ECE 3600 Transmission Line notes

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Figure 5.1
Typical ACSR conductor

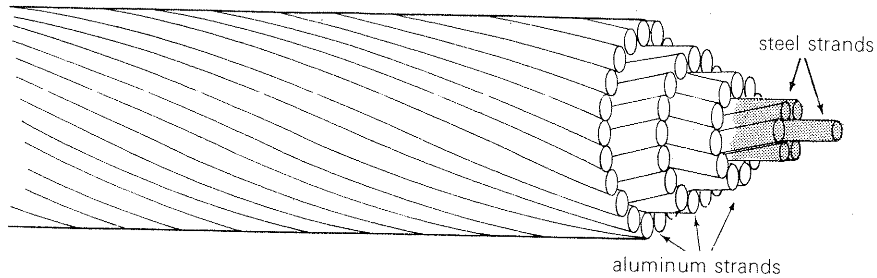


Table A.4 Characteristics of aluminum cable, steel, reinforced (Aluminum Company of America)

bigger

smaller

Code Word	Circular Mils Aluminum	Aluminum			Steel		Outside Diameter (Inches)	Copper Equivalent* Circular Mils or A.W.G.	Ultimate Strength (Pounds)	Weight (Pounds per Mile)	Geometric Mean Radius at 60 Hz (Feet)	Approx. Current Carrying Capacity† (Amps)	Resistance r_a Resistance (Ohms per Conductor per Mile) 25°C (77°F) Small Currents				50°C (122°F) Current Approx. 75% Capacity‡				x_a Inductive Reactance (Ohms per Conductor per Mile at 1 Ft Spacing All Currents)	x'_a Shunt Capacitive Reactance (Megohms per Conductor per Mile at 1 Ft Spacing)
				Strand Diameter (Inches)		Strand Diameter (Inches)							dc	25 Hz	50 Hz	60 Hz	dc	25 Hz	50 Hz	60 Hz	60 Hz	60 Hz
Joree	2 515 000	76	...	0.1819	19	0.0849	1.880		61 700		0.0621									0.0450	0.337	0.0755
Thrasher	2 312 000	76	..	0.1744	19	0.0814	1.802		57 300		0.0595									0.0482	0.342	0.0767
Kiwi	2 167 000	72	4	0.1735	7	0.1157	1.735		49 800		0.0570									0.0511	0.348	0.0778
Bluebird	2 156 000	84	4	0.1602	19	0.0961	1.762		60 300		0.0588									0.0505	0.344	0.0774
Chukar	1 781 000	84	4	0.1456	19	0.0874	1.602		51 000		0.0534									0.0598	0.355	0.0802
Falcon	1 590 000	54	3	0.1716	19	0.1030	1.545	1 000 000	56 000	10 777	0.0520	1 380	0.0587	0.0588	0.0590	0.0591	0.0646	0.0656	0.0675	0.0684	0.359	0.0814
Parrot	1 510 500	54	3	0.1673	19	0.1004	1.506	950 000	53 200	10 237	0.0507	1 340	0.0618	0.0619	0.0621	0.0622	0.0680	0.0690	0.0710	0.0720	0.362	0.0821
Plover	1 431 000	54	3	0.1628	19	0.0977	1.465	900 000	50 400	9 699	0.0493	1 300	0.0652	0.0653	0.0655	0.0656	0.0718	0.0729	0.0749	0.0760	0.365	0.0830
Martin	1 351 000	54	3	0.1582	19	0.0949	1.424	850 000	47 600	9 160	0.0479	1 250	0.0691	0.0692	0.0694	0.0695	0.0761	0.0771	0.0792	0.0803	0.369	0.0838
Pheasant	1 272 000	54	3	0.1535	19	0.0921	1.382	800 000	44 800	8 621	0.0465	1 200	0.0734	0.0735	0.0737	0.0738	0.0808	0.0819	0.0840	0.0851	0.372	0.0847
Grackle	1 192 500	54	3	0.1486	19	0.0892	1.338	750 000	43 100	8 082	0.0450	1 160	0.0783	0.0784	0.0786	0.0788	0.0862	0.0872	0.0894	0.0906	0.376	0.0857
Finch	1 113 000	54	3	0.1436	19	0.0862	1.293	700 000	40 200	7 544	0.0435	1 110	0.0839	0.0840	0.0842	0.0844	0.0924	0.0935	0.0957	0.0969	0.380	0.0867
Curlew	1 033 500	54	3	0.1384	7	0.1384	1.246	650 000	37 100	7 019	0.0420	1 060	0.0903	0.0905	0.0907	0.0909	0.0994	0.1005	0.1025	0.1035	0.385	0.0878
Cardinal	954 000	54	3	0.1329	7	0.1329	1.196	600 000	34 200	6 479	0.0403	1 010	0.0979	0.0980	0.0981	0.0982	0.1078	0.1088	0.1118	0.1128	0.390	0.0890
Canary	900 000	54	3	0.1291	7	0.1291	1.162	566 000	32 300	6 112	0.0391	970	0.104	0.104	0.104	0.104	0.1145	0.1155	0.1175	0.1185	0.393	0.0898
Crane	874 500	54	3	0.1273	7	0.1273	1.146	550 000	31 400	5 940	0.0386	950	0.107	0.107	0.107	0.108	0.1178	0.1188	0.1218	0.1228	0.395	0.0903
Condor	795 000	54	3	0.1214	7	0.1214	1.093	500 000	28 500	5 399	0.0368	900	0.117	0.118	0.118	0.119	0.1288	0.1308	0.1358	0.1378	0.401	0.0917
Drake	795 000	26	2	0.1749	7	0.1360	1.108	500 000	31 200	5 770	0.0375	900	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.399	0.0912
Mallard	795 000	30	2	0.1628	19	0.0977	1.140	500 000	38 400	6 517	0.0393	910	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.393	0.0904
Crow	715 500	54	3	0.1151	7	0.1151	1.036	450 000	26 300	4 859	0.0349	830	0.131	0.131	0.131	0.132	0.1442	0.1452	0.1472	0.1482	0.407	0.0932
Starling	715 500	26	2	0.1659	7	0.1290	1.051	450 000	28 100	5 193	0.0355	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.405	0.0928
Redwing	715 500	30	2	0.1544	19	0.0926	1.081	450 000	34 600	5 865	0.0372	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.399	0.0920
Flamingo	666 600	54	3	0.1111	7	0.1111	1.000	419 000	24 500	4 527	0.0337	800	0.140	0.140	0.141	0.141	0.1541	0.1571	0.1591	0.1601	0.412	0.0943
Rook	636 000	54	3	0.1085	7	0.1085	0.977	400 000	23 600	4 319	0.0329	770	0.147	0.147	0.148	0.148	0.1618	0.1638	0.1678	0.1688	0.414	0.0950
Grosbeak	636 000	26	2	0.1564	7	0.1216	0.990	400 000	25 000	4 616	0.0335	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.412	0.0946
Egret	636 000	30	2	0.1456	19	0.0874	1.019	400 000	31 500	5 213	0.0351	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.406	0.0937
Peacock	605 000	54	3	0.1059	7	0.1059	0.953	380 500	22 500	4 109	0.0321	750	0.154	0.155	0.155	0.155	0.1695	0.1715	0.1755	0.1775	0.417	0.0957
Squab	605 000	26	2	0.1525	7	0.1186	0.966	380 500	24 100	4 391	0.0327	760	0.154	0.154	0.154	0.154	0.1700	0.1720	0.1720	0.1720	0.415	0.0953
Dove	556 500	26	2	0.1463	7	0.1138	0.927	350 000	22 400	4 039	0.0313	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.420	0.0965
Eagle	556 500	30	2	0.1362	7	0.1362	0.953	350 000	27 200	4 588	0.0328	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.415	0.0957
Hawk	477 000	26	2	0.1355	7	0.1054	0.858	300 000	19 430	3 462	0.0290	670	0.196	0.196	0.196	0.196	0.216				0.430	0.0988
Hen	477 000	30	2	0.1261	7	0.1261	0.883	300 000	23 300	3 933	0.0304	670	0.196	0.196	0.196	0.196	0.216				0.424	0.0980
Ibis	397 500	26	2	0.1236	7	0.0961	0.783	250 000	16 190	2 885	0.0265	590	0.235				0.259				0.441	0.1015
Lark	397 500	30	2	0.1151	7	0.1151	0.806	250 000	19 980	3 277	0.0278	600	0.235		Same as dc		0.259		Same as dc		0.435	0.1006
Linnet	336 400	26	2	0.1138	7	0.0855	0.721	4/0	14 050	2 442	0.0244	530	0.278				0.306				0.451	0.1039
Oriole	336 400	30	2	0.1059	7	0.1059	0.741	4/0	17 040	2 774	0.0255	530	0.278				0.306				0.445	0.1032
Ostrich	300 000	26	2	0.1074	7	0.0835	0.680	188 700	12 650	2 178	0.0230	490	0.311				0.342				0.458	0.1057
Piper	300 000	30	2	0.1000	7	0.1000	0.700	188 700	15 430	2 473	0.0241	500	0.311				0.342				0.462	0.1049
Partridge	266 800	26	2	0.1013	7	0.0788	0.642	3/0	11 250	1 936	0.0217	460	0.350				0.385				0.465	0.1074

