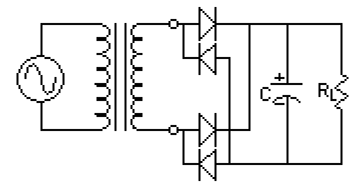
Transformer Examples

Ex.1 a) An ideal transformer has 360 turns on the primary winding and 36 turns on the secondary. If the primary is connected across a 120 V (rms) generator, what is the rms output voltage?

$$120 \cdot \text{volt} \cdot \frac{36}{360} = 12 \cdot \text{volt}$$

b) If you used a full-wave rectifier and a capacitor to make a DC power supply with this transformer, what DC voltage should you get?



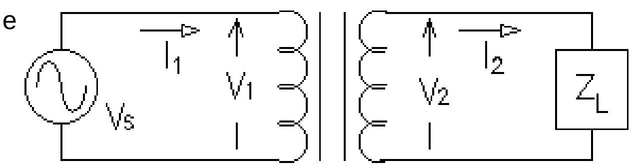
$$12 \cdot V \cdot \sqrt{2} - 2 \cdot 0.7 \cdot V = 15.6 \cdot V$$ less under load

peak 2 diodes

Ex.2 A transformer has $N_1 = 320$ turns and $N_2 = 1000$ turns. If the input voltage is $v(t) = (255\ V)\cos(\omega t)$, what rms voltage is developed across the secondary coil?

$$\frac{255 \cdot \text{volt}}{\sqrt{2}} \cdot \frac{1000}{320} = 563 \cdot \text{volt}$$

Ex.3 A transformer is rated at 480V / 120V, 1.2kVA. Assume the transformer is ideal and all voltages and currents are RMS.



a) What is the current rating of the primary?

$$\frac{1.2 \cdot kVA}{480 \cdot V} = 2.5 \cdot A$$

$|\mathbf{Z_L}| = 20 \cdot \Omega$

$pf := 75 \cdot \%$ lagging

$\mathbf{V_L} := 110 \cdot V$

b) What is the current rating of the secondary? $\frac{1.2 \cdot kVA}{120 \cdot V} = 10 \cdot A$

c) The secondary has 100 turns of wire. How many turns does the primary have?

$$N_2 := 100 \qquad N_1 := \frac{480 \cdot V}{120 \cdot V} \cdot N_2 \qquad N_1 = 400 \text{ turns}$$

d) $\mathbf{V_L} := 110 \cdot V$ How big is the source voltage ($|\mathbf{V_S}|$)?

$$\mathbf{V_S} := \frac{N_1}{N_2} \cdot \mathbf{V_L} \qquad \mathbf{V_S} = 440 \cdot V$$

e) The secondary load ($\mathbf{Z_L}$) has a magnitude of 20 Ω at a power factor of 75%. Find the secondary current, $\mathbf{I_2}$ (magnitude **and <u>angle</u>**).

$pf := 75 \cdot \%$

$$\mathbf{I_2} = \frac{\mathbf{V_L}}{20 \cdot \Omega} = 5.5 \cdot A \qquad pf = 0.75 \qquad \text{acos}(pf) = 41.4 \cdot \text{deg} \qquad \mathbf{I_2} = 5.5A\ \underline{/-41.4^o}$$

f) Find the primary current, $\mathbf{I_1}$ (magnitude **and <u>angle</u>**).

$$\mathbf{I_1} = \frac{100}{400} \cdot 5.5 \cdot A = 1.375 \cdot A \qquad \text{acos}(pf) = 41.4 \cdot \text{deg} \qquad \mathbf{I_1} = 1.375A\ \underline{/-41.4^o}$$

Transformer is ideal, so angle is exactly the same as the load.

g) How much average power does the load dissipate?

$$P_L = |\mathbf{V_2}| \cdot |\mathbf{I_2}| \cdot pf = 110 \cdot V \cdot 5.5 \cdot A \cdot 75 \cdot \% = 453.8 \cdot \text{watt}$$

h) How much average power does the power source ($\mathbf{V_S}$) supply?

