Think for a moment about systems which have transformers and impedances. We've learned that transformers don't just transform voltages and currents-- they also transform impedances (by the turns ratio squared). When there are impedances on both sides of a transformer, it's usually easier to transform impedances in such a way that the ideal part of the transformer can be eliminated from the calculations. Otherwise, you'll have to solve multiple equations simultaneously.

But, there is another way...

We could express our voltages as percentages of the transformer ratings instead of their actual values. As an example, what if the primary voltage was 90\% of the rated primary voltage, then the secondary would also be 90\% of the rated secondary voltage.

\[
I_1 \, V_1 = 90\% \cdot V_{\text{rated}} = V_{\text{pu}} \cdot V_{\text{rated}}
\]

\[
I_2 \, V_2 = 90\% \cdot V_{\text{rated}} = V_{\text{pu}} \cdot V_{\text{rated}}
\]

Let's find the rated currents so that the currents can also be expressed as percentages of rated values.

\[
I_{1\text{rated}} = \frac{S_{\text{rated}}}{V_{1\text{rated}}} \quad I_{2\text{rated}} = \frac{S_{\text{rated}}}{V_{2\text{rated}}}
\]

And a "rated" impedance

\[
Z_{2\text{rated}} = \frac{V_{2\text{rated}}}{I_{2\text{rated}}} = \frac{V_{2\text{rated}}^2}{S_{\text{rated}}}
\]

If this "rated" impedance hooked to the secondary, then the rated voltage would make the rated current flow.

So let's say the actual \( Z_2 \) is 50\% greater than the "rated" impedance

\[
Z_2 = 150\% \cdot Z_{2\text{rated}} = Z_{\text{pu}} \cdot Z_{2\text{rated}}
\]

We could now calculate the current as a percentage of the rated current

\[
I_{2} = 60\% \cdot I_{2\text{rated}} = 60\% \cdot I_{2\text{rated}}
\]

And the primary current is also 60\% of the rated primary current

\[
I_{1} = 60\% \cdot I_{1\text{rated}} = I_{\text{pu}} \cdot I_{1\text{rated}}
\]

And the upshot is, if we work with these percentages rather than the actual values, the percentages are the same on both sides of the transformer and the transformer "disappears". Notice that the percentages are the \( V_{\text{pu}}, I_{\text{pu}} \) and \( Z_{\text{pu}} \) values. These "per unit" values may be complex.

I wouldn't recommend using this method for a circuit with a single transformer. But, in circuits with many transformers, like the power grid, this method becomes very... powerful. There are a few differences:

1. Use "base" values rather than a single transformer's ratings, although \( V_{\text{base}} \) values almost always match the transformer ratings.

2. The per-unit method is almost exclusively used in 3-phase systems, so a few of the base relationships include

\[
3 \quad \text{and} \quad \sqrt{3}.
\]
Base Values \( S_{\text{base}}, V_{\text{base}}, I_{\text{base}}, \text{and } Z_{\text{base}} \)

At least two base values must be specified in order to find all the other base values. The power base \( (S_{\text{base}}) \) is the most universal base value since it isn’t changed by transformers-- it’s the same across the entire system. Usually a power company will use a nice round number, like 10MVA or 100MVA. The second most common base is the voltage \( (V_{\text{base}}) \) which will, of course, change at each transformer. \( I_{\text{base}} \) and \( Z_{\text{base}} \) are calculated from the first two and also change at each transformer. All these base values only need to be calculated once for a given \( S_{\text{base}} \) and system configuration. Once the base values are established, all load and power-flow calculations can be made much more easily and quickly.

Per unit analysis is usually done on 3-phase systems and then on a per-phase basis, BUT, \( S_{\text{base}} \) and \( V_{\text{base}} \) are 3-phase and line-to-line respectively, so calculations of \( I_{\text{base}} \) and \( Z_{\text{base}} \) will need to take that into account. \( I_{\text{base}} \) is line current and \( Z_{\text{base}} \) is for a Y-connected load. Other bases are sometimes defined, but it should be clear that they are not the normal bases of a 3-phase system.

If per-unit values are given by the manufacturer of a generator, transformer or other device, then the manufacturer will use the device ratings as base values. For those devices, the per-unit impedances will have to be converted from the device’s \( S_{\text{rated}} \) (or \( S_{\text{base}} \)) to the \( S_{\text{base}} \) of the system.