*Based on copper 97%, aluminum 61% conductivity.
†For conductor at 75°C, air at 25°C, wind 1.4 miles per hour (2 ft/sec), frequency = 60 Hz.
‡"Current Approx. 75% Capacity" is 75% of the "Approx. Current Carrying Capacity in Amps" and is approximately the current which will produce 50°C conductor temp. (25°C rise) with 25°C air temp., wind 1.4 miles per hour.

ρ : 100% = $1.72 \times 10^{-8}\ \Omega \cdot m$, 10.37 cmil/ft

Cu $1.77 \times 10^{-8}\ \Omega \cdot m$

Al $2.83 \times 10^{-8}\ \Omega \cdot m$

Other conductor types include the all-aluminum conductor (AAC), all-aluminum-alloy conductor (AAAC), aluminum conductor alloy-reinforced (ACAR), and aluminum-clad steel conductor (Alumoweld). There is also a conductor known as "expanded ACSR," which has a filler such as fiber or paper between the aluminum and steel strands. The filler increases the conductor diameter, which reduces the electric field at the conductor surface, to control corona.

EHV lines often have more than one conductor per phase; these conductors are called a *bundle*. The 765-kV line in Figure 5.2 has four conductors per phase, and the 345-kV double-circuit line in Figure 5.3 has two conductors per phase. Bundle conductors have a lower electric field strength at the conductor surfaces, thereby controlling corona. They also have a smaller series reactance.