$$P_S = P_L = 454 \cdot \text{watt}$$

i) What is the load as seen by $\mathbf{V_S}$? (magnitude **and <u>angle</u>**)

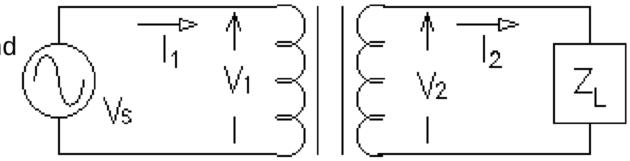
$$\left(\frac{400}{100}\right)^2 \cdot 20 \cdot \Omega = 320 \cdot \Omega \qquad \text{acos}(pf) = 41.4 \cdot \text{deg} \qquad \mathbf{Z_{eq}} = 320\Omega\ \underline{/41.4^o}$$

OR: $\frac{440 \cdot V}{1.375 \cdot A} = 320 \cdot \Omega\ \underline{/\ 0 - -41.4^o}$

Transformer Examples p2

Ex.4 A transformer is rated at 480V/240V, 1.2kVA.
Assume the transformer is ideal and all voltages and currents are RMS.

How much power does the load consume?



load

$|\mathbf{V_S}| = 440 \cdot V$ $\quad |\mathbf{Z_L}| = 16 \cdot \Omega$

$pf := 80 \cdot \%$ lagging

$\mathbf{V_L} := \mathbf{V_S} \cdot \left(\frac{240}{480}\right) \qquad |\mathbf{V_L}| = 220 \cdot V$

$I_2 := \frac{|\mathbf{V_L}|}{|\mathbf{Z_L}|} \qquad P_L := |\mathbf{V_L}| \cdot I_2 \cdot pf \qquad P_L = 2.42 \cdot kW$

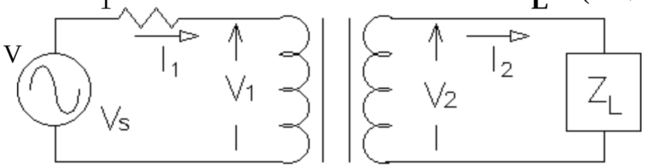
Ex.5 The transformer shown in the circuit below is ideal. It is rated at 220/110 V, 200 VA, 60 Hz

Find the following:

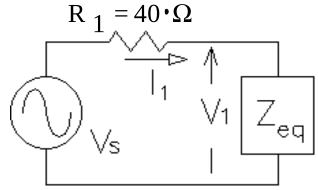
a) The primary current (magnitude).

$|\mathbf{I_1}| = ?$

$R_1 := 40 \cdot \Omega \qquad \mathbf{Z_L} := (15 + 20 \cdot j) \cdot \Omega \qquad V_S := 120 \cdot V$



$R_1 = 40 \cdot \Omega$



$\mathbf{Z_{eq}} = \left(\frac{N_1}{N_2}\right)^2 \cdot \mathbf{Z_L}$

$\mathbf{Z_{eq}} := \left(\frac{220}{110}\right)^2 \cdot \mathbf{Z_L}$

$\mathbf{Z_{eq}} = 60 + 80j \cdot \Omega$

$R_1 + \mathbf{Z_{eq}} = 100 + 80j \cdot \Omega$

$I_1 := \frac{V_S}{\sqrt{100^2 + 80^2} \cdot \Omega} \qquad I_1 = 0.937 \cdot A$

b) The primary voltage (magnitude).

$|\mathbf{V_1}| = ?$

$V_1 := I_1 \cdot \sqrt{60^2 + 80^2} \cdot \Omega \qquad V_1 = 93.7 \cdot V$

c) The secondary voltage (magnitude).

$|\mathbf{V_2}| = ?$

$V_2 = \frac{110}{220} \cdot V_1 = 46.85 \cdot V$

d) The power supplied by the source.

