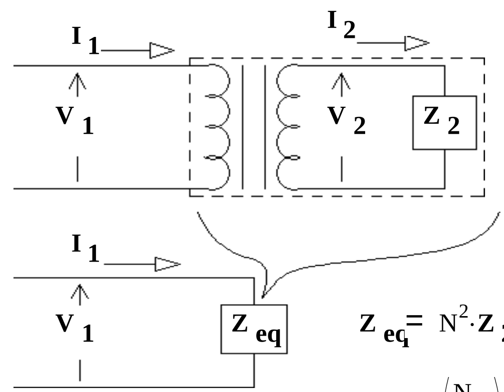
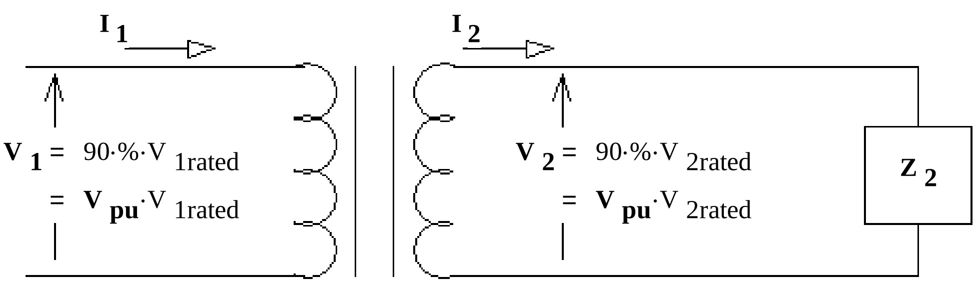
ECE 3600 Per-Unit Notes

Think for a moment about systems which have transformers and impedances. We've learned that transformers don't just transform voltages and currents-- they also transform impedances (by the turns ratio squared). When there are impedances on both sides of a transformer, it's usually easier to transform impedances in such a way that the ideal part of the transformer can be eliminated from the calculations. Otherwise, you'll have to solve multiple equations simultaneously.



$$\mathbf{Z}_{eq} = N^2 \cdot \mathbf{Z}_2 = \left(\frac{N_1}{N_2}\right)^2 \cdot \mathbf{Z}_2$$

But, there is another way...
We could express our voltages as percentages of the transformer ratings instead of their actual values. As an example, what if the primary voltage was 90% of the rated primary voltage, then the secondary would also be 90% of the rated secondary voltage.



Let's find the rated currents so that the currents can also be expressed as percentages of rated values.

$$I_{1rated} = \frac{S_{rated}}{V_{1rated}} \qquad I_{2rated} = \frac{S_{rated}}{V_{2rated}}$$

And a "rated" impedance $Z_{2rated} = \frac{V_{2rated}}{I_{2rated}} = \frac{V_{2rated}^{\,2}}{S_{rated}}$

If this "rated" impedance hooked to the secondary, then the rated voltage would make the rated current flow.

So let's say the actual $\mathbf{Z}_2$ is 50% greater than the "rated" impedance $\mathbf{Z}_2 = 150\cdot\%\cdot Z_{2rated} = \mathbf{Z}_{pu}\cdot Z_{2rated}$

We could now calculate the current as a percentage of the rated current $\frac{\mathbf{V}_{pu}}{\mathbf{Z}_{pu}} = \frac{90\cdot\%}{150\cdot\%} = 60\cdot\% = \mathbf{I}_{pu}$

$\mathbf{I}_2$ is 60% of the rated secondary current $\mathbf{I}_2 = \mathbf{I}_{pu}\cdot I_{2rated} = 60\cdot\%\cdot I_{2rated}$

And the primary current is also 60% of the rated primary current $\mathbf{I}_1 = 60\cdot\%\cdot I_{1rated} = \mathbf{I}_{pu}\cdot I_{1rated}$

And $\frac{\mathbf{V}_{pu}}{\mathbf{Z}_{pu}} = \frac{90\cdot\%}{150\cdot\%} = 60\cdot\% = \mathbf{I}_{pu}$

And the upshot is, if we work with these percentages rather than the actual values, the percentages are the same on both sides of the transformer and the transformer "disappears". Notice that the percentages are the $\mathbf{V}_{pu}$, $\mathbf{I}_{pu}$ and $\mathbf{Z}_{pu}$ values. These "per unit" values may be complex.

