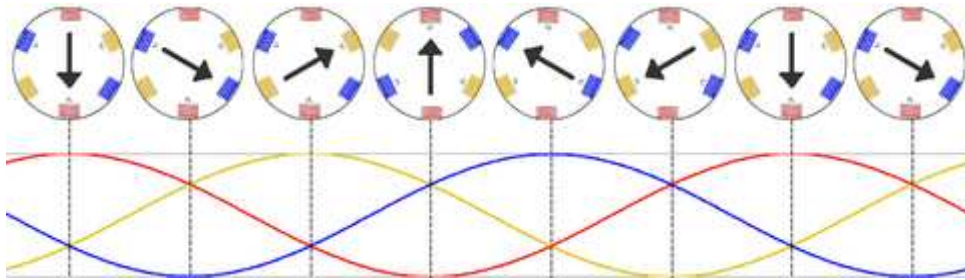


# Induction Motors

## Three-Phase, Rotating Magnetic Field

The magnetic flux inside the motor rotates at the "synchronous" speed.



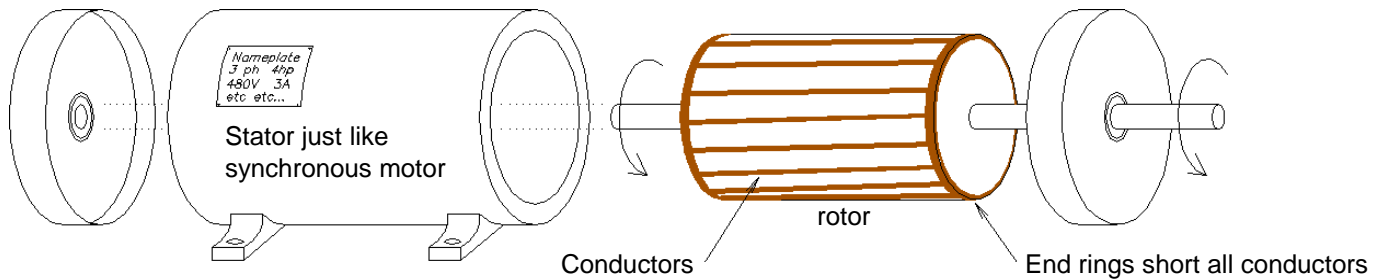
2-pole, 3-phase stator winding and the rotating flux



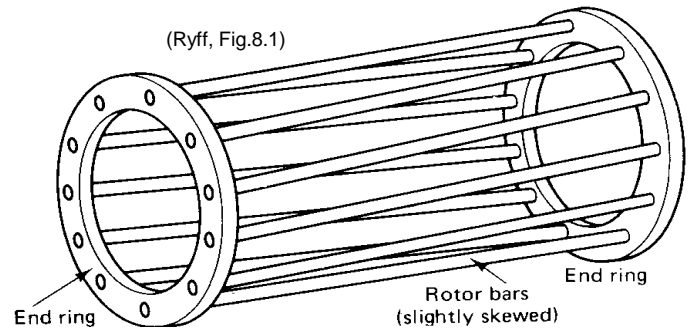
simplified 2-pole

If you place a rotor with a magnet or electromagnet inside this rotating magnetic flux, it would rotate at the synchronous speed. This is the way a synchronous motor works.

If you place a rotor with wire loops (typically a squirrel-cage winding) inside this rotating magnetic flux, voltages and currents would be induced in these windings as in the secondary of a transformer. These currents would, in turn, create a magnetic field which would tend to follow the rotating magnetic field and cause the rotor to turn. It would NOT rotate at the synchronous speed because the closer it gets, the smaller the induced currents in the rotor windings. (If the rotor ever did turn at the synchronous speed, the induced currents and torque would be zero.) This is the way an induction motor works.



A typical rotor of an induction motor includes a number of thick conductors called "rotor bars". Voltage is induced in these bars because the rotor normally turns at speed which is slower than the synchronous speed (the speed of the rotating flux caused by the stator windings). By Lenz's law, the resulting currents will flow in a direction to oppose the change that caused them. Thus the interaction between the induced current and the rotating flux provides the motor torque.



For more pictures of induction motors and a far more detailed description of how they work, see chapter 7 in your textbook.

## Slip (s)

The synchronous speed is the speed of the rotating magnetic field caused by the stator windings.

$$\omega_{sync} = \frac{4 \cdot \pi \cdot f}{N_{poles}} \quad \frac{\text{rad}}{\text{sec}} \quad f = \text{electrical frequency (60Hz in US)}$$

$$\omega = n \cdot \left( 2 \cdot \pi \cdot \frac{\text{rad}}{\text{rev}} \right) \cdot \left( \frac{\text{min}}{60 \cdot \text{sec}} \right) \quad \text{rpm}$$