$P_S = ?$

$P_S = I_1^2 \cdot 100 \cdot \Omega = 87.8 \cdot W$

e) Is this transformer operating within its ratings? Show your evidence.

$I_{1max} = \frac{200 \cdot VA}{220 \cdot V} = 0.909 \cdot A \quad < \quad I_1 = 0.937 \cdot A$ ALWAYS CHECK CURRENT

NO

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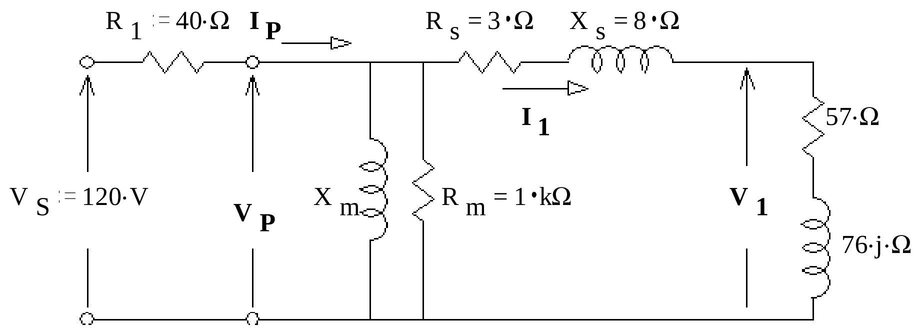
Ex.6 Repeat Ex.5 with a non-ideal transformer whose characteristics are shown below.

$R_m := 1\cdot k\Omega$ $\quad X_m := 400\cdot\Omega$ $\quad R_s := 3\cdot\Omega$ $\quad X_s := 8\cdot\Omega$ $\quad N := 1.95$



$$\mathbf{Z_{eq}} := (1.95)^2\cdot(15+20\cdot j)\cdot\Omega$$

$$\mathbf{Z_{eq}} = 57.037 + 76.05j \cdot\Omega$$

Find the following:

a) The primary current (magnitude).

$|\mathbf{I_P}| = ?$

$$R_s + X_s\cdot j + \mathbf{Z_{eq}} = 60.037 + 84.05j \cdot\Omega$$

$$\frac{1}{\dfrac{1}{X_m\cdot j} + \dfrac{1}{R_m} + \dfrac{1}{(60+84\cdot j)\cdot\Omega}} = 43.689 + 68.412j \cdot\Omega$$

$$R_1 + (43.689 + 68.412\cdot j)\cdot\Omega = 83.689 + 68.412j \cdot\Omega$$

$$\mathbf{I_P} := \frac{120\cdot V}{(83.689+68.412\cdot j)\cdot\Omega} \qquad \mathbf{I_P} = 0.86 - 0.703j \cdot A$$

$$|\mathbf{I_P}| = 1.11\cdot A$$

b) The primary voltage (magnitude).

$|\mathbf{V_P}| = ?$

$$\mathbf{V_P} := \mathbf{I_P}\cdot(43.689 + 68.412\cdot j)\cdot\Omega \qquad \mathbf{V_P} = 85.619 + 28.105j \cdot V$$

$$|\mathbf{V_P}| = 90.114\cdot V$$

c) The secondary voltage (magnitude).

$|\mathbf{V_2}| = ?$

$$\mathbf{I_1} := \frac{\mathbf{V_P}}{(60+84\cdot j)\cdot\Omega} \qquad \mathbf{I_1} = 0.704 - 0.517j \cdot A$$

$$\mathbf{V_1} := \mathbf{I_1}\cdot(57+76\cdot j)\cdot\Omega \qquad \mathbf{V_1} = 79.375 + 24.026j \cdot V$$

$$|\mathbf{V_1}| = 82.931\cdot V \qquad |\mathbf{V_2}| = \frac{1}{N}\cdot 82.931\cdot V = 42.529\cdot V$$

OR, simply:

$$I_1 := \frac{90.114\cdot V}{\sqrt{60^2+84^2}\cdot\Omega} \qquad I_1 = 0.873\cdot A \qquad V_2 = \frac{I_1\cdot\sqrt{57^2+76^2}\cdot\Omega}{1.95} = 42.529\cdot V$$

d) The power supplied by the source.

