Table A.4 Characteristics of aluminum cable, steel, reinforced (Aluminum Company of America).

<table>
<thead>
<tr>
<th>Code Word</th>
<th>Circular Mil</th>
<th>Copper</th>
<th>Aluminum</th>
<th>Steel</th>
<th>Copper Equivalent</th>
<th>Ultimate</th>
<th>Weight</th>
<th>Approx.</th>
<th>Geometric</th>
<th>50°C</th>
<th>77°F</th>
<th>100°F</th>
<th>122°F</th>
<th>158°F</th>
<th>200°F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>50°C × 10600 A/W</td>
<td>Strength</td>
<td>(Pounds</td>
<td>Current</td>
<td>Mean</td>
<td>(Amps)</td>
<td>(Amps)</td>
<td>(Amps)</td>
<td>(Amps)</td>
<td>(Amps)</td>
<td>(Amps)</td>
</tr>
<tr>
<td>25°C</td>
<td>75°C</td>
<td>80°C</td>
<td>75°C</td>
<td>80°C</td>
<td>75°C</td>
<td>80°C</td>
<td>75°C</td>
<td>80°C</td>
<td>75°C</td>
<td>80°C</td>
<td>75°C</td>
<td>80°C</td>
<td>75°C</td>
<td>80°C</td>
<td>75°C</td>
</tr>
<tr>
<td>1100</td>
<td>1200</td>
<td>1300</td>
<td>1400</td>
<td>1500</td>
<td>1600</td>
<td>1700</td>
<td>1800</td>
<td>1900</td>
<td>2000</td>
<td>2100</td>
<td>2200</td>
<td>2300</td>
<td>2400</td>
<td>2500</td>
<td>2600</td>
</tr>
<tr>
<td>75</td>
<td>80</td>
<td>75</td>
<td>80</td>
<td>75</td>
<td>80</td>
<td>75</td>
<td>80</td>
<td>75</td>
<td>80</td>
<td>75</td>
<td>80</td>
<td>75</td>
<td>80</td>
<td>75</td>
<td>80</td>
</tr>
<tr>
<td>1.75</td>
<td>2.00</td>
<td>2.25</td>
<td>2.50</td>
<td>2.75</td>
<td>3.00</td>
<td>3.25</td>
<td>3.50</td>
<td>3.75</td>
<td>4.00</td>
<td>4.25</td>
<td>4.50</td>
<td>4.75</td>
<td>5.00</td>
<td>5.25</td>
<td>5.50</td>
</tr>
</tbody>
</table>

*Based on copper 97%, aluminum 61% conductivity.

** Conduc. for 75°C, air at 25°C, wind 1 mile per hour (2.25 m/s), frequency = 60Hz.

Other conductor types include the all-aluminum conductor (AAC), all-aluminum-alloy conductor (AAAC), aluminum conductor alloy-reinforced (ACAR), and aluminum-clad steel conductor (Alumoweld). There is also a conductor known as "expanded ACSR," which has a filler such as fiber or paper between the aluminum and steel strands. The filler increases the conductor diameter, which reduces the electric field at the conductor surface, to control corona.

EHV lines often have more than one conductor per phase; these conductors are called a bundle. The 765-kV line in Figure 5.2 has four conductors per phase, and the 345-kV double-circuit line in Figure 5.3 has two conductors per phase. Bundle conductors have a lower electric field strength at the conductor surface, thereby controlling corona. They also have a smaller series reactance.