I wouldn't recommend using this method for a circuit with a single transformer. But.., in circuits with many transformers, like the power grid, this method becomes very .. well.. powerful. There are a few differences:

1. Use "base" values rather than a single transformer's ratings, although V_{base} values almost always match the transformer ratings.
2. The per-unit method is almost exclusively used in 3-phase systems, so a few of the base relationships include 3 and $\sqrt{3}$.

Base Values S_{base}, V_{base}, I_{base}, and Z_{base}

At least two base values must be specified in order to find all the other base values. The power base (S_{base}) is the most universal base value since it isn't changed by transformers-- it's the same across the entire system. Usually a power company will use a nice round number, like 10MVA or 100MVA. The second most common base is the voltage (V_{base}) which will, of course, change at each transformer. I_{base}, and Z_{base} are calculated from the first two and also change at each transformer. All these base values only need to be calculated once for a given S_{base} and system configuration. Once the base values are established, all load and power-flow calculations can be made much more easily and quickly.

Per unit analysis is usually done on 3-phase systems and then on a per-phase basis, BUT, S_{base} and V_{base} are 3-phase and line-to-line respectively, so calculations of I_{base}, and Z_{base} will need to take that into account. I_{base} is line current and Z_{base} is for a Y-connected load. Other bases are sometimes defined, but it should be clear that they are not the normal bases of a 3-phase system

If per-unit values are given by the manufacturer of a generator, transformer or other device, then the manufacturer will use the device ratings as base values. For those devices, the per-unit impedances will have to be converted from the device's S_{rated} (or S_{base}) to the S_{base} of the system.

Starting bases

S_{base} = The 3-phase power base of the system, usually a nice round number, like 10MVA or 100MVA

$P_{base} = Q_{base} = S_{base}$ don't need to be separately defined

V_{base} = The nominal V_L (V_{LL}) in each region of the power system, where regions are regions are separated by transformers

$V_{base} = V_L = V_{LL}$ nominal in each region

Finding the other bases

$$I_{base} = \frac{\left(\frac{S_{base}}{3}\right)}{\left(\frac{V_{base}}{\sqrt{3}}\right)} = \frac{S_{base}}{\sqrt{3}\cdot V_{base}}$$

= The base for line current = current in one phase of a Y-connected load or device.

\ most / common way to calculate

$$Z_{base} = \frac{\left(\frac{V_{base}}{\sqrt{3}}\right)}{I_{base}} = \frac{V_{base}}{\sqrt{3}\cdot I_{base}} = \frac{\left(\frac{V_{base}}{\sqrt{3}}\right)^2}{\left(\frac{S_{base}}{3}\right)} = \frac{V_{base}^{\ 2}}{S_{base}}$$

\ most / common way to calculate

$R_{base} = X_{base} = Z_{base}$ don't need to be separately defined

It is possible (but not recommended) to define the following: $S_{1\phi\_base} = \frac{S_{base}}{3}$ and $V_{LN\_base} = \frac{V_{base}}{\sqrt{3}}$

In which case: $I_{base} = \frac{S_{1\phi\_base}}{V_{LN\_base}}$ and $Z_{base} = \frac{V_{LN\_base}}{I_{base}} = \frac{V_{LN\_base}^{\ 2}}{S_{1\phi\_base}}$

Base changes

Per-unit impedances given by a manufacturer of a generator, transformer or other device must be converted from the device ratings (S_{rated} and V_{rated}) to the system base values (S_{base} and V_{base}).

$$Z_{pu} = Z_{pu\_device}\cdot\frac{S_{base}\cdot\left(V_{rated}\right)^2}{S_{rated}\cdot\left(V_{base}\right)^2}$$