$P_S = ?$

$$P_S = 120\cdot V\cdot \mathrm{Re}(\mathbf{I_P}) = 103.14\cdot W$$

e) Is this transformer operating within its ratings? Show your evidence.

$$I_{2max} = \frac{200\cdot VA}{110\cdot V} = 1.818\cdot A \quad > \quad |\mathbf{I_1}|\cdot N = 1.702\cdot A$$

YES

f) Find the efficiency, assuming that the only useful output is from $\mathbf{Z_L}$.

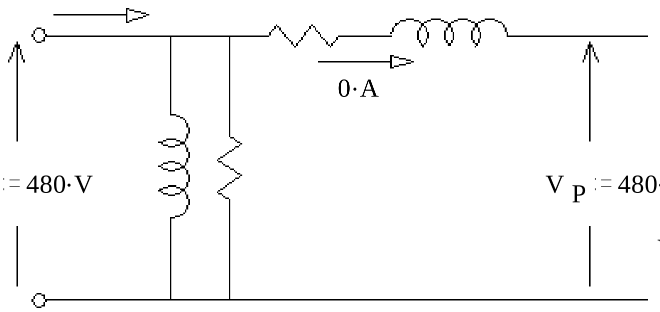
$$\eta = \frac{(|\mathbf{I_1}|)^2\cdot 57\cdot\Omega}{103.14\cdot W}\cdot 100\cdot\% = 42.12\cdot\%$$

Ex.7 Find the voltage regulation and full-load efficiency of the transformer with the following ratings and characteristics.

Rated at 480/120 V, 2 kVA, 60 Hz $R_m := 8.4\cdot k\Omega$ $X_m := 2\cdot k\Omega$ $R_s := 5\cdot\Omega$ $X_s := 15\cdot\Omega$

$V_{Prated} := 480\cdot V$ $V_{Srated} := 120\cdot V$ $S_{rated} := 2000\cdot VA$

No load:



$0\cdot A$

$V_S := 480\cdot V$ $V_P := 480\cdot V$ $V_2 := 120\cdot V$

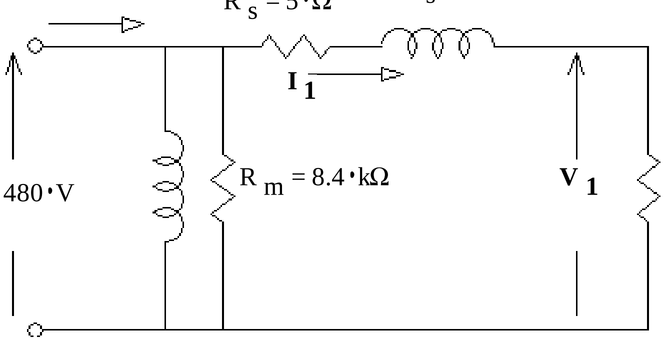
$V_{NL1} := 480$

Full load: $R_L := \frac{(120\cdot V)^2}{2000\cdot W}$ $R_L = 7.2\cdot\Omega$

Find the actual turns ratio: (See page 3 of notes)

$R_x := \frac{V_{Prated}^2}{S_{rated}} - 2\cdot R_s$ $R_{eq} := \frac{R_x + \sqrt{R_x^2 - 4\cdot\left(R_s^2 + X_s^2\right)}}{2}$ Turns ratio $N := \sqrt{\frac{R_{eq}}{R_L}}$ $N = 3.778$

$R_s = 5\cdot\Omega$ $j\cdot X_s = 15j\cdot\Omega$



$\mathbf{I_1}$

$V_S = 480\cdot V$ $R_m = 8.4\cdot k\Omega$ $\mathbf{V_1}$ $R_{eq} = 102.767\cdot\Omega$