Starting bases

\[
S_{\text{base}} = \text{The 3-phase power base of the system, usually a nice round number, like 10MVA or 100MVA} \\
P_{\text{base}} = Q_{\text{base}} = S_{\text{base}} \\
V_{\text{base}} = \text{The nominal } V_L (V_{LL}) \text{ in each region of the power system, where regions are regions are separated by transformers} \\
V_{\text{base}} = V_L = V_{LL} \text{ nominal in each region}
\]

Finding the other bases

\[
I_{\text{base}} = \left( \frac{S_{\text{base}}}{3} \right)^{1/3} \sqrt{3} V_{\text{base}}^{-1/3} = \frac{S_{\text{base}}}{3 \sqrt[3]{3} V_{\text{base}}} = \text{The base for line current = current in one phase of a Y-connected load or device.} \\
R_{\text{base}} = X_{\text{base}} = Z_{\text{base}} \\
I_{\text{base}} \text{ most / common way to calculate} \\
Z_{\text{base}} = \left( \frac{V_{\text{base}}}{3} \right)^{1/3} \frac{1}{I_{\text{base}}} \sqrt{3} = \frac{V_{\text{base}}^{2/3}}{S_{\text{base}}^{1/3}} = \frac{V_{\text{base}}^{2}}{S_{\text{base}}} \text{ / most / common way to calculate} \\
R_{\text{base}} = X_{\text{base}} = Z_{\text{base}} \\
Z_{\text{base}} \text{ most / common way to calculate}
\]

It is possible (but not recommended) to define the following: \( S_{1\phi_{\text{base}}} = \frac{S_{\text{base}}}{3} \) and \( V_{\text{LN}_{\text{base}}} = \frac{V_{\text{base}}}{\sqrt{3}} \)

In which case: \( I_{\text{base}} = \frac{S_{1\phi_{\text{base}}}}{V_{\text{LN}_{\text{base}}}} \) and \( Z_{\text{base}} = \frac{V_{\text{LN}_{\text{base}}}}{I_{\text{base}}} = \frac{V_{\text{LN}_{\text{base}}}}{S_{1\phi_{\text{base}}}} \)

Base changes

Per-unit impedances given by a manufacturer of a generator, transformer or other device must be converted from the device ratings \( (S_{\text{rated}} \text{ and } V_{\text{rated}}) \) to the system base values \( (S_{\text{base}} \text{ and } V_{\text{base}}) \).

\[
Z_{\text{pu}} = Z_{\text{pu}_{\text{device}}} \frac{S_{\text{base}}}{S_{\text{rated}}} \left( \frac{V_{\text{rated}}}{V_{\text{base}}} \right)^2 \text{ OR, more commonly} \quad Z_{\text{pu}} = Z_{\text{pu}_{\text{device}}} \frac{S_{\text{base}}}{S_{\text{rated}}} \left( \frac{V_{\text{base}}}{V_{\text{rated}}} \right)^2
\]

When device \( V_{\text{rated}} = V_{\text{base}} \)
Expressing values as "per-unit", pu

(Multiply by 100% to express as %, otherwise, use "pu" as the units)

\[ S_{pu} = \frac{S_{3\phi}}{S_{base}} \]
\[ P_{pu} = \frac{P_{3\phi}}{S_{base}} \]
\[ Q_{pu} = \frac{Q_{3\phi}}{S_{base}} \]
\[ V_{pu} = \frac{V_{L}}{V_{base}} \]
\[ I_{pu} = \frac{I_{L}}{I_{base}} \]

resistance \( R_{pu} = \frac{R}{Z_{base}} \)
conductance \( G = \frac{1}{R} \)
\( G_{pu} = \frac{1}{R_{pu}} \)

reactance \( X_{pu} = \frac{X}{Z_{base}} \)
susceptance \( B = \frac{1}{X} \)
\( B_{pu} = \frac{1}{X_{pu}} \)

impedance \( Z_{pu} = \frac{Z}{Z_{base}} \)
admittance \( Y = \frac{1}{Z} \)
\( Y_{pu} = \frac{1}{Z_{pu}} \)

common in power-flow calculations

The \( V_{pu}, I_{pu}, Z_{pu} \) and \( S_{pu} \) values are not affected by transformers.

The voltage, current and impedance bases WILL change at each transformer. The power base will NOT.

Please note that:

\[ S_{pu} = \frac{S_{3\phi}}{S_{base}} = \frac{S_{1\phi}}{S_{1\phi_{base}}} \]
\[ V_{pu} = \frac{V_{L}}{V_{base}} = \frac{V_{LN}}{V_{LN_{base}}} \]

so these calculations (and the respective bases) are rarely needed or wanted

Example

a) Find the bases for a simple system with two transformers.