$$n_{sync} = \frac{120 \cdot f}{N_{poles}} \quad n \text{ denotes rpm}$$

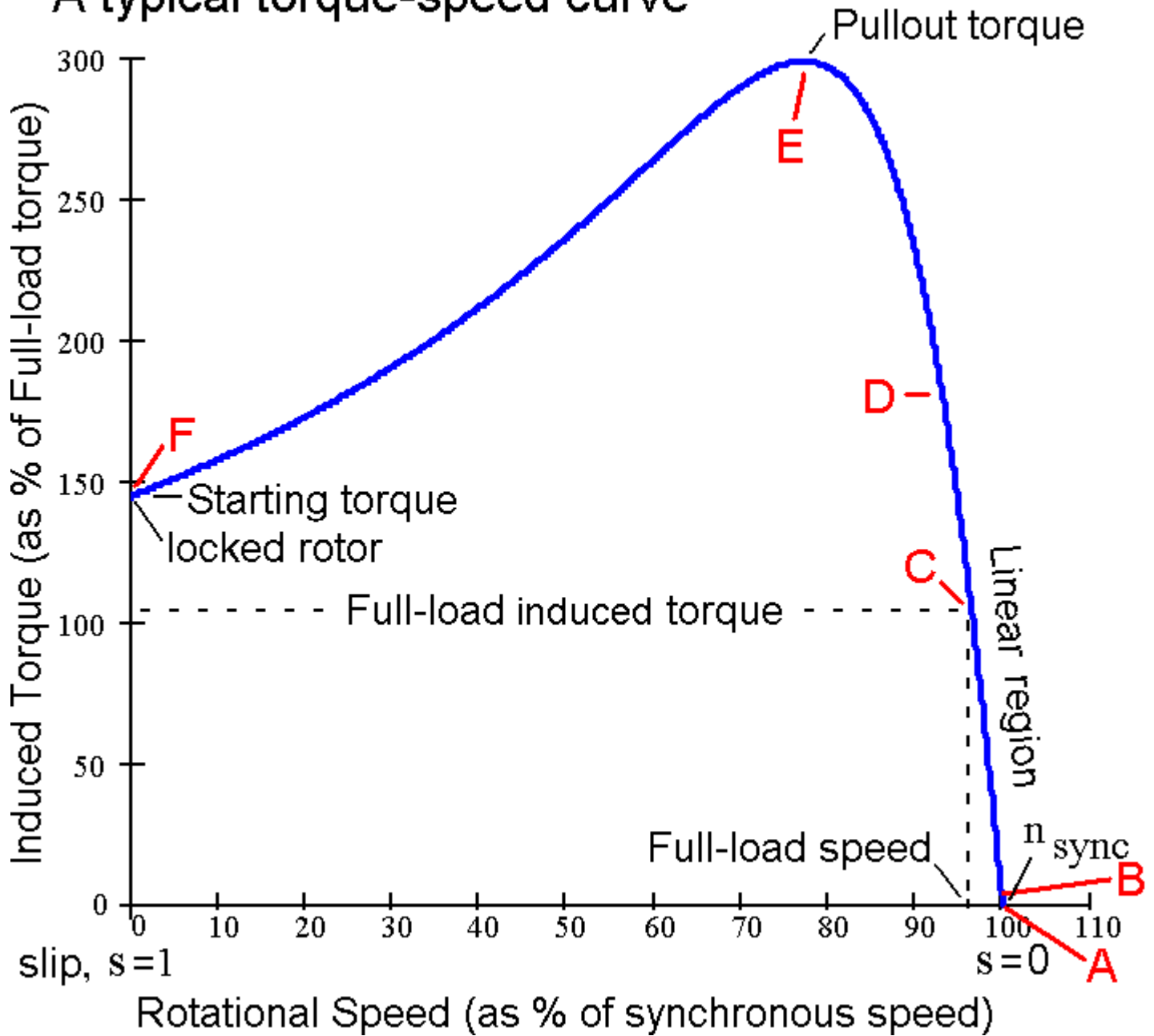
Slip (s) indicates how much slower the rotor is spinning.

$$n_m = \text{the mechanical speed of the rotor} = (1 - s) \cdot n_{sync} \quad n_{slip} = n_{sync} - n_m = s \cdot n_{sync}$$

$$\text{slip } s = \frac{n_{slip}}{n_{sync}} = \frac{n_{sync} - n_m}{n_{sync}}$$

$$f_{rot} = s \cdot 60 \cdot \text{Hz} = \text{frequency of voltages induced on the rotor}$$

### A typical torque-speed curve



Look at the typical torque-speed curve of an induction motor shown above. It shows induced torque as a function of the rotor speed. Induced torque is the torque produced by the motor internally. It includes both the torque needed to overcome mechanical losses inside the motor and the torque needed to rotate whatever mechanical load may be coupled to the motor shaft (the shaft torque). The internal losses are friction and wind resistance (windage). The windage can be significant since most rotors include some rudimentary fan blades to move cooling air through the motor.

Let's look at some specific points and areas on the curve (Letters A through F, going backwards over the curve):

A The Motor cannot operate here.  $n_m = n_{sync}$        $s = 0$        $f_{rot} = 0\text{-Hz}$        $\tau_{ind} = 0\text{-N}\cdot\text{m}$

The rotor is spinning at the synchronous speed, the speed of the rotating magnetic field produced by the stator windings. The rotor conductors do not experience any change in the magnetic field, so have no induced voltage or current and the induced torque on the rotor is zero. The motor CANNOT run at this speed because it cannot overcome the friction within the motor. It MUST run at a slower speed.

B No-load Condition  $P_{out} = 0$        $\tau_{shaft} = 0$        $\tau_{ind} = \tau_{mech\_loss}$

typical numbers:  $n_m > 99\% \cdot n_{sync}$        $s < 0.01$        $f_{rot} < 0.6 \cdot \text{Hz}$

The rotor speed is slightly less than the synchronous speed, typically within 1%. The magnetic field cuts the rotor conductors just fast enough to induce just enough rotor voltage and current to produce just enough torque to overcome friction. There is no additional torque to turn any load with the motor shaft.

C The Full-load Condition  $P_{out} = \text{Rated power of motor (often hp)}$        $\tau_{shaft} = \frac{P_{rated}}{\omega_{rated}}$        $\tau_{ind} = \tau_{shaft} + \tau_{mech\_loss}$

typical numbers:  $n_m > 95\% \cdot n_{sync}$        $s < 0.05$        $f_{rot} < 3 \cdot \text{Hz}$

The rotor spins at the rated speed of the motor, typically still within 5% of the synchronous speed. The magnetic field cuts the rotor conductors fast enough to induce enough rotor voltage and current to deliver the full, rated power to a load *and* overcome friction and windage. Since these ratings are usually shown on the "nameplate" attached to the motor, this is also called "nameplate operation". It is rare that the load will be just right to utilize the motor at its rated power, but we will often use the rated, or nameplate, numbers as a known operation point to make other calculations. Figure 7-37 in your textbook shows a motor nameplate.

B to C Normal Operation  $P_{out} = \tau_{shaft} \cdot \omega_m$

The motor is working within its ratings. Its output power is determined by the mechanical load.

A to D The Linear Region

The curve is almost a straight line in this region. If you know one or two points of operation, you can use methods of proportionality to find other points.

E Pullout Torque

If you load down the motor too much, it will eventually stall. This stall will happen suddenly when the shaft torque reaches the pullout torque.