ECE 3600 Transmission Line notes
## Table 5.1  Typical transmission-line characteristics [1,2]

<table>
<thead>
<tr>
<th>NOMINAL VOLTAGE</th>
<th>NUMBER OF STRINGS PER PHASE</th>
<th>NUMBER OF STANDARD INSULATOR DISCS PER SUSPENSION STRING</th>
<th>TYPE</th>
<th>NUMBER</th>
<th>DIAMETER (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>69</td>
<td>1</td>
<td>4 to 6</td>
<td>Steel</td>
<td>0.1 or 2</td>
<td>-</td>
</tr>
<tr>
<td>138</td>
<td>1</td>
<td>8 to 11</td>
<td>Steel</td>
<td>0.1 or 2</td>
<td>-</td>
</tr>
<tr>
<td>230</td>
<td>1</td>
<td>12 to 21</td>
<td>Steel or ACSR</td>
<td>1 or 2</td>
<td>1.1 to 1.5</td>
</tr>
<tr>
<td>345</td>
<td>1</td>
<td>18 to 21</td>
<td>Alumoweld</td>
<td>2</td>
<td>0.87 to 1.5</td>
</tr>
<tr>
<td>345</td>
<td>1 and 2</td>
<td>18 to 21</td>
<td>Alumoweld</td>
<td>2</td>
<td>0.87 to 1.5</td>
</tr>
<tr>
<td>500</td>
<td>2 and 4</td>
<td>24 to 27</td>
<td>Alumoweld</td>
<td>2</td>
<td>0.98 to 1.5</td>
</tr>
<tr>
<td>500</td>
<td>2 and 4</td>
<td>24 to 27</td>
<td>Alumoweld</td>
<td>2</td>
<td>0.98 to 1.5</td>
</tr>
<tr>
<td>765</td>
<td>2 and 4</td>
<td>30 to 35</td>
<td>Alumoweld</td>
<td>2</td>
<td>0.98</td>
</tr>
</tbody>
</table>

**Figure 5.5**

Cut-away view of a standard insulator disc for suspension insulator strings.

(Courtesy of Ohio Brass)
Table 5.2
Comparison of SI and English units for calculating conductor resistance

<table>
<thead>
<tr>
<th>QUANTITY</th>
<th>SYMBOL</th>
<th>SI UNITS</th>
<th>ENGLISH UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistivity</td>
<td>( \rho )</td>
<td>( \Omega \text{m} )</td>
<td>( \Omega \text{ cmil/ft} )</td>
</tr>
<tr>
<td>Length</td>
<td>( l )</td>
<td>( \text{m} )</td>
<td>( \text{ft} )</td>
</tr>
<tr>
<td>Cross-sectional area</td>
<td>( A )</td>
<td>( \text{m}^2 )</td>
<td>( \text{cmil} )</td>
</tr>
<tr>
<td>dc resistance</td>
<td>( R_{dc} = \frac{\rho}{A} )</td>
<td>( \Omega )</td>
<td>( \Omega )</td>
</tr>
</tbody>
</table>

Resistivity depends on the conductor metal. Annealed copper is the international standard for measuring resistivity \( \rho \) (or conductivity \( \sigma \), where \( \sigma = \frac{1}{\rho} \)). Resistivity of conductor metals is listed in Table 5.3. As shown, hard-drawn aluminum, which has 61% of the conductivity of the international standard, has a resistivity at 20°C of 17.00 \( \Omega \text{-cmil/ft} \) or \( 2.83 \times 10^{-8} \Omega \text{m} \).

Table 5.3
% Conductivity, resistivity, and temperature constant of conductor metals

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>% CONDUCTIVITY</th>
<th>( \rho_{20\degree C} )</th>
<th>( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Omega \text{m} \times 10^{-8} )</td>
<td>( \Omega \text{-cmil/ft} )</td>
<td>°C</td>
</tr>
<tr>
<td>Copper:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annealed</td>
<td>100%</td>
<td>1.72</td>
<td>10.37</td>
</tr>
<tr>
<td>Hard-drawn</td>
<td>97.3%</td>
<td>1.77</td>
<td>10.66</td>
</tr>
<tr>
<td>Aluminum</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hard-drawn</td>
<td>61%</td>
<td>2.83</td>
<td>17.00</td>
</tr>
<tr>
<td>Brass</td>
<td>20–27%</td>
<td>6.4–8.4</td>
<td>38–51</td>
</tr>
<tr>
<td>Bronze</td>
<td>9–13%</td>
<td>13–18</td>
<td>78–108</td>
</tr>
<tr>
<td>Iron</td>
<td>17.2%</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>Silver</td>
<td>108%</td>
<td>1.59</td>
<td>9.6</td>
</tr>
<tr>
<td>Sodium</td>
<td>40%</td>
<td>4.3</td>
<td>26</td>
</tr>
<tr>
<td>Steel</td>
<td>2 to 14%</td>
<td>12 to 88</td>
<td>72–530</td>
</tr>
</tbody>
</table>