NOMINAL VOLTAGE	PHASE CONDUCTORS				
	NUMBER OF CONDUCTORS PER BUNDLE	ALUMINUM CROSS-SECTION AREA PER CONDUCTOR (ACSR)	BUNDLE SPACING	MINIMUM CLEARANCES 0.3048m = 1 ft	
				PHASE-TO-PHASE	PHASE-TO-GROUND
kV		kcmil	cm	m	m
~~46kV~~ 115kV 69	1	–	–	–	–
138	1	300–700	–	4 to 5	–
230	1	400–1000	–	6 to 9	–
345	1	2000–2500	–	6 to 9	7.6 to 11
345	2	800–2200	45.7 (18")	6 to 9	7.6 to 11
500	2	2000–2500	45.7	9 to 11	9 to 14
500	3	900–1500	45.7	9 to 11	9 to 14
765	4	900–1300	45.7	13.7 (45')	12.2 (40')

3.281 ft = 1m

NOMINAL VOLTAGE	SUSPENSION INSULATOR STRING		SHIELD WIRES (lightning protection)		
	NUMBER OF STRINGS PER PHASE	NUMBER OF STANDARD INSULATOR DISCS PER SUSPENSION STRING	TYPE	NUMBER	DIAMETER
kV					cm
69	1	4 to 6	Steel	0,1 or 2	–
138	1	8 to 11	Steel	0,1 or 2	–
230	1	12 to 21	Steel or ACSR	1 or 2	1.1 to 1.5
345	1	18 to 21	Alumoweld	2	0.87 to 1.5
345	1 and 2	18 to 21	Alumoweld	2	0.87 to 1.5
500	2 and 4	24 to 27	Alumoweld	2	0.98 to 1.5
500	2 and 4	24 to 27	Alumoweld	2	0.98 to 1.5
765	2 and 4	30 to 35	Alumoweld	2	0.98

Table 5.1 Typical transmission-line characteristics [1, 2]

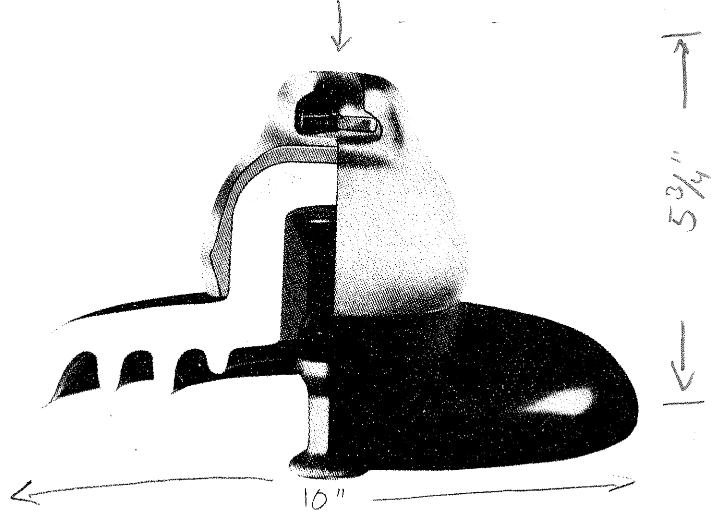


Figure 5.5

Cut-away view of a standard insulator disc for suspension insulator strings

(Courtesy of Ohio Brass)

Table 5.2

Comparison of SI and English units for calculating conductor resistance

QUANTITY	SYMBOL	SI UNITS	ENGLISH UNITS
Resistivity	ρ	Ωm	Ω-cmil/ft
Length	ℓ	m	ft
Cross-sectional area	A	m^2	cmil
dc resistance	$R_{dc} = \frac{\rho \ell}{A}$	Ω	Ω

Resistivity depends on the conductor metal. Annealed copper is the international standard for measuring resistivity ρ (or conductivity σ, where $\sigma = 1/\rho$). Resistivity of conductor metals is listed in Table 5.3. As shown, hard-drawn aluminum, which has 61% of the conductivity of the international standard, has a resistivity at 20°C of 17.00 Ω-cmil/ft or 2.83×10^{-8} Ωm.

Table 5.3

% Conductivity, resistivity, and temperature constant of conductor metals

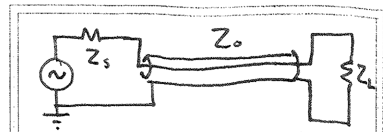
MATERIAL	% CONDUCTIVITY	$\rho_{20°C}$ RESISTIVITY AT 20°C		T TEMPERATURE CONSTANT
		$\Omega m \times 10^{-8}$	Ω-cmil/ft	°C
Copper:				
Annealed	100%	1.72	10.37	234.5
Hard-drawn	97.3%	1.77	10.66	241.5
Aluminum				
Hard-drawn	61%	2.83	17.00	228.1
Brass	20–27%	6.4–8.4	38–51	480
Bronze	9–13%	13–18	78–108	1980
Iron	17.2%	10	60	180
Silver	108%	1.59	9.6	243
Sodium	40%	4.3	26	207
Steel	2 to 14%	12 to 88	72–530	180–980

Conductor resistance depends on the following factors:

1. Spiraling + 1-2% resistance
2. Temperature $\rho_{T_2} = \rho_{20}\left(\frac{T_2 + T}{20°C + T}\right)$
3. Frequency ("skin effect") ~ + 3%
4. Current magnitude—magnetic conductors

Characteristic impedance

From Wikipedia, the free encyclopedia

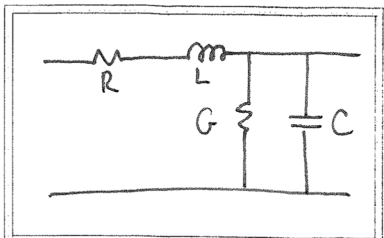


Schematic representation of a transmission line, showing the characteristic impedance Z_0.