$N^2\cdot R_L = 102.767\cdot\Omega$

NOT: $\left(\frac{480}{120}\right)^2\cdot R_L = 115.2\cdot\Omega$

$\mathbf{I_1} := \frac{V_S}{R_{eq} + R_s + 15j\cdot\Omega}$ $\mathbf{I_1} = 4.369 - 0.608j\ \cdot A$

$\mathbf{V_1} := \mathbf{I_1}\cdot R_{eq}$ $\mathbf{V_1} = 449.031 - 62.5j\ \cdot V$

$V_{FL1} := \left|\mathbf{V_1}\right|$ $V_{FL1} = 453.359\cdot V$

OR, simply:

$I_1 := \frac{V_S}{\sqrt{\left(R_{eq} + R_s\right)^2 + X_s^2}}$ $I_1 = 4.412\cdot A$

$V_{FL1} = I_1\cdot R_{eq} = 453.359\cdot V$

Voltage regulation: $\%VR = \frac{V_{no\_load} - V_{full\_load}}{V_{full\_load}}\cdot 100\cdot\% = \frac{V_S - V_{FL1}}{V_{FL1}}\cdot 100\cdot\% = 5.876\cdot\%$

Efficiency $\eta = \frac{P_{out}}{P_{in}}\cdot 100\cdot\% = \frac{P_{out}}{P_{out} + P_{losses}}\cdot 100\cdot\%$

$P_{out} := I_1^2\cdot R_{eq}$ $P_{out} = 2\cdot kW$

$P_{losses} := I_1^2\cdot 5.\cdot\Omega + \frac{(480\cdot V)^2}{8400\cdot\Omega}$ $P_{losses} = 0.125\cdot kW$

$\eta = \frac{P_{out}}{P_{out} + P_{losses}}\cdot 100\cdot\% = 94.129\cdot\%$

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Ex.8 A 500/100-V, 2.5-kVA transformer is subjected to an OC test and a SC test with the results below.

a) Draw a model of this transformer and find the values of all the elements of the model, including the turns ratio.

During the open-circuit test: $I_{OC} := 0.5\cdot A$ $\quad P_{OC} := 150\cdot W$ $\quad V_{sec} := 108\cdot V$

During the short-circuit test: $V_{SC} := 22\cdot V$ $\quad P_{SC} := 70\cdot W$

$S_{rated} := 2.5\cdot kVA$ $\quad V_{Prated} := 500\cdot V$ $\quad I_{Prated} := \dfrac{S_{rated}}{V_{Prated}}$ $\quad I_{Prated} = 5\cdot A$

Open-circuit test used to find R_m and X_m and turns ratio.

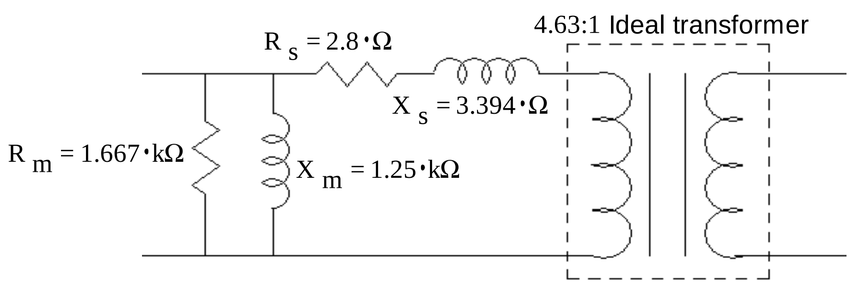
$V_{OC} := V_{Prated}$ $\quad R_m := \dfrac{V_{OC}^2}{P_{OC}}$ $\quad R_m = 1.667\cdot k\Omega$

$Q_{OC} := \sqrt{(V_{OC}\cdot I_{OC})^2 - P_{OC}^2}$ $\quad Q_{OC} = 200\cdot VAR$ $\quad X_m := \dfrac{V_{OC}^2}{Q_{OC}}$ $\quad X_m = 1.25\cdot k\Omega$