F Locked Rotor or Stall Condition  $n_m = 0$        $s = 1$        $f_{rot} < 60 \cdot \text{Hz}$        $\tau_{ind} = \tau_{start}$        $P_{out} = 0$

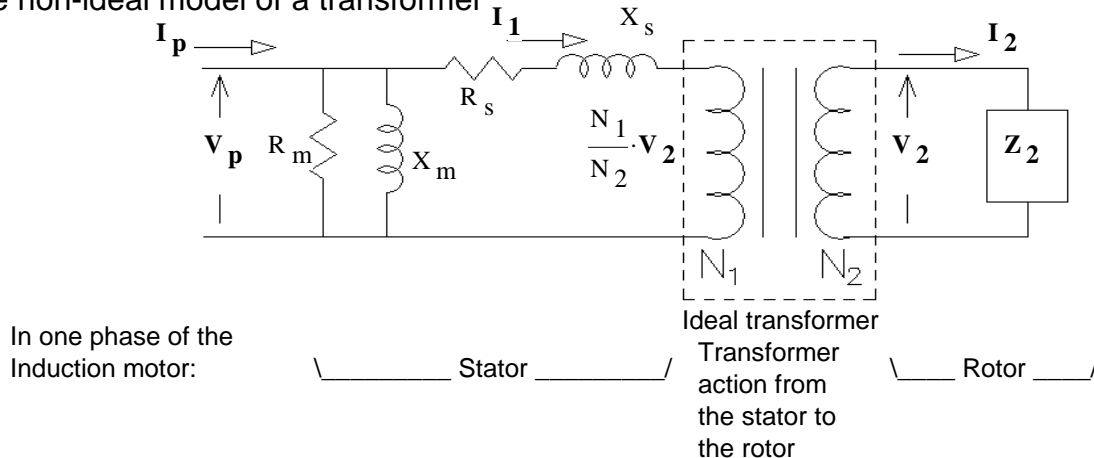
This is also the starting point of the motor when first turned on, but then it's a transitory condition. The starting current can be very high and, in large motors, may require us to use special starting methods.

E to F Unstable Region

The motor transitions through this region very quickly. Either accelerating when starting or decelerating when stalling. When starting, most of the power converted from electrical to mechanical ( $P_{conv}$ ) goes to accelerating the mass of the rotor and whatever is attached to it.

A to F The shape of this curve is dictated by the internal electrical workings of the motor and the transformer-like interaction of the stator with the rotor. It is particularly dependent on the resistance and reactance of the rotor. We'll explore this next with the electrical model of the motor.

Recall the non-ideal model of a transformer

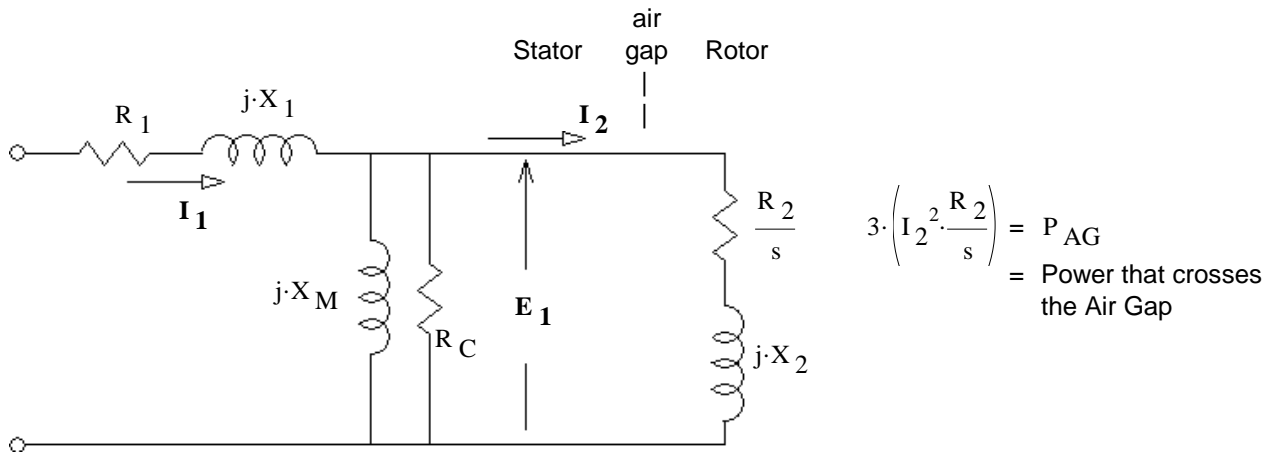


across the air gap of the motor.

Based on the non-ideal model of a transformer with a number of modifications.

1.  $X_m$  and  $R_m$  are moved to the right of  $R_s$  and  $X_s$  because the air gap makes  $X_m$  a much smaller value and  $X_s$  much larger. Most are renamed.
2. The rotor resistance and reactance are moved to the primary side as  $R_2$  and  $X_2$ .
3. The resistance is shown as  $R_2/s$  because the voltage on the rotor depends on the slip but it's easier to think of the resistor as inversely dependent on slip.
4. The reactance would also be inversely dependent on slip, except the rotor reactance itself depends on the electrical frequency on the rotor like:  $X_{rot} = \omega_{rot} \cdot L_{rot} = s \cdot \omega_{sync} \cdot L_{rot}$ . The two effects cancel each other out.

Of particular interest is the power used by  $R_2/s$  because that's the power transferred from the stator to the rotor. It represents both the power needed to turn the rotor and power lost to  $I^2R$  losses in the rotor wires.



$$3 \cdot \left( I_2^2 \cdot \frac{R_2}{s} \right) = P_{AG}$$

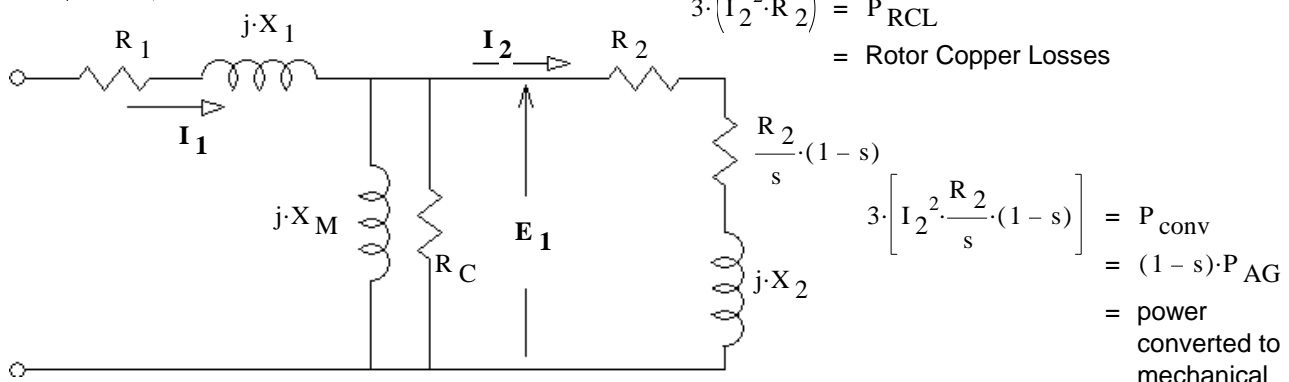
= Power that crosses the Air Gap

= Stator Copper Losses

$$3 \cdot (I_1^2 \cdot R_1) = P_{SCL}$$

$$3 \cdot (I_2^2 \cdot R_2) = P_{RCL}$$

= Rotor Copper Losses



$$3 \cdot \left[ I_2^2 \cdot \frac{R_2}{s} \cdot (1-s) \right] = P_{conv}$$

=  $(1-s) \cdot P_{AG}$   
= power converted to mechanical

$$3 \cdot \frac{E_1^2}{R_C} = P_{core}$$

= core loss

$$P_{out} = P_{conv} - P_{mech} - P_{misc}$$

mechanical losses

induced torque =  $\tau_{ind} = \frac{P_{conv}}{\omega_m}$  OR:  $\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$  (N.m)