Conductor resistance depends on the following factors:
1. Spiraling + 1–2% resistance
2. Temperature
3. Frequency ("skin effect") \( \sim + 3\% \)
4. Current magnitude—magnetic conductors

Characteristic impedance

The characteristic impedance or surge impedance of a uniform transmission line, usually written \( Z_0 \), is the ratio of the amplitudes of a single pair of voltage and current waves propagating along the line in the absence of reflections. The SI unit of characteristic impedance is the ohm. The characteristic impedance of a lossless transmission line is purely real, that is, there is no imaginary component (\( Z_0 = |Z_0| + j0 \)). Characteristic impedance appears like a resistance in this case, such that power generated by a source on one end of an infinitely long lossless transmission line is dissipated through the line but is not dissipated in the line itself. A transmission line of finite length (lossless or lossy) that is terminated at one end with a resistor equal to the characteristic impedance (\( Z_L = Z_0 \)) appears like an infinitely long transmission line to the source.
Transmission line model

Applying the transmission line model based on the telegrapher's equations, the general expression for the characteristic impedance of a transmission line is:

\[ Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \]

where

- \( R \) is the resistance per unit length,
- \( L \) is the inductance per unit length,
- \( G \) is the conductance of the dielectric per unit length,
- \( C \) is the capacitance per unit length,
- \( j \) is the imaginary unit, and
- \( \omega \) is the angular frequency.

The voltage and current phasors on the line are related by the characteristic impedance as:

\[ \frac{V^+}{I^+} = Z_0 = -\frac{V^-}{I^-} \]

where the superscripts + and − represent forward- and backward-traveling waves, respectively.

Lossless line

For a lossless line, \( R \) and \( G \) are zero so the equation for characteristic impedance reduces to

\[ Z_0 = \sqrt{\frac{L}{C}} \]

Surge Impedance Loading

In electric power transmission, the characteristic impedance of a transmission line is expressed in terms of the **surge impedance loading (SIL)**, or natural loading, being the MW loading at which reactive power is neither produced nor absorbed:

\[ SIL = \left( \frac{V_{L-L}}{Z_0} \right)^2 = 3 \frac{V_{L}^2}{Z_0} = 3 \frac{V_{R}^2}{Z_0} \]

in which \( V_{L-L} \) is the line-to-line voltage in volts.

Loaded below its SIL, a line supplies lagging reactive power to the system, tending to raise system voltages. Above it, the line absorbs reactive power, tending to depress the voltage. The Ferranti effect describes the voltage gain towards the remote end of a very lightly loaded (or open ended) transmission line. Underground cables normally have a very low characteristic impedance, resulting in an SIL that is typically in excess of the thermal limit of the cable. Hence a cable is almost always a source of lagging reactive power.
### Table 4-1
Transmission Line Parameters with Bundled Conductors (except at 230 kV) at 60 Hz [2, 6]

<table>
<thead>
<tr>
<th>Nominal Voltage</th>
<th>$R(\Omega/km)$</th>
<th>$\omega L(\Omega/km)$</th>
<th>$\omega C(\mu \Omega/km)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>230 kV</td>
<td>0.055</td>
<td>0.489</td>
<td>3.373</td>
</tr>
<tr>
<td>345 kV</td>
<td>0.037</td>
<td>0.376</td>
<td>4.518</td>
</tr>
<tr>
<td>500 kV</td>
<td>0.029</td>
<td>0.326</td>
<td>5.220</td>
</tr>
<tr>
<td>765 kV</td>
<td>0.013</td>
<td>0.339</td>
<td>4.988</td>
</tr>
</tbody>
</table>