The **characteristic impedance** or **surge impedance** of a uniform transmission line, usually written Z_0, is the ratio of the amplitudes of a *single* pair of voltage and current waves propagating along the line in the absence of reflections. The SI unit of characteristic impedance is the ohm. The characteristic impedance of a lossless transmission line is purely real, that is, there is no imaginary component ($Z_0 = |Z_0| + j0$). Characteristic impedance appears like a resistance in this case, such that power generated by a source on one end of an infinitely long lossless transmission line is dissipated *through* the line but is not dissipated *in* the line itself. A transmission line of finite length (lossless or lossy) that is terminated at one end with a resistor equal to the characteristic impedance ($Z_L = Z_0$) appears like an infinitely long transmission line to the source.

Transmission line model

Applying the transmission line model based on the telegrapher's equations, the general expression for the characteristic impedance of a transmission line is:



Schematic representation of the elementary components of a transmission line.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

where

R is the resistance per unit length,
L is the inductance per unit length,
G is the conductance of the dielectric per unit length,
C is the capacitance per unit length,
j is the imaginary unit, and
ω is the angular frequency.

The voltage and current phasors on the line are related by the characteristic impedance as:

$$\frac{V^+}{I^+} = Z_0 = -\frac{V^-}{I^-}$$

where the superscripts $+$ and $-$ represent forward- and backward-traveling waves, respectively.

Lossless line

For a lossless line, R and G are Zero so the equation for characteristic impedance reduces to

$$Z_0 = \sqrt{\frac{L}{C}}.$$

Surge Impedance Loading

In electric power transmission, the characteristic impedance of a transmission line is expressed in terms of the **surge impedance loading (SIL)**, or natural loading, being the MW loading at which reactive power is neither produced nor absorbed:

$$SIL = \frac{(V_{\mathrm{L-L}})^2}{Z_0} = 3\,\frac{V_{LN}^2}{Z_0} = 3\,\frac{V_R^2}{Z_0}$$

in which $V_{\mathrm{L-L}}$ is the line-to-line voltage in volts.

Loaded below its SIL, a line supplies lagging reactive power to the system, tending to raise system voltages. Above it, the line absorbs reactive power, tending to depress the voltage. The Ferranti effect describes the voltage gain towards the remote end of a very lightly loaded (or open ended) transmission line. Underground cables normally have a very low characteristic impedance, resulting in an SIL that is typically in excess of the thermal limit of the cable. Hence a cable is almost always a source of lagging reactive power.

ECE 3600 Transmission Line Typical Values

Table 4-1
Transmission Line Parameters with Bundled Conductors (except at 230 kV) at 60 Hz [2, 6]

10^{-6} Siemens

Nominal Voltage	$R(\Omega/km)$	$\omega L(\Omega/km)$	$\omega C(\mu\mho/km)$
230 kV	0.055	0.489	3.373
345 kV	0.037	0.376	4.518
500 kV	0.029	0.326	5.220
765 kV	0.013	0.339	4.988

Table 4-2
Surge Impedance and Three-Phase Surge Impedance Loading [2, 6]

Nominal Voltage	$Z_c(\Omega)$	$SIL(MW)$ (MVA)	for 1 SIL I_L(A)
230 kV	375	140 MW	350A
345 kV	280	425 MW	710A
500 kV	250	1000 MW	1160A
765 kV	255	2300 MW	1740A

Table 4-3
Loadability of Transmission Lines [6]

	Line Length (km)	Limiting Factor	Multiple of SIL
short	0 - 80 km 50 mi	Thermal	> 3
medium	80 - 240 km 50 - 150 mi	5% Voltage Drop	1.5 - 3
Long 240 km +	240 - 480 km 150 - 300 mi	Stability	1.0 – 1.5

Typical values for transmission lines taken from:
First Course on Power Systems by Ned Mohan

ECE 3600 Lumped-Parameter Transmission Line Models

c

Long-length Lines: over 240 km (150 miles)

(over 200 mi in some texts)

Need:

		Units				Units
line length:	len , d	m or km		stick to the same unit length for all parameters miles may also be used		
Resistance per unit length:	r	$\frac{\Omega}{m}$ or $\frac{\Omega}{km}$				
Inductance per unit length:	l	$\frac{H}{m}$ or $\frac{H}{km}$	OR	Inductive reactance per unit length:	x	$\frac{\Omega}{m}$ or $\frac{\Omega}{km}$
Capacitance per unit length:	c	$\frac{F}{m}$ or $\frac{F}{km}$	OR	Capacitance admittance per unit length:	y	$\frac{S}{m}$ or $\frac{S}{km}$
Conductance to ground:	g	$\frac{S}{m}$ or $\frac{S}{km}$		Common assumption: $g := 0\cdot\frac{S}{km}$		$S :=$ siemens

Find:

		Units
Surge impedance:	$\mathbf{Z}_{\mathbf{c}} = \sqrt{\frac{j\cdot x + r}{j\cdot y + g}}$	Ω
Propagation constant:	$\gamma = \sqrt{(j\cdot x + r)\cdot(j\cdot y + g)}$	$\frac{1}{m}$ or $\frac{1}{km}$