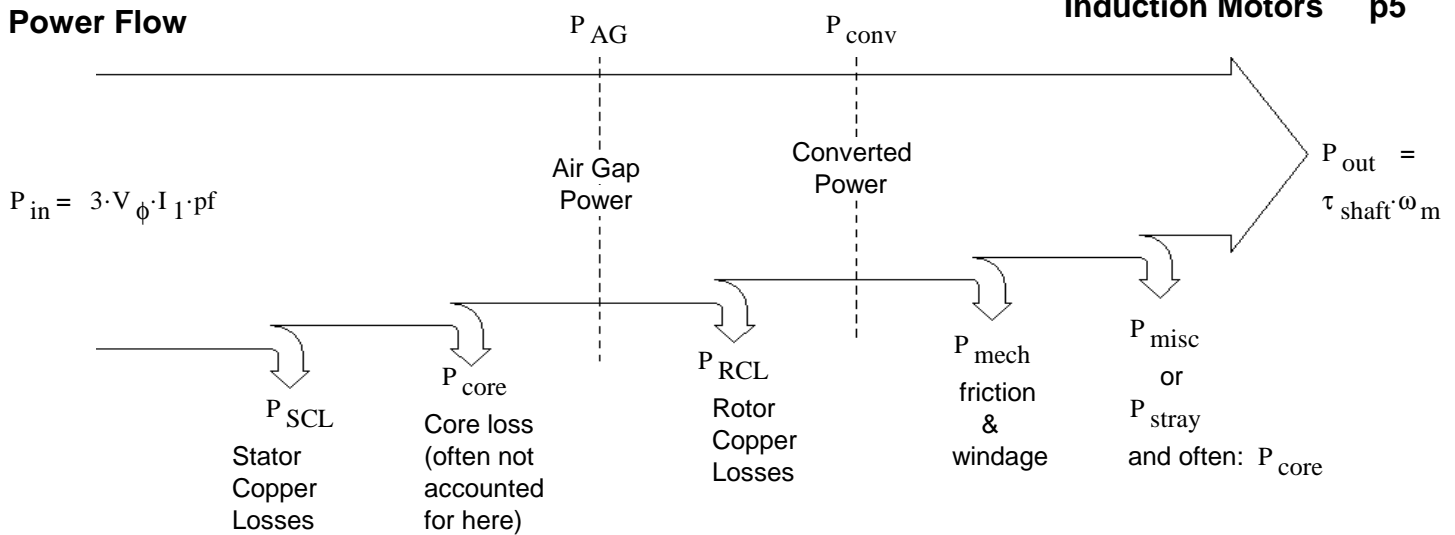
load torque =  $\tau_{load} = \frac{P_{out}}{\omega_m}$

Starting Current (Locked rotor)

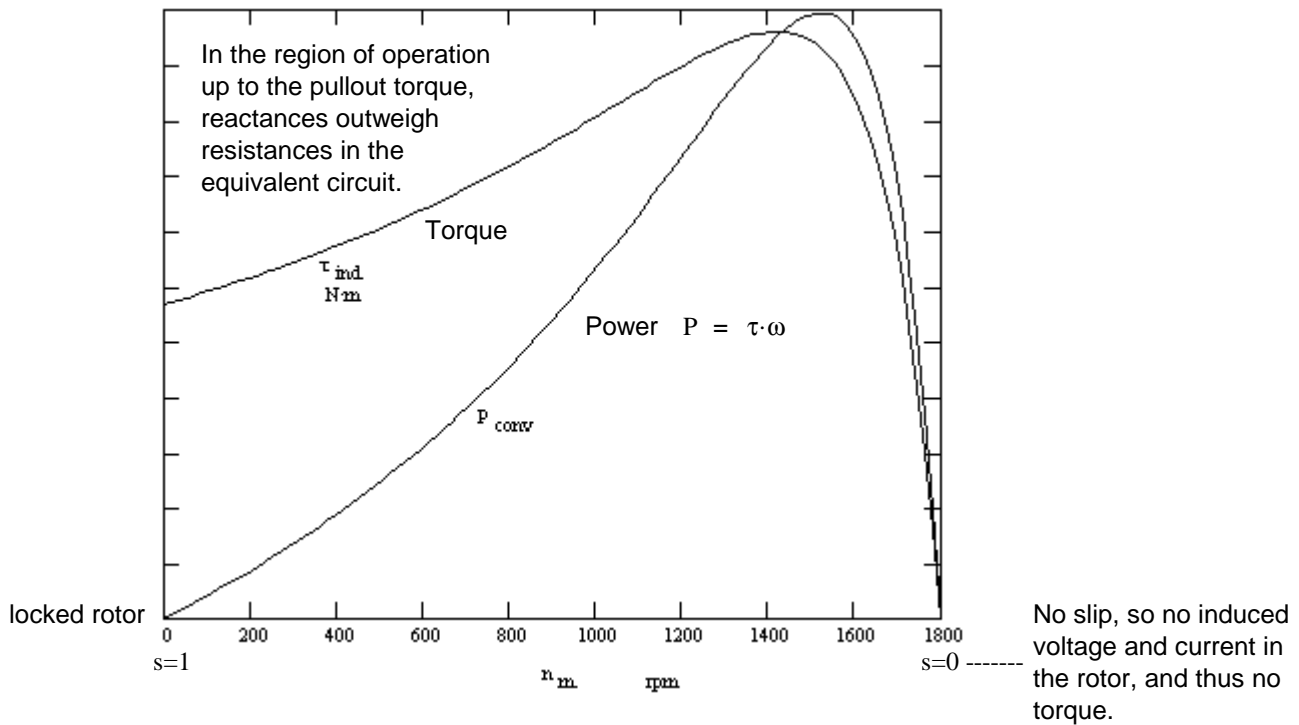
$$Z_{eq} = R_1 + jX_1 + \frac{1}{\frac{1}{jX_M} + \frac{1}{R_C} + \frac{1}{jX_2 + R_2}}$$