### Table 4-2
Surge Impedance and Three-Phase Surge Impedance Loading [2, 6]

<table>
<thead>
<tr>
<th>Nominal Voltage</th>
<th>$Z_s(\Omega)$</th>
<th>SIL (MW)</th>
<th>( I_{th}(A) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>230 kV</td>
<td>375</td>
<td>140 MW</td>
<td>350A</td>
</tr>
<tr>
<td>345 kV</td>
<td>280</td>
<td>425 MW</td>
<td>710A</td>
</tr>
<tr>
<td>500 kV</td>
<td>250</td>
<td>1000 MW</td>
<td>1160A</td>
</tr>
<tr>
<td>765 kV</td>
<td>255</td>
<td>2300 MW</td>
<td>1740A</td>
</tr>
</tbody>
</table>

### Table 4-3
Loadability of Transmission Lines [6]

<table>
<thead>
<tr>
<th>Line Length (km)</th>
<th>Limiting Factor</th>
<th>Multiple of SIL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short</td>
<td>0 - 80 km</td>
<td>Thermal</td>
</tr>
<tr>
<td>Medium</td>
<td>80 - 240 km</td>
<td>5% Voltage Drop</td>
</tr>
<tr>
<td>Long</td>
<td>240 - 480 km</td>
<td>Stability</td>
</tr>
</tbody>
</table>

Typical values for transmission lines taken from: *First Course on Power Systems* by Ned Mohan
Long-length Lines: over 240 km (150 miles) (over 200 mi in some texts)

Need:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line length</td>
<td>len, d m or km</td>
<td>stick to the same unit length for all parameters</td>
</tr>
<tr>
<td>Resistance per unit length</td>
<td>r Ω/m or Ω/km</td>
<td>miles may also be used</td>
</tr>
<tr>
<td>Inductance per unit length</td>
<td>l H/m or H/km</td>
<td>OR Inductive reactance per unit length: x Ω/m or Ω/km</td>
</tr>
<tr>
<td>Capacitance per unit length</td>
<td>c F/m or F/km</td>
<td>OR Capacitance admittance per unit length: y S/m or S/km</td>
</tr>
<tr>
<td>Conductance to ground</td>
<td>g S/m or S/km</td>
<td>Common assumption: g := 0 S/km S := siemens</td>
</tr>
</tbody>
</table>

Find:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Units</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge impedance:</td>
<td>Zc</td>
<td>Ω</td>
</tr>
<tr>
<td>Propagation constant:</td>
<td>γ</td>
<td>1/m or 1/km</td>
</tr>
<tr>
<td>Series impedance:</td>
<td>Z series</td>
<td>Ω</td>
</tr>
<tr>
<td>Shunt admittance:</td>
<td>Y shunt</td>
<td>S or 1/Ω</td>
</tr>
<tr>
<td>Shunt impedance:</td>
<td>2·Z shunt</td>
<td>S or 1/Ω</td>
</tr>
</tbody>
</table>

Model:

```
I_S       \\
V_S       \\
          2·Z_shunt
          \\
Z_series  \\
          \\
I_R       \\
V_R       \\
          2·Z_shunt
          \\
neutral  \\
          \\
2·Z_shunt
          \\
```