<u>If your calculator doesn't have hyperbolic trig functions</u>

Series impedance $\quad \mathbf{Z}_{\mathbf{series}} = \mathbf{Z}_{\mathbf{c}}\cdot\sinh(\gamma\cdot len) = \mathbf{Z}_{\mathbf{c}}\cdot\frac{e^{\gamma\cdot len} - e^{-\gamma\cdot len}}{2} \quad \Omega$

Shunt admittance: $\quad \frac{\mathbf{Y}_{\mathbf{shunt}}}{2} = \frac{1}{\mathbf{Z}_{\mathbf{c}}}\cdot\tanh\left(\gamma\cdot\frac{len}{2}\right) = \frac{1}{\mathbf{Z}_{\mathbf{c}}}\cdot\frac{e^{\gamma\cdot\frac{len}{2}} - e^{-\gamma\cdot\frac{len}{2}}}{e^{\gamma\cdot\frac{len}{2}} + e^{-\gamma\cdot\frac{len}{2}}} = \frac{1}{\mathbf{Z}_{\mathbf{c}}}\cdot\frac{\sqrt{e^{\gamma\cdot len}} - \sqrt{e^{-(\gamma\cdot len)}}}{\sqrt{e^{\gamma\cdot len}} + \sqrt{e^{-(\gamma\cdot len)}}} \quad \Omega$

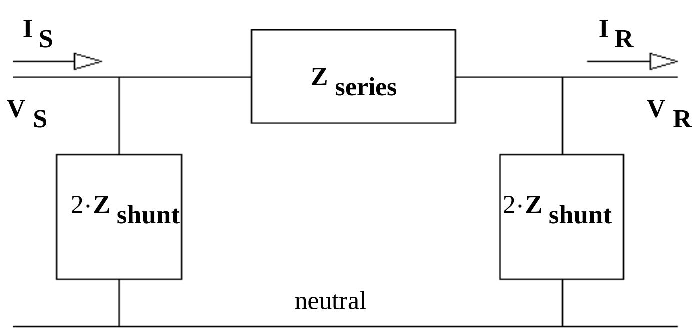
OR

Shunt impedance: $\quad 2\cdot\mathbf{Z}_{\mathbf{shunt}} = \frac{\mathbf{Z}_{\mathbf{c}}}{\tanh\left(\gamma\cdot\frac{len}{2}\right)} \quad$ S or $\frac{1}{\Omega}$

<u>If your calculator can't handle complex exponents</u>

$$e^{(a+b\cdot j)} = e^{a}\cdot e^{b\cdot j} = e^{a}\ \underline{/b \text{ (in radians)}}$$

Model:



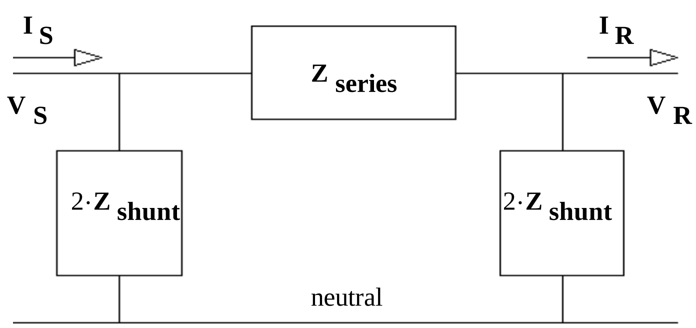
Medium-length Lines: 80 - 240 km (50 to 150 miles) (100 - 200 mi in some texts)

Need:

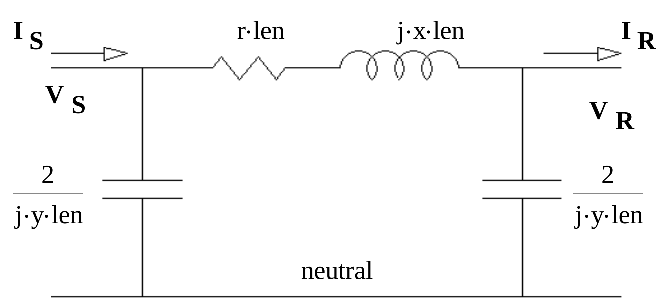
		Units				Units
line length:	len , d	m or km		stick to the same unit length for all parameters miles may also be used		
Resistance per unit length:	r	$\frac{\Omega}{m}$ or $\frac{\Omega}{km}$				
Inductance per unit length:	l	$\frac{H}{m}$ or $\frac{H}{km}$	OR	Inductive reactance per unit length:	x	$\frac{\Omega}{m}$ or $\frac{\Omega}{km}$
Capacitance per unit length:	c	$\frac{F}{m}$ or $\frac{F}{km}$	OR	Capacitance admittance per unit length:	y	$\frac{S}{m}$ or $\frac{S}{km}$
Conductance to ground:	g	$\frac{S}{m}$ or $\frac{S}{km}$		Common assumption: $g := 0 \cdot \frac{S}{km}$		

Find:

			Units
Surge Impedance:	$\mathbf{Z}_{\mathbf{c}} = \sqrt{\frac{x}{y}}$	Only needed if load is in terms of SIL	Ω
Series Resistance:	$R_{line} = r \cdot len$		Ω
Series impedance	$\mathbf{Z}_{\mathbf{series}} = (r + j \cdot x) \cdot len$		Ω
Shunt admittance:	$\frac{\mathbf{Y}_{\mathbf{shunt}}}{2} = j \cdot y \cdot \frac{len}{2}$		S or $\frac{1}{\Omega}$
OR			
Shunt impedance:	$2 \cdot \mathbf{Z}_{\mathbf{shunt}} = \frac{2}{j \cdot y \cdot len}$		Ω



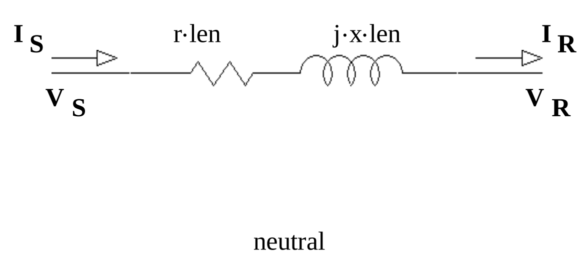
OR:



Short-length Lines: less than 80km (50 mi)

(less than 100 mi in some texts)

Same as above but without the capacitors



ECE 3600 Transmission Line Examples

b

Ex1. A 500 kV transmission line is 500 km long and has the line parameters shown below. Use the long-length model to find $\mathbf{V_S}$ and $\mathbf{I_S}$ if the line is loaded to 900 MVA and $|\mathbf{V_{RLL}}|$ is 490 kV. Assume the phase angle of $\mathbf{V_R}$ is 0^o and assume load pf = 1.

$len := 500 \cdot km \qquad V_{RLL} := 490 \cdot kV \qquad \mathbf{V_R} := \frac{V_{RLL}}{\sqrt{3}} \qquad S_{1\phi} := \frac{900 \cdot MVA}{3}$

$r := 0.029 \cdot \frac{\Omega}{km}$ Assume: $g := 0 \cdot \frac{S}{km}$

Note: These are typical values for a 500 kV transmission line

$x := 0.326 \cdot \frac{\Omega}{km} \qquad y := 5.220 \cdot 10^{-6} \cdot \frac{S}{km}$

Long-length line model:

Surge Impedance: $\mathbf{Z_c} := \sqrt{\frac{j \cdot x + r}{j \cdot y + g}}$ $\qquad \mathbf{Z_c} = 250.151 - 11.104j \cdot \Omega$

Propagation constant: $\gamma := \sqrt{(j \cdot x + r) \cdot (j \cdot y + g)}$ $\qquad \gamma = 5.797 \cdot 10^{-5} + 1.306 \cdot 10^{-3} j \cdot \frac{1}{km}$

Series impedance: $\mathbf{Z_{series}} := \mathbf{Z_c} \cdot \sinh(\gamma \cdot len)$ $\qquad \mathbf{Z_{series}} = 12.508 + 151.772j \cdot \Omega$

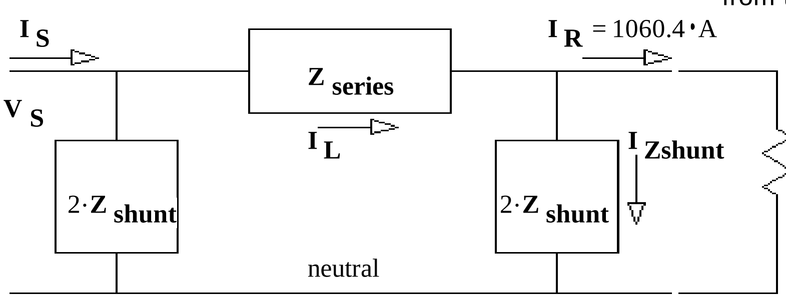
Shunt admittance: $\mathbf{Y_{shunt}} := \frac{2}{\mathbf{Z_c}} \cdot \tanh\left(\gamma \cdot \frac{len}{2}\right)$ $\qquad \frac{\mathbf{Y_{shunt}}}{2} = 4.49 \cdot 10^{-6} + 1.353 \cdot 10^{-3} j \cdot S$
(Not used in my solution)

Shunt impedance: $\mathbf{Z_{shunt}} := \frac{\mathbf{Z_c}}{2 \cdot \tanh\left(\gamma \cdot \frac{len}{2}\right)}$ $\qquad 2 \cdot \mathbf{Z_{shunt}} = 2.451 - 738.924j \cdot \Omega$

Solve circuit:

$\mathbf{I_R} := \frac{S_{1\phi}}{|\mathbf{V_R}|}$ (Not complex in this case because pf = 1 otherwise include a phase angle calculated from the pf or load other information)

$\mathbf{I_R} = 1060.4 \cdot A$



$\mathbf{V_R} = 282.902 \cdot kV$

$\mathbf{I_{Zshunt}} := \frac{\mathbf{V_R}}{2 \cdot \mathbf{Z_{shunt}}}$

$\mathbf{I_{Zshunt}} = 1.27 + 382.852j \cdot A$

$\mathbf{I_L} := \mathbf{I_{Zshunt}} + \mathbf{I_R}$ $\qquad \mathbf{I_L} = 1.062 \cdot 10^{3} + 382.852j \cdot A$

$\mathbf{V_S} := \mathbf{V_R} + \mathbf{I_L} \cdot \mathbf{Z_{series}}$ $\qquad \mathbf{V_S} = 2.381 \cdot 10^{5} + 1.659 \cdot 10^{5} j \cdot V$ $\qquad |\mathbf{V_S}| = 290.192 \cdot kV \qquad \arg(\mathbf{V_S}) = 34.874 \cdot deg$