Starting current:  $I_{LR} = \frac{V_\phi}{Z_{eq}}$

## Power Flow



Typical torque-speed and power-speed curves for a 4-pole Induction motor

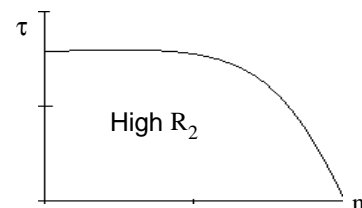
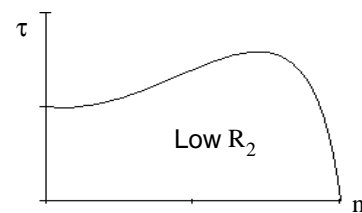


## $R_2$ Affects Curve Shapes

These curves are greatly affected by the rotor resistance ( $R_2$  in our model). See Fig 7-20 in the textbook.

Low $R_2$ :	Higher efficiency at normal running	good
	Higher available power	good
	Better speed regulation at normal running	good
	Lower startup torque	bad
	Higher startup current	bad

High $R_2$ :	Lower efficiency at normal running	bad
	Less available power	bad
	More speed variance at normal running	bad
	Higher startup torque	good
	Lower startup current	good

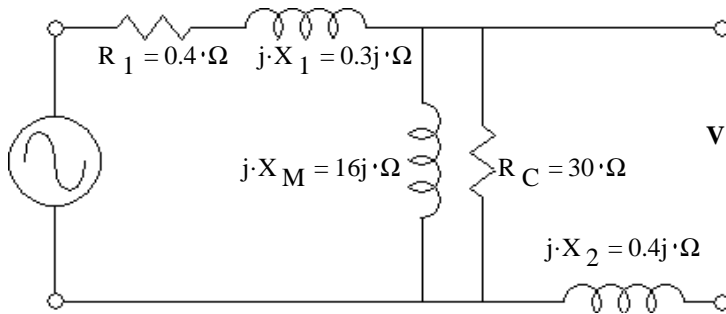


# Torque-Speed Characteristics derived from the Model

Remove the resistor that changes with  $s$  and create a Thevenin equivalent of the rest of the model.  
 Best illustrated with actual numbers

$$V_\phi := \frac{208 \cdot V}{\sqrt{3}} \quad R_1 := 0.4 \cdot \Omega \quad R_C := 30 \cdot \Omega \quad R_2 := 0.2 \cdot \Omega \quad N_{poles} := 4 \quad n_{sync} := \frac{7200 \cdot rpm}{N_{poles}}$$

$$X_1 := 0.3 \cdot \Omega \quad X_M := 16 \cdot \Omega \quad X_2 := 0.4 \cdot \Omega \quad \omega_{sync} := \frac{240 \cdot \pi \cdot rad}{N_{poles} \cdot sec}$$



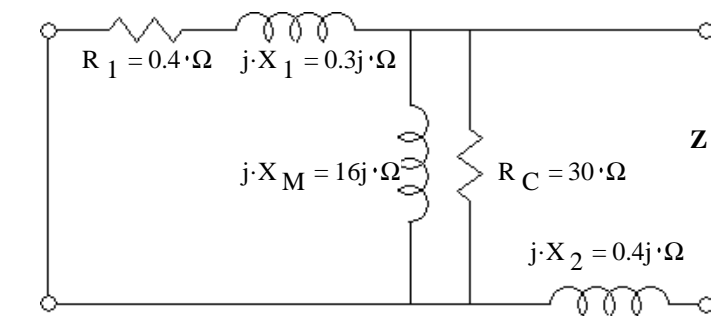
$$V_{Th} := V_\phi \cdot \frac{\left( \frac{1}{j \cdot X_M} + \frac{1}{R_C} \right)}{R_1 + j \cdot X_1 + \left( \frac{1}{j \cdot X_M} + \frac{1}{R_C} \right)}$$

$$V_{Th} = 116.331 + 1.691j \cdot V$$

$$V_{Th} := |V_{Th}|$$

$$V_{Th} = 116.343 \cdot V$$

$$\arg(V_{Th}) = 0.833 \cdot deg$$



$$Z_{Th} := \frac{1}{\frac{1}{R_1 + j \cdot X_1} + \frac{1}{j \cdot X_M} + \frac{1}{R_C}} + j \cdot X_2$$

$$Z_{Th} = 0.383 + 0.696j \cdot \Omega$$

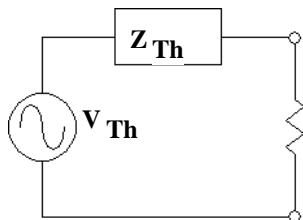
$$|Z_{Th}| = 0.7948 \cdot \Omega$$

$$\arg(Z_{Th}) = 61.169 \cdot deg$$

$$R_{Th} := \text{Re}(Z_{Th})$$

$$R_{Th} = 0.383 \cdot \Omega$$

## Maximum Induced Torque $\tau_{max}$



Maximum  $P_{AG}$  occurs at the  $s$  that makes

$$\frac{R_2}{s} = |Z_{Th}|$$

and since:  $\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$  that's also the  $s$  for maximum torque:

$$s_{max} = \frac{R_2}{|Z_{Th}|} = 0.252$$

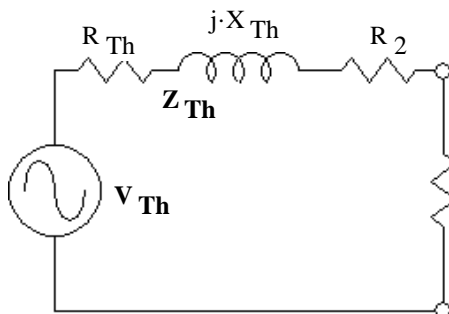
$$\tau_{max} = \frac{3 \cdot I^2 \cdot \left( \frac{R_2}{s_{max}} \right)}{\omega_{sync}} = \frac{3}{\omega_{sync}} \cdot \left[ \frac{V_{Th}}{|Z_{Th} + \frac{R_2}{s_{max}}|} \right]^2 \cdot \left( \frac{R_2}{s_{max}} \right) = \frac{3}{\omega_{sync}} \cdot \left( \frac{V_{Th}}{|Z_{Th} + |Z_{Th}||} \right)^2 \cdot |Z_{Th}|$$