OR, more commonly

$$Z_{pu} = Z_{pu\_device}\cdot\frac{S_{base}}{S_{rated}}$$

When device $V_{rated} = V_{base}$

<u>Expressing values as "per-unit", pu</u> (Multiply by 100% to express as %, otherwise, use "pu" as the units)

$$S_{pu} = \frac{S_{3\phi}}{S_{base}} \qquad P_{pu} = \frac{P_{3\phi}}{S_{base}} \qquad Q_{pu} = \frac{Q_{3\phi}}{S_{base}}$$

$$V_{pu} = \frac{V_L}{V_{base}} \qquad I_{pu} = \frac{I_L}{I_{base}}$$

resistance	$R_{pu} = \frac{R}{Z_{base}}$	conductance	$G = \frac{1}{R}$	$G_{pu} = \frac{1}{R_{pu}}$
reactance	$X_{pu} = \frac{X}{Z_{base}}$	susceptance	$B = \frac{1}{X}$	$B_{pu} = \frac{1}{X_{pu}}$
impedance	$Z_{pu} = \frac{Z}{Z_{base}}$	admittance	$Y = \frac{1}{Z}$	$Y_{pu} = \frac{1}{Z_{pu}}$ common in power-flow calculations

The V_{pu}, I_{pu}, Z_{pu} and S_{pu} values are not affected by transformers.

The voltage, current and impedance bases WILL change at each transformer. The power base will NOT.

Please note that: $S_{pu} = \frac{S_{3\phi}}{S_{base}} = \frac{S_{1\phi}}{S_{1\phi\_base}}$ and $V_{pu} = \frac{V_L}{V_{base}} = \frac{V_{LN}}{V_{LN\_base}}$

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so these calculations (and the respective bases) are rarely needed or wanted

Example

a) Find the bases for a simple system with two transformers.

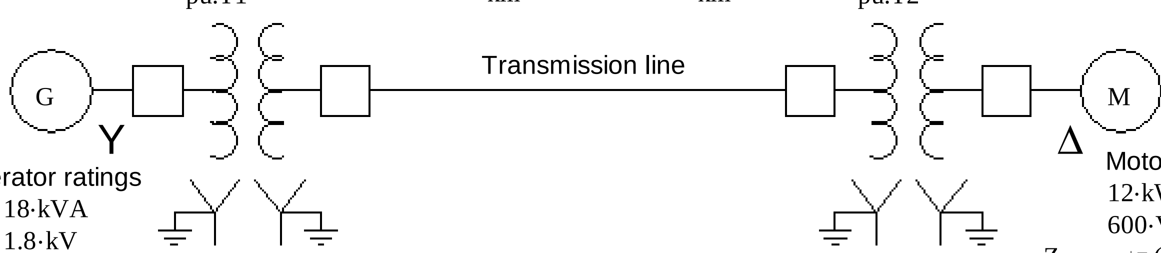
Since this is a very small, self-contained system, let's use an unusually small $S_{base} := 18 \cdot kVA$

Transmission line length = $len := 50 \cdot km$

1.8kV / 4kV
$18 \cdot kVA$
$Z_{pu.T1} := 0.1j \cdot pu$

$r := 0.3 \cdot \frac{\Omega}{km}$ $\qquad \omega l := 0.8 \cdot \frac{\Omega}{km}$

4kV / 500V
$15 \cdot kVA$
$Z_{pu.T2} := 9j \cdot \%$



Generator ratings
$18 \cdot kVA$
$1.8 \cdot kV$
$Z_{pu.G} := 1.5j \cdot \%$
actual voltage
$1732 \cdot V$

Motor ratings
$12 \cdot kW$
$600 \cdot V$
$Z_{pu.M} := (0.958 + 0.2j) \cdot pu$
per phase
at full load

Region 1	Region 2	Region 3
$V_{base1} := 1.8 \cdot kV$	$V_{base2} := 4 \cdot kV$	$V_{base3} := 500 \cdot V$

Set the V_{base} values in each region by looking at the transformer ratings.

b) Find the other bases.

Region 1

$$I_{base1} := \frac{S_{base}}{\sqrt{3}\cdot V_{base1}}$$

$$I_{base1} = 5.774\cdot A$$

$$Z_{base1} := \frac{\left(\frac{V_{base1}}{\sqrt{3}}\right)}{I_{base1}}$$

$$= \frac{{V_{base1}}^2}{S_{base}} = 180\cdot\Omega$$

Region 2

$$I_{base2} := \frac{S_{base}}{\sqrt{3}\cdot V_{base2}}$$

$$I_{base2} = 2.598\cdot A$$

$$Z_{base2} := \frac{{V_{base2}}^2}{S_{base}}$$

$$Z_{base2} = 888.889\cdot\Omega$$

Region 3

$$I_{base3} := \frac{S_{base}}{\sqrt{3}\cdot V_{base3}}$$

$$I_{base3} = 20.785\cdot A$$

$$Z_{base3} := \frac{{V_{base3}}^2}{S_{base}}$$

$$Z_{base3} = 13.889\cdot\Omega$$

c) Find all the impedances in per-unit form. Make base changes as necessary.