If your calculator doesn't have hyperbolic trig functions

```
\[ Z_{\text{series}} = Z_c \cdot \sinh(γ \cdot \text{len}) = Z_c \cdot \frac{e^{γ \cdot \text{len}} - e^{-γ \cdot \text{len}}}{2} \]
```

```
\[ Y_{\text{shunt}} = \frac{1}{Z_c} \cdot \tanh \left( \frac{γ \cdot \text{len}}{2} \right) = \frac{1}{Z_c} \cdot \frac{e^{γ \cdot \text{len}/2} - e^{-γ \cdot \text{len}/2}}{e^{γ \cdot \text{len}/2} + e^{-γ \cdot \text{len}/2}} = \frac{1}{Z_c} \cdot \frac{\sqrt{e^{γ \cdot \text{len}} - e^{-γ \cdot \text{len}}}}{\sqrt{e^{γ \cdot \text{len}} + e^{-γ \cdot \text{len}}}} \]
```

If your calculator can't handle complex exponents

```
e^{a+bj} = e^a \cdot e^{bj} = e^a / b \text{ (in radians)}
```
**Medium-length Lines:** 80 - 240 km (50 to 150 miles) (100 - 200 mi in some texts)

Need:
- Line length: \( l \) or \( d \) \( \text{m} \) or \( \text{km} \) stick to the same unit length for all parameters
- Resistance per unit length: \( r \) \( \frac{\Omega}{\text{m}} \) or \( \frac{\Omega}{\text{km}} \)
- Inductance per unit length: \( l \) \( \frac{H}{\text{m}} \) or \( \frac{H}{\text{km}} \) OR Inductive reactance per unit length: \( x \) \( \frac{\Omega}{\text{m}} \) or \( \frac{\Omega}{\text{km}} \)
- Capacitance per unit length: \( c \) \( \frac{F}{\text{m}} \) or \( \frac{F}{\text{km}} \) OR Capacitance admittance per unit length: \( y \) \( \frac{S}{\text{m}} \) or \( \frac{S}{\text{km}} \)
- Conductance to ground: \( g \) \( \frac{S}{\text{m}} \) or \( \frac{S}{\text{km}} \) Common assumption: \( g := 0 \cdot \frac{S}{\text{km}} \)

Find:
- Surge Impedance: \( Z_c = \frac{x}{y} \) Only needed if load is in terms of SIL
- Series Resistance: \( R_{\text{line}} = r \cdot l \) \( \Omega \)
- Series Impedance: \( Z_{\text{series}} = (r + jx) \cdot l \) \( \Omega \)
- Shunt admittance: \( Y_{\text{shunt}} = jy \cdot l \) \( \frac{S}{\text{m}} \) or \( \frac{1}{\Omega} \)
- OR
- Shunt Impedance: \( 2Z_{\text{shunt}} = \frac{2}{jy \cdot l} \) \( \Omega \)

**Short-length Lines:** less than 80km (50 mi) (less than 100 mi in some texts)

Same as above but without the capacitors

ECE 3600 Transmission Line notes p7
Ex1. A 500 kV transmission line is 500 km long and has the line parameters shown below. Use the long-length model to find $V_S$ and $I_S$ if the line is loaded to 900 MVA and $|V_{RLL}|$ is 490 kV. Assume the phase angle of $V_R$ is 0° and assume load $pf = 1$.

Long-length line model:

Surge Impedance: \[ Z_c := \frac{j \cdot x + r}{j \cdot y + g} \]
\[ Z_c = 250.151 - 11.104j \ \Omega \]

Propagation constant: \[ \gamma := \sqrt{(j \cdot x + r) \cdot (j \cdot y + g)} \]
\[ \gamma = 5.797 \times 10^{-5} + 1.306 \times 10^{-3}j \ \cdot \frac{1}{\text{km}} \]

Series impedance:
\[ Z_{series} = Z_c \cdot \sinh(\gamma \cdot \text{len}) \]
\[ Z_{series} = 12.508 + 151.772j \ \Omega \]

Shunt admittance:
\[ Y_{shunt} := \frac{2}{Z_c} \cdot \tanh\left(\frac{\gamma \cdot \text{len}}{2}\right) \]
\[ Y_{shunt} = 4.49 \times 10^{-6} + 1.353 \times 10^{-3}j \ \cdot S \]