After some messy algebra: 
$$\tau_{max} = \frac{3}{\omega_{sync}} \cdot \frac{V_{Th}^2}{2 \cdot (R_{Th} + |Z_{Th}|)} = 91.437 \cdot N \cdot m$$

## Starting Torque ( $s = 1$ )

$$\frac{3 \cdot I^2 \cdot R_2}{\omega_{sync}} = \frac{3}{\omega_{sync}} \cdot \left( \frac{V_{Th}}{|Z_{Th} + R_2|} \right)^2 \cdot R_2 = 52.228 \cdot N \cdot m$$

## Maximum Power Converted to Mechanical $P_{conv}$



Maximum  $P_{conv}$  occurs at the  $s$  that makes

$$\frac{R_2}{s} \cdot (1 - s) = |Z_{Th} + R_2|$$

$$s_{maxP} = \frac{R_2}{|Z_{Th} + R_2| + R_2} = 0.1805$$

After similar messy algebra...

$$P_{conv,max} = 3 \cdot \frac{V_{Th}^2}{2 \cdot (R_{Th} + R_2 + |Z_{Th} + R_2|)} = 13.613 \cdot kW$$

Now to get plots, let  $s$  change over a range  $i := 0..1100$

$s_i := \frac{i}{500} - .995$  Notice it's not allowed to be 0

$s_{\max\tau} := \frac{R_2}{|Z_{Th}|}$        $n_{\max\tau} := (1 - s_{\max\tau}) \cdot n_{\text{sync}}$

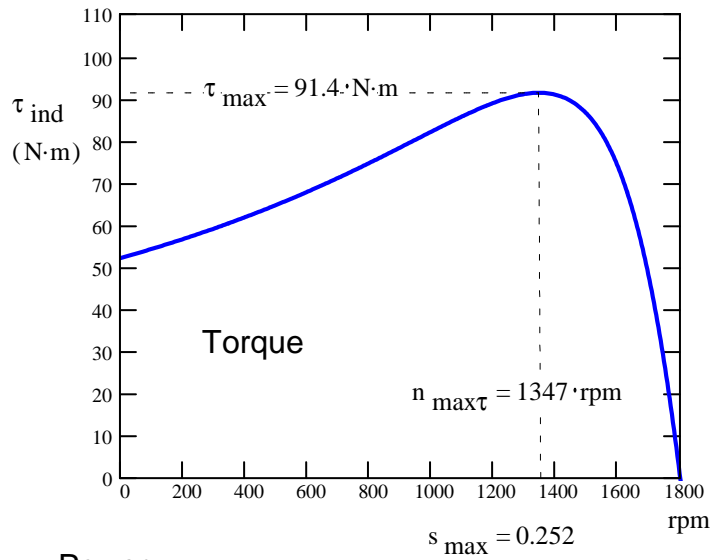
$\tau_{\max} := \frac{3}{\omega_{\text{sync}}} \cdot \frac{V_{Th}^2}{2 \cdot (R_{Th} + |Z_{Th}|)}$

$s_{\max P} := \frac{R_2}{|Z_{Th} + R_2| + R_2}$        $n_{\max P} := (1 - s_{\max P}) \cdot n_{\text{sync}}$

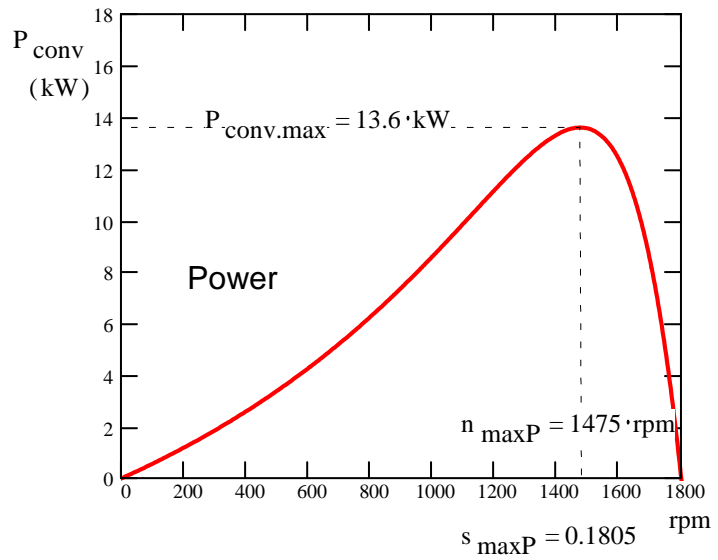
$P_{\text{conv.max}} := 3 \cdot \frac{V_{Th}^2}{2 \cdot (R_{Th} + R_2 + |Z_{Th} + R_2|)}$

$n_{m_i} := (1 - s_i) \cdot n_{\text{sync}}$        $I_{2_i} := \frac{V_{Th}}{Z_{Th} + \frac{R_2}{s_i}}$        $P_{AG_i} := 3 \cdot \left[ \left( |I_{2_i}| \right)^2 \cdot \frac{R_2}{s_i} \right]$        $\tau_{\text{ind}_i} := \frac{P_{AG_i}}{\omega_{\text{sync}}}$        $P_{\text{conv}_i} := (1 - s_i) \cdot P_{AG_i}$