Turns ratio: $N := \dfrac{V_{Prated}}{V_{sec}}$ $\quad N = 4.63$

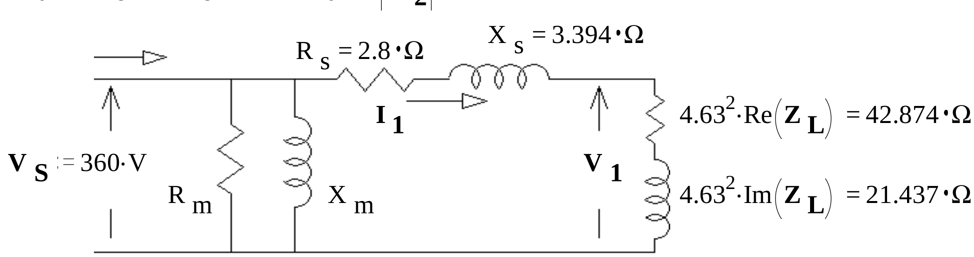
Short-circuit test used to find R_s and X_s $\quad I_{SC} := I_{Prated}$ $\quad R_s := \dfrac{P_{SC}}{I_{SC}^2}$ $\quad R_s = 2.8\cdot\Omega$

$Q_{SC} := \sqrt{(V_{SC}\cdot I_{SC})^2 - P_{SC}^2}$ $\quad Q_{SC} = 0.085\cdot kVAR$ $\quad X_s := \dfrac{Q_{SC}}{I_{SC}^2}$ $\quad X_s = 3.394\cdot\Omega$



b) The transformer is connected to a primary source voltage of 360V and loaded with $\mathbf{Z_L} := (2 + 1\cdot j)\cdot\Omega$

Find the secondary voltage. Magnitude only. $|\mathbf{V_2}|$ = ?



$$|\mathbf{V_1}| = V_1 := \mathbf{V_S}\cdot\frac{\sqrt{(42.874\cdot\Omega)^2 + (21.437\cdot\Omega)^2}}{\sqrt{(R_s + 42.874\cdot\Omega)^2 + (X_s + 21.437\cdot\Omega)^2}} \qquad V_1 = 331.935\cdot V$$

$$|\mathbf{V_2}| = V_2 = \frac{V_1}{4.63} = 71.69\cdot V$$

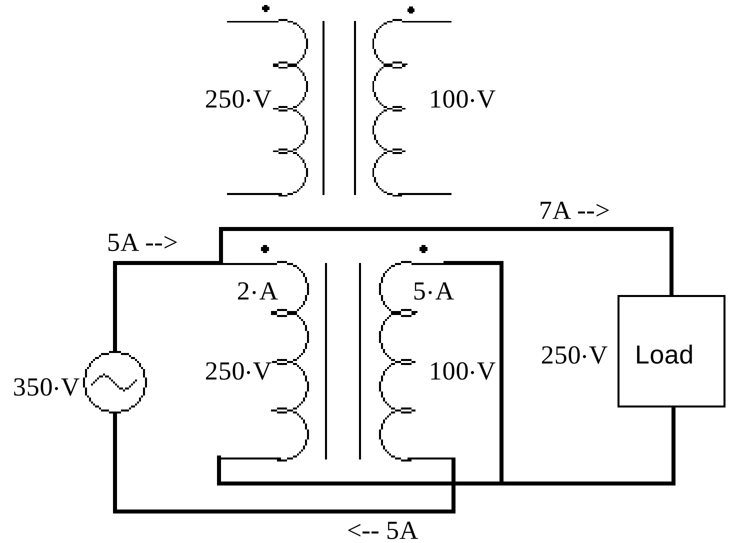
c) Is this transformer operating within its ratings? Show all evidence and calculate needed to to determine this.

$$|\mathbf{I_2}| = I_2 := \frac{\mathbf{V_S}}{\sqrt{(R_s + 42.874\cdot\Omega)^2 + (X_s + 21.437\cdot\Omega)^2}}\cdot N \qquad I_2 = 32.059\cdot A > I_{Srated} = \frac{S_{rated}}{100\cdot V} = 25\cdot A$$

NO !