$\mathbf{I_{ZshuntS}} := \frac{\mathbf{V_S}}{2 \cdot \mathbf{Z_{shunt}}}$ $\qquad \mathbf{I_{ZshuntS}} = -223.48 + 322.934j \cdot A$ $\qquad |\sqrt{3} \cdot \mathbf{V_S}| = 502.628 \cdot kV$

$\mathbf{I_S} := \mathbf{I_{ZshuntS}} + \mathbf{I_L}$ $\qquad \mathbf{I_S} = 838.23 + 705.786j \cdot A$ $\qquad |\mathbf{I_S}| = 1096 \cdot A \qquad \arg(\mathbf{I_S}) = 40.097 \cdot deg$

Ex 2. A 345 kV transmission line is 220 km long and has the line parameters shown below.
Find $\mathbf{V_S}$ and $\mathbf{I_S}$ if the line is loaded to 800MVA with pf = 91% lagging. $|\mathbf{V_{RLL}}|$ is 510 kV. $\quad pf := 0.91$

$len := 220 \cdot km \qquad V_{RLL} := 510 \cdot kV \qquad \mathbf{V_R} := \frac{V_{RLL}}{\sqrt{3}}$

Assume the phase angle of $\mathbf{V_R}$ is 0^o if $\mathbf{V_R}$ is given

$r := 0.037 \cdot \frac{\Omega}{km}$ $\qquad$ Assume: $g := 0 \cdot \frac{S}{km}$

Note: These are typical values for a 345 kV transmission line

$x := 0.376 \cdot \frac{\Omega}{km} \qquad y := 4.518 \cdot 10^{-6} \cdot \frac{S}{km}$

Medium-length line model:

Series impedance: $\mathbf{Z_{series}} := (r + j \cdot x) \cdot len \qquad \mathbf{Z_{series}} = 8.14 + 82.72j \cdot \Omega$

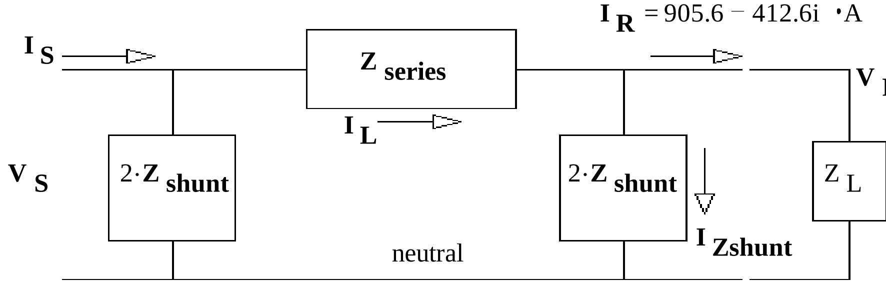
Shunt admittance: $\mathbf{Y_{shunt}} := j \cdot y \cdot len \qquad \frac{\mathbf{Y_{shunt}}}{2} = 496.98j \cdot \mu S$
Not used in my solution

Shunt impedance: $\mathbf{Z_{shunt}} := \frac{1}{j \cdot y \cdot len} \qquad 2 \cdot \mathbf{Z_{shunt}} = -2.012 \cdot 10^3 j \cdot \Omega$

Solve circuit:

$S_{1\phi} := \frac{800 \cdot MW}{3 \cdot pf} \qquad \mathbf{I_R} := \frac{S_{1\phi}}{|\mathbf{V_R}|} \cdot e^{-j \cdot acos(pf)}$ (Negative phase angle because the pf is lagging)

$\mathbf{I_R} = 905.6 - 412.6i \cdot A$



$\mathbf{V_R} = 294.449 \cdot kV$ $\quad 0^o$ reference, only because this is a given number

$\mathbf{I_{Zshunt}} := \frac{\mathbf{V_R}}{2 \cdot \mathbf{Z_{shunt}}} \qquad \mathbf{I_{Zshunt}} = 146.335j \cdot A$

$\mathbf{I_L} := \mathbf{I_{Zshunt}} + \mathbf{I_R} \qquad \mathbf{I_L} = 905.647 - 266.29j \cdot A$

$\mathbf{V_S} := \mathbf{V_R} + \mathbf{I_L} \cdot \mathbf{Z_{series}} \qquad \mathbf{V_S} = 3.238 \cdot 10^5 + 7.275 \cdot 10^4 j \cdot V \qquad |\mathbf{V_S}| = 331.918 \cdot kV \qquad arg(\mathbf{V_S}) = 12.66 \cdot deg$

Line voltage: $|\sqrt{3} \cdot \mathbf{V_S}| = 574.9 \cdot kV$

power angle = δ = $arg(\mathbf{V_S}) - arg(\mathbf{V_R}) = 12.66 \cdot deg$

$\mathbf{I_{ZshuntS}} := \frac{\mathbf{V_S}}{2 \cdot \mathbf{Z_{shunt}}} \qquad \mathbf{I_{ZshuntS}} = -36.154 + 160.946j \cdot A$

$\mathbf{I_S} := \mathbf{I_{ZshuntS}} + \mathbf{I_L} \qquad \mathbf{I_S} = 869.493 - 105.344j \cdot A \qquad |\mathbf{I_S}| = 876 \cdot A \qquad arg(\mathbf{I_S}) = -6.908 \cdot deg$

Ex3. A 230 kV transmission line has the following length and line parameters.