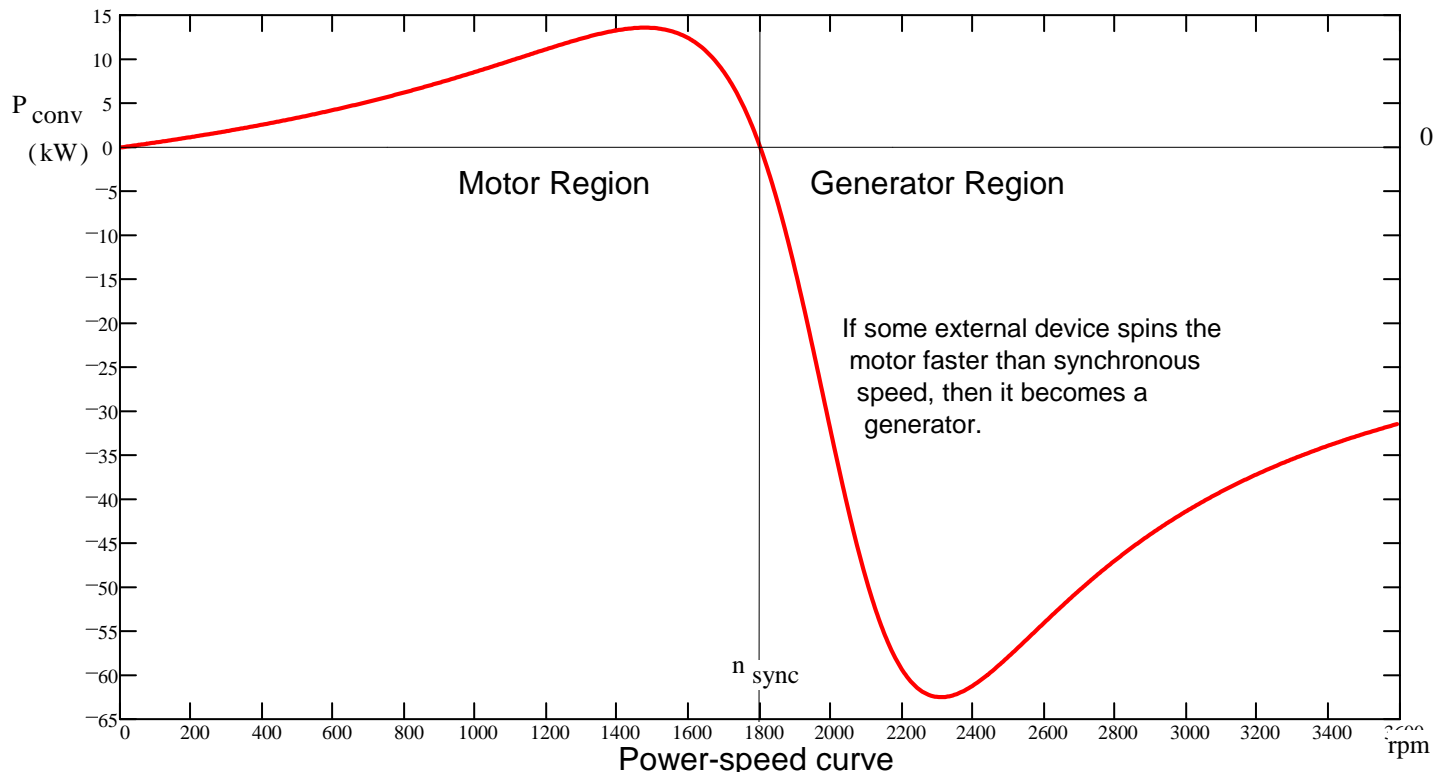
Torque-speed curve



Power-speed curve



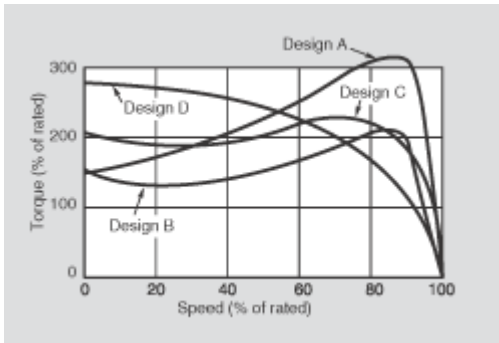
Power



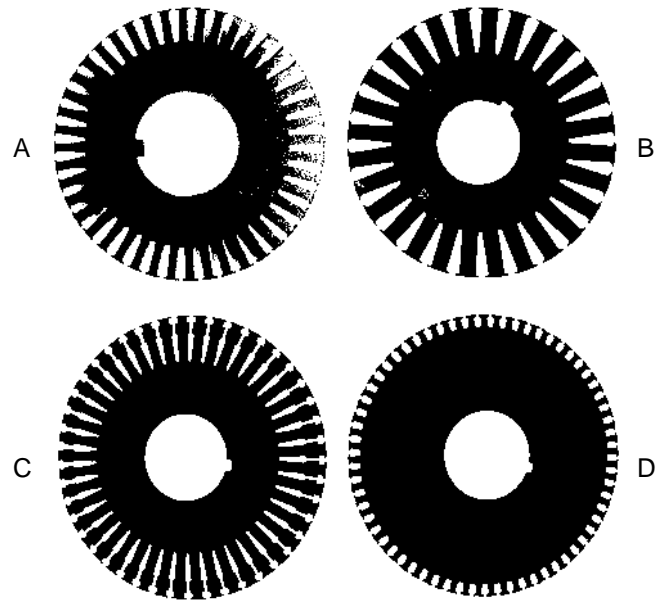
Power-speed curve

## Induction Motors p8

By manipulating the design of the rotor cage, you can mix some of the benefits of both small and large  $R_2$  values.



NEMA Designs, A, B, C, D



Rotor cross-sections

For more information about these different designs and how this works, see section 7.6 in your textbook.

### Wound-Rotor

See Figures 7-4 and 7-5 in your textbook

The squirrel-cage winding allows no control over the ever-important  $R_2$  once the motor has been constructed. Another way to build an induction motor is to replace the few thick conductors of the squirrel-cage with many repeated thinner conductors wound on the rotor. Then you could allow that smaller current to flow through some slip rings to additional resistance outside of the motor. This is the wound-rotor induction motor. Added resistance would reduce the starting current and increase the starting torque-- both improvements. Shorting the added resistance once the motor has reached operating speed would improve the motor's efficiency, power, and speed regulation. You could achieve similar results right on the rotor with one or more centrifugal switches.

### Starting Induction Motors

Starting an induction motor from a stopped condition can require enormous surge currents. Much of the fuss over  $R_2$  comes from starting issues. All sorts of methods have been used to limit the starting current. Most involve starting at a lower voltage and then switching to a higher voltage once the motor is near its operating speed. The initial lower voltage can be achieved by adding resistance, using transformers or autotransformers, or using a Y connection at starting and a  $\Delta$  connection at run. All involve switches like those shown in Figure 7-28 in your textbook. (Please note that the symbols we might take for capacitors are actually switch contacts.)

All these methods that require switches and relays are losing popularity now that Variable-Frequency Drives (VFDs) are becoming cheaper and more reliable. See the next section.

### Speed Control of Induction Motors

There is really only one good method to control the speed of an induction motor-- the Variable-Frequency Drive (VFD). A VFD takes in AC power, rectifies it to make DC, and then creates a different frequency using power electronics. This effectively changes the  $n_{sync}$ .

At lower frequencies than the motor is rated for, the VFD must also decrease the voltage to avoid saturating the motor core.