No changes necessary

$$Z_{pu.G} = 0.015j\cdot pu$$

$$Z_{pu.T1} = 0.1j\cdot pu$$

Transmission line

$$Z_{pu.TL} := \frac{\left(0.3\cdot\frac{\Omega}{km} + 0.8\cdot j\cdot\frac{\Omega}{km}\right)\cdot len}{Z_{base2}}$$

$$Z_{pu.TL} = 1.687 + 4.5j\ \cdot\%$$

Transformer 2

$$S_{rated\_T2} := 15\cdot kVA$$

$$Z_{pu.T2} := 0.09j\cdot\frac{S_{base}}{S_{rated\_T2}}$$

$$Z_{pu.T2} = 0.108j\cdot pu$$

Motor

$$S_{rated\_M} := 12\cdot kVA$$

$$Z_{pu.M} = 0.958 + 0.2j\ \cdot pu$$

$$Z_{pu.M} := Z_{pu.M}\cdot\frac{S_{base}\cdot(600\cdot V)^2}{S_{rated\_T2}\cdot {V_{base3}}^2}$$

$$Z_{pu.M} = 1.655 + 0.346j\ \cdot pu$$

at full load

ALL calculations made to this point **ONLY need to be made ONCE!!**

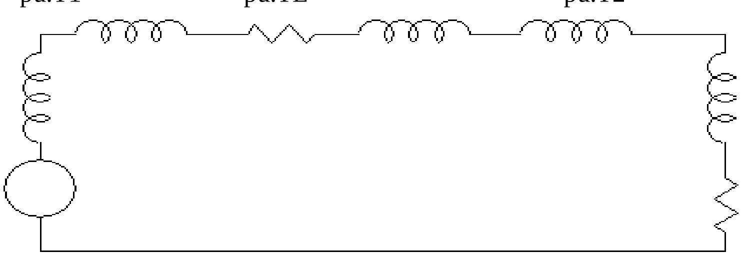
With the exception of the equivalent resistance of the motor. That will depend on the mechanical load placed on the motor.

d) Make a per-phase drawing that could be used to make current calculations.

$Z_{pu.T1} = 0.1j\cdot pu$ $\quad Z_{pu.TL} = 1.687 + 4.5j\ \cdot\%$ $\quad Z_{pu.T2} = 0.108j\cdot pu$

$Z_{pu.G} = 1.5j\cdot\%$

$\frac{1732\cdot V}{V_{base1}} = 0.962\cdot pu$



$0.24j\cdot pu$

$1.15\cdot pu$ Will change as the motor load changes

e) Calculate the currents and motor power when the motor in under full load.

The per-unit current: $I_{pu} := \frac{0.962\cdot pu}{\left[\sqrt{(0.017 + 1.15)^2 + (0.015 + 0.1 + 0.045 + 0.108 + 0.24)^2}\right]\cdot pu}$ $\quad I_{pu} = 0.756\cdot pu$

Region 1

$$I_{L1} = I_{pu}\cdot I_{base1} = 4.364\cdot A$$

Region 2

$$I_{L2} = I_{pu}\cdot I_{base2} = 1.964\cdot A$$

Region 3

$$I_{L3} = I_{pu}\cdot I_{base3} = 15.71\cdot A$$

Motor Power $\quad P_{Mpu} := {I_{pu}}^2\cdot 1.15 \qquad P_{Mpu} = 0.657\cdot pu$

$P_M := P_{Mpu}\cdot S_{base} \qquad P_M = 11.825\cdot kW$

A little less than the full load of 12kW because the voltage is a little low.

Notice that once the bases and per-unit impedances are established, the actual calculations are very easy!

Note: In these notes, I have retained "pu" in the subscripts of my variables as well as using "pu" or % as a unit of the number. If this seems redundant to you, it is. Most texts drop the "pu" in the variable subscripts.