Shunt impedance:
\[ Z_{shunt} := \frac{Z_c}{2 \cdot \tanh\left(\frac{\gamma \cdot \text{len}}{2}\right)} \]
\[ Z_{shunt} = 2.451 - 738.924j \ \Omega \]

Solve circuit:

\[ I_R := \sqrt{\frac{S}{1\phi} \cdot \frac{|V_R|}{Z_{series}}} \]
\[ I_R = 1060.4A \]
\[ V_R = 282.902 \text{ kV} \]
\[ I_{Zshunt} := \frac{V_R}{2 \cdot Z_{shunt}} \]
\[ I_{Zshunt} = 1.27 + 382.852j \ \cdot A \]

\[ I_L := I_{Zshunt} + I_R \]
\[ I_L = 1.062 \times 10^3 + 382.852j \ \cdot A \]

\[ V_S := V_R + I_L \cdot Z_{series} \]
\[ V_S = 2.381 \times 10^5 + 1.659 \times 10^5 j \ \cdot V \]
\[ |V_S| = 290.192 \text{ kV} \quad \arg(V_S) = 34.874 \text{ deg} \]

\[ I_{ZshuntS} := \frac{V_S}{2 \cdot Z_{shunt}} \]
\[ I_{ZshuntS} = -223.48 + 322.934j \ \cdot A \]
\[ |\sqrt{3} \cdot V_S| = 502.628 \text{ kV} \]

\[ I_S := I_{ZshuntS} + I_L \]
\[ I_S = 838.23 + 705.786j \ \cdot A \]
\[ |I_S| = 1096 \text{ A} \quad \arg(I_S) = 40.097 \text{ deg} \]
Ex 2. A 345 kV transmission line is 220 km long and has the line parameters shown below. Find \( V_S \) and \( I_S \) if the line is loaded to 800 MVA with \( \text{pf} = 91\% \) lagging. \( V_{RLL} \) is 510 kV. \( \text{pf} := 0.91 \)

\[
\begin{align*}
\text{len} & := 220\text{-km} \\
V_{RLL} & := 510\text{-kV} \\
V_R & := \frac{V_{RLL}}{\sqrt{3}} \\
\text{Assume the phase angle of } V_R & \text{ is } 0^\circ \text{ if } V_R \text{ is given} \\
r & := 0.037\frac{\Omega}{\text{km}} \\
x & := 0.376\frac{\Omega}{\text{km}} \\
y & := 4.518 \times 10^{-6}\frac{S}{\text{km}} \\
\text{Medium-length line model:} \\
\text{Series impedance:} & \quad Z_{\text{series}} := (r + j \times x) \cdot \text{len} \\
\text{Shunt admittance:} & \quad Y_{\text{shunt}} := j \times y \times \text{len} \\
& \quad \text{Not used in my solution} \\
\text{Shunt impedance:} & \quad Z_{\text{shunt}} := \frac{1}{j \times y \times \text{len}} \\
\end{align*}
\]

Solve circuit:

\[
\begin{align*}
V_S & := V_R + I_L Z_{\text{series}} \\
V_S & = 3.238 \times 10^5 + 7.275 \times 10^4 \text{j} \cdot \text{V} \\
|V_S| & = 331.918 \text{ kV} \quad \arg(V_S) = 12.66^\circ \text{deg} \\
\end{align*}
\]

Line voltage: \( |\sqrt{3} \cdot V_S| = 574.9 \text{ kV} \)

\[
\begin{align*}
\text{power angle} & = \delta = \arg(V_S) - \arg(V_R) = 12.66^\circ \text{deg} \\
\end{align*}
\]