$\text{len} := 150\cdot\text{km}$ $\quad r := 0.06\cdot\frac{\Omega}{\text{km}}$ $\quad x := 0.5\cdot\frac{\Omega}{\text{km}}$ $\quad g := 0\cdot\frac{S}{\text{km}}$ $\quad y := 4\cdot10^{-6}\cdot\frac{S}{\text{km}}$

Medium-length line model:

Series impedance: $\mathbf{Z}_{\text{series}} := (r + j\cdot x)\cdot\text{len}$ $\quad \mathbf{Z}_{\text{series}} = 9 + 75j \cdot\Omega$

Shunt admittance: $\mathbf{Y}_{\text{shunt}} := j\cdot y\cdot\text{len}$ $\quad \frac{\mathbf{Y}_{\text{shunt}}}{2} = 0.3j\cdot\text{mS}$

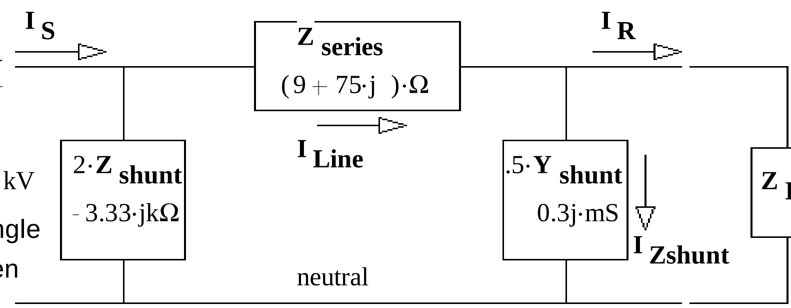
Shunt impedance: $\mathbf{Z}_{\text{shunt}} := \frac{1}{j\cdot y\cdot\text{len}}$ $\quad 2\cdot\mathbf{Z}_{\text{shunt}} = -3.333j\cdot\text{k}\Omega$

a) The load is 250Ω with a power factor of 0.87, leading. Find the line current, $\mathbf{I}_{\text{Line}}$.

$\mathbf{V}_S := \frac{230\cdot\text{kV}}{\sqrt{3}}$

$\mathbf{V}_S = 132.79\cdot\text{kV}$

Assume the phase angle of $\mathbf{V}_S$ is 0° if $\mathbf{V}_S$ is given



$Z_L := 250\cdot\Omega$

$\text{pf} := 0.87$

$\mathbf{Z}_L := Z_L\cdot e^{-j\cdot\text{acos}(\text{pf})}$

$\mathbf{Z}_L = 217.5 - 123.263j \cdot\Omega$ $= 250\Omega\,\underline{/\ 29.54^\circ}$

$\mathbf{Z} := \mathbf{Z}_{\text{series}} + \frac{1}{\frac{\mathbf{Y}_{\text{shunt}}}{2} + \frac{1}{\mathbf{Z}_L}}$ $\quad \mathbf{Z} = 210.467 - 56.544j \cdot\Omega$ $= 219.7\Omega\,\underline{/\ -15.04^\circ}$

$\mathbf{I}_{\text{Line}} := \frac{\mathbf{V}_S}{\mathbf{Z}}$ $\quad \mathbf{I}_{\text{Line}} = 588.459 + 158.096j \cdot\text{A}$ $= 609.3\text{A}\,\underline{/\ 15.04^\circ}$

b) Find the load line voltage. $\quad \mathbf{I}_{\text{Line}}\cdot\mathbf{Z}_{\text{series}} = -6.561 + 45.557j \cdot\text{kV}$

$\mathbf{V}_R := \mathbf{V}_S - \mathbf{I}_{\text{Line}}\cdot\mathbf{Z}_{\text{series}}$ $\quad \mathbf{V}_R = 139.352 - 45.557j \cdot\text{kV}$ $= 146.6\text{kV}\,\underline{/\ -18.1^\circ}$

Receiving line voltage = $\left|\sqrt{3}\cdot\mathbf{V}_R\right| = 253.9\cdot\text{kV}$

Notice that $|\mathbf{V}_R|$ is bigger than $|\mathbf{V}_S|$, this can happen when the receiving-end power factor is leading.

c) What is the "power angle" (δ)? $\quad \delta = -\arg(\mathbf{V}_R) = 18.104\cdot\text{deg}$

d) How much power is delivered to the load?

$I_R := \frac{|\mathbf{V}_R|}{|\mathbf{Z}_L|}$ $\quad P_L = 3\cdot|\mathbf{V}_R|\cdot I_R\cdot\text{pf} = 224.4\cdot\text{MW}$

Power estimate for the same $|\mathbf{V}_R|$ and $|\mathbf{V}_S|$, but neglecting the line resistance: $\quad \simeq 3\cdot\frac{|\mathbf{V}_S|\cdot|\mathbf{V}_R|\cdot\sin(18.1\cdot\text{deg})}{|\mathbf{Z}_{\text{series}}|} = 240\cdot\text{MW}$

e) Express this loading in terms of SIL

Surge Impedance: $\mathbf{Z}_c := \sqrt{\frac{x}{y}}$ $\quad \mathbf{Z}_c = 353.6\cdot\Omega$ $\quad \frac{\mathbf{Z}_c}{Z_L} = 1.414$ SIL load

Not asked for in this class