 1.414$ SIL load

Not asked for in this class