\[
\begin{align*}
I_{\text{Zshunt}} & := \frac{V_R}{2 \times Z_{\text{shunt}}} \\
I_{\text{Zshunt}} & = 146.335j \cdot \text{A} \\
I_L & := I_{\text{Zshunt}} + I_R \\
I_L & = 905.647 - 266.29j \cdot \text{A} \\
V_S & := V_R + I_L Z_{\text{series}} \\
I_{\text{ZshuntS}} & := \frac{V_S}{2 \times Z_{\text{shunt}}} \\
I_{\text{ZshuntS}} & = -36.154 + 160.946j \cdot \text{A} \\
I_S & := I_{\text{ZshuntS}} + I_L \\
I_S & = 869.493 - 105.344j \cdot \text{A} \\
|I_S| & = 876 \cdot \text{A} \quad \arg(I_S) = -6.908^\circ \text{deg} \\
\end{align*}
\]
Ex3. A 230 kV transmission line has the following length and line parameters.

\[ \text{len} := 150 \text{ km} \]
\[ r := 0.06 \frac{\Omega}{\text{km}} \]
\[ x := 0.5 \frac{\Omega}{\text{km}} \]
\[ g := 0 \frac{S}{\text{km}} \]
\[ y := 4 \times 10^6 \frac{S}{\text{km}} \]

Medium-length line model:

Series impedance:
\[ Z_{\text{series}} := (r + j \cdot x) \cdot \text{len} \]
\[ Z_{\text{series}} = 9 + 75j \cdot \Omega \]

Shunt admittance:
\[ Y_{\text{shunt}} := j \cdot y \cdot \text{len} \]
\[ Y_{\text{shunt}} = 0.3j \cdot mS \]

Shunt impedance:
\[ Z_{\text{shunt}} := \frac{1}{j \cdot y \cdot \text{len}} \]
\[ 2 \cdot Z_{\text{shunt}} = -3.33j \cdot k\Omega \]

a) The load is 250 Ω with a power factor of 0.87, leading. Find the line current, \( I_{\text{Line}} \).

\[ V_S := \frac{230 \text{ kV}}{\sqrt{3}} \]
\[ V_S = 132.79 \text{ kV} \]

Assume the phase angle of \( V_S \) is 0° if \( V_S \) is given

\[ Z := Z_{\text{series}} + \frac{1}{Y_{\text{shunt}}} + \frac{1}{Z_L} \]
\[ Z = 210.467 - 56.544j \cdot \Omega = 219.7 \Omega / -15.04° \]

\[ I_{\text{Line}} := \frac{V_S}{Z} \]
\[ I_{\text{Line}} = 588.459 + 158.096j \cdot A = 609.3A / 15.04° \]

b) Find the load line voltage.

\[ I_{\text{Line}} \cdot Z_{\text{series}} = -6.561 + 45.557j \cdot kV \]
\[ V_R := V_S - I_{\text{Line}} \cdot Z_{\text{series}} \]
\[ V_R = 139.352 - 45.557j \cdot kV = 146.6kV / -18.1° \]

Receiving line voltage = \[ \sqrt{3} \cdot V_R \] = 253.9 kV

Notice that \( |V_R| \) is bigger than \( |V_S| \), this can happen when the receiving-end power factor is leading.

c) What is the "power angle" (\( \delta \))? \[ \delta = - \arg(V_R) = 18.104 \cdot \text{deg} \]

d) How much power is delivered to the load?

\[ I_R := \frac{|V_R|}{|Z_L|} \]
\[ P_L = 3 \cdot |V_R| \cdot I_R \cdot \text{pf} = 224.4 \cdot \text{MW} \]

Power estimate for the same \( |V_R| \) and \( |V_S| \), but neglecting the line resistance:
\[ \approx 3 \cdot |V_S| \cdot |V_R| \cdot \sin(18.1 \cdot \text{deg}) \cdot \frac{1}{|Z_{\text{series}}|} = 240 \cdot \text{MW} \]

e) Express this loading in terms of SIL.

Surge Impedance:
\[ Z_c := \frac{x}{\sqrt{y}} \]
\[ Z_c = 353.6 \Omega \]
\[ \frac{Z_c}{Z_L} = 1.414 \]

Not asked for in this class