Since this is a very small, self-contained system, let's use an unusually small \( S_{base} := 18\text{-kVA} \)

Transmission line length \( \text{len} := 50\text{-km} \)

\[
\begin{align*}
1.8\text{kV} / 4\text{kV} & \quad 4\text{kV} / 500\text{V} \\
18\text{kVA} & \quad 15\text{kVA} \\
Z_{pu,T1} := 0.1j\text{pu} & \quad Z_{pu,T2} := 9j\% \\
r := 0.3\Omega \text{ km} & \quad \omega l := 0.8\Omega \text{ km} \\
\end{align*}
\]

Generator ratings
18-kVA
1.8-kV
\( Z_{pu,G} := 1.5j\% \)
actual voltage
1732-V

\[
\begin{align*}
\text{Region 1} & \quad V_{base1} := 1.8\text{-kV} \\
\text{Region 2} & \quad V_{base2} := 4\text{-kV} \\
\text{Region 3} & \quad V_{base3} := 500\text{-V} \\
\end{align*}
\]

Set the \( V_{base} \) values in each region by looking at the transformer ratings.
b) Find the other bases.

Region 1

\[ I_{\text{base}1} = \frac{S_{\text{base}}}{\sqrt{3}V_{\text{base}1}} \]

\[ I_{\text{base}1} = 5.774 \cdot \text{A} \]

\[ Z_{\text{base}1} = \frac{V_{\text{base}1}}{I_{\text{base}1}} = \frac{180}{5.774} \cdot \Omega \]

Region 2

\[ I_{\text{base}2} = \frac{S_{\text{base}}}{\sqrt{3}V_{\text{base}2}} \]

\[ I_{\text{base}2} = 2.598 \cdot \text{A} \]

\[ Z_{\text{base}2} = \frac{V_{\text{base}2}}{S_{\text{base}}} = \frac{888.889}{2.598} \cdot \Omega \]

Region 3

\[ I_{\text{base}3} = \frac{S_{\text{base}}}{\sqrt{3}V_{\text{base}3}} \]

\[ I_{\text{base}3} = 20.785 \cdot \text{A} \]

\[ Z_{\text{base}3} = \frac{V_{\text{base}3}}{S_{\text{base}}} = \frac{13.889}{20.785} \cdot \Omega \]

c) Find all the impedances in per-unit form. Make base changes as necessary.

No changes necessary

Transmission line

\[ Z_{\text{pu.G}} = 0.015j \cdot \text{pu} \]

\[ Z_{\text{pu.TL}} = 0.1j \cdot \text{pu} \]

\[ Z_{\text{pu.TL}} = 1.687 + 4.5j \cdot \% \]

Motor

\[ S_{\text{rated.M}} = 12 \cdot \text{kVA} \]

\[ Z_{\text{pu.M}} = 0.958 + 0.2j \cdot \text{pu} \]

\[ Z_{\text{pu.M}} = 1.655 + 0.346j \cdot \text{pu} \]

at full load

ALL calculations made to this point ONLY need to be made ONCE!!

With the exception of the equivalent resistance of the motor. That will depend on the mechanical load placed on the motor.

d) Make a per-phase drawing that could be used to make current calculations.

\[ Z_{\text{pu.T1}} = 0.1j \cdot \text{pu} \]

\[ Z_{\text{pu.TL}} = 1.687 + 4.5j \cdot \% \]

\[ Z_{\text{pu.T2}} = 0.108j \cdot \text{pu} \]

\[ Z_{\text{pu.G}} = 1.5j \cdot \% \]

\[ \frac{1732 \cdot V}{V_{\text{base1}}} = 0.962 \cdot \text{pu} \]

\[ 0.24j \cdot \text{pu} \]

\[ 1.15 \cdot \text{pu} \]

Will change as the motor load changes

e) Calculate the currents and motor power when the motor is under full load.

The per-unit current:

\[ I_{\text{pu}} = \frac{0.962 \cdot \text{pu}}{\sqrt{(0.017 + 1.15)^2 + (0.015 + 0.1 + 0.045 + 0.108 + 0.24)^2}} \]

\[ I_{\text{pu}} = 0.756 \cdot \text{pu} \]

Region 1

\[ I_{L1} = I_{\text{pu}} \cdot I_{\text{base1}} = 4.364 \cdot \text{A} \]

Region 2

\[ I_{L2} = I_{\text{pu}} \cdot I_{\text{base2}} = 1.964 \cdot \text{A} \]

Region 3

\[ I_{L3} = I_{\text{pu}} \cdot I_{\text{base3}} = 15.71 \cdot \text{A} \]

Motor Power

\[ P_{\text{pu}} = I_{\text{pu}}^2 \cdot 1.15 \]

\[ P_{\text{pu}} = 0.657 \cdot \text{pu} \]

\[ P_{\text{M}} = 11.825 \cdot \text{kW} \]

A little less than the full load of 12kW because the voltage is a little low.

Notice that once the bases and per-unit impedances are established, the actual calculations are very easy!

Note: In these notes, I have retained "pu" in the subscripts of my variables as well as using "pu" or % as a unit of the number. If this seems redundant to you, it is. Most texts drop the "pu" in the variable subscripts.