Ex.9 You have a 250/100-V, 500-VA transformer.

a) Show the necessary connections to use this transformer to transform 350 V to 250 V. Also show the 350-V source and the load.



b) Connected this way, determine the maximum power that could be converted from 350 V to 250 V without overloading the transformer.

ratings: $\dfrac{500\cdot VA}{250\cdot V} = 2\cdot A \qquad \dfrac{500\cdot VA}{100\cdot V} = 5\cdot A$

new VA rating and maximum power: $5\cdot A\cdot 350\cdot V = 1.75\cdot kVA$

OR: $7\cdot A\cdot 250\cdot V = 1.75\cdot kVA$

$1.75\cdot kW$

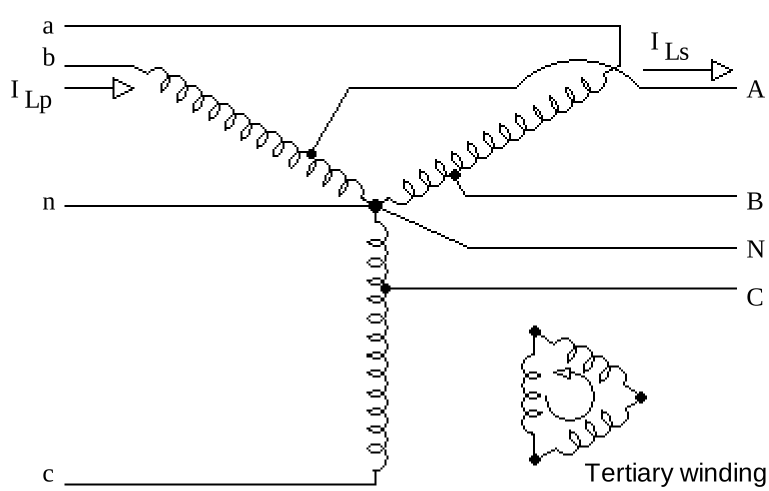
c) Besides the right impedance magnitude, what other characteristic must the load posses in order to actually use this much power?

Load must be purely resistive (power factor is 1).

d) Could this transformer also be used to transform 280 V to 200 V ? If yes, what is the maximum power that could be transformed?

Same connections as above Maximum power: $5\cdot A\cdot 280\cdot V = 1.4\cdot kW$

Ex.10 A 345kV/138kV, 750-MVA transformer is shown.



a) What is the purpose of the tertiary winding?

To allow 3rd harmonic currents to flow without affecting currents outside the transformer.

b) Find the maximum I_{Lp} and I_{Ls}.

$$I_{Lp.rated} := \frac{\left(\frac{750\cdot MVA}{3}\right)}{\left(\frac{345\cdot kV}{\sqrt{3}}\right)} \qquad I_{Lp.rated} = 1255\cdot A$$

$$I_{Ls.rated} := \frac{345}{138}\cdot I_{Lp.rated} \qquad I_{Ls.rated} = 3138\cdot A$$

c) Find the currents flowing in the transformer when operated at rated capacity.

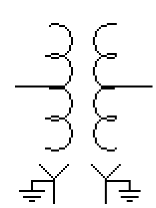
Current from primary terminal to the tap: $I_p = I_{Lp.rated} = 1255\cdot A$

Current from neutral to the tap: $I_p = I_{Ls.rated} - I_{Lp.rated} = 1883\cdot A$

Current from tap to secondary ouput of the transformer: $I_s = I_{Ls.rated} = 3138\cdot A$

d) At what fraction of the total turns is the tap located? $\dfrac{138}{345} = 0.4 = \dfrac{4}{10}$ OR at 40%

e) What one-line symbol would be used for this transformer?



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