At higher frequencies than the motor is rated for, the VFD doesn't increase the voltage to avoid damaging the motor insulation. Centrifugal forces can also damage the motor at higher than rated speed.

Figure 7-29c (p334) in your textbook shows a family of torque-speed curves for a 4-pole motor controlled by a VFD. Also, watch the animation presented in class.

Variable-Frequency Drives are becoming so affordable and reliable that you will see more and more of them, even just to limit the starting current.

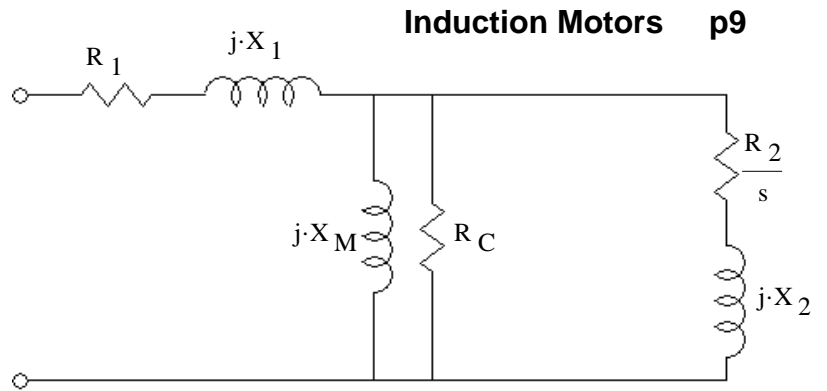
VFDs are so effective that induction motors are now routinely used in place of DC motors where speed control is essential. DC motors used to be the only practical choice for those applications.



## Determining Circuit Model Parameters

Standard Circuit Model or one phase of an induction motor.

How do we find  $R_1$   $R_C$   $R_2$   
 $X_1$   $X_M$   $X_2$   
 from measurements ??



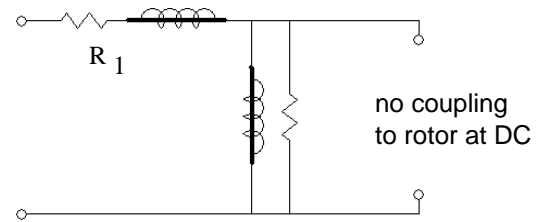
Turn to section 7.9 in your textbook. Follow along as we go through this section.

Refer to IEEE Standard 112 for more information if you ever have to make these measurements for real.

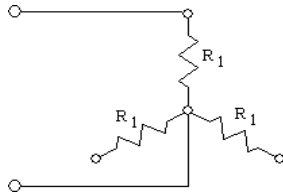
### DC test to find $R_1$

Use DC voltage and current to find  $R_1$ . With DC input, the model reduces to:

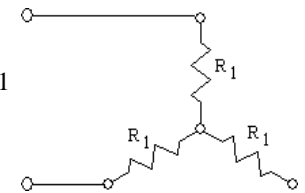
The book recommends making this measurement at rated current, so that the winding will be close to normal temperature. Be mindful of the connection that you have to the motor.



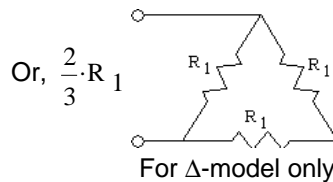
If it isn't a Y-connected motor with an available neutral connection, like this:



Then you may be measuring  $2 \cdot R_1$



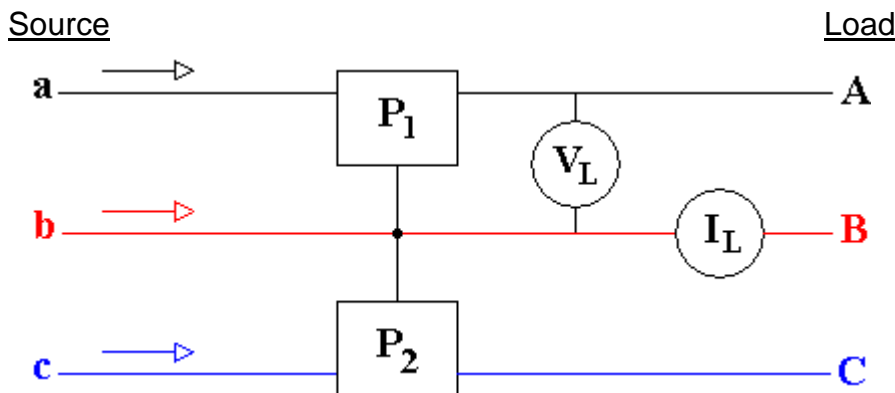
For actual Y-connected motor or Y-model of a  $\Delta$ -connected motor.



Note: All of the calculations made in section 7.9 assume a Y-connected motor (or a Y-model of a  $\Delta$ -connected motor).

### Measuring 3-phase Power with Two Wattmeters (Balanced)

With the wattmeters connected as shown in Figure 7-33:



$$P_{3\phi} = P_1 + P_2 = \sqrt{3} \cdot V_L \cdot I_L \cdot \cos(\theta)$$

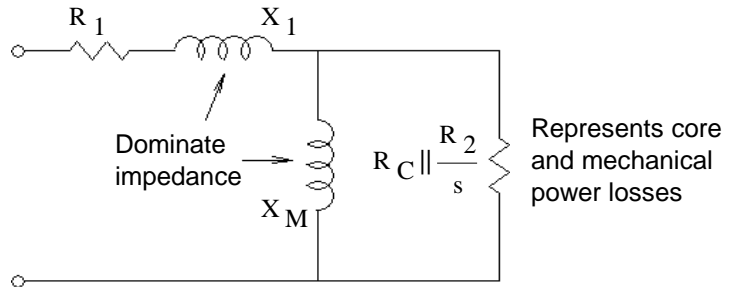
$$Q_{3\phi} = \sqrt{3} \cdot (P_2 - P_1) = \sqrt{3} \cdot V_L \cdot I_L \cdot \sin(\theta)$$

$$S_{3\phi} = \sqrt{P_{3\phi}^2 + Q_{3\phi}^2} = \sqrt{3} \cdot V_L \cdot I_L$$

**No-Load Test** Shaft is free-running--not coupled to any load.

Run motor at rated voltage,  
Measure:  $P_{3\phi}$ ,  $I_{\phi}$ , and  $V_{\phi}$

After several steps of simplification, see Fig. 7-33



$$P_{3\phi} = P_{SCL} + P_{core} + P_{FW} + P_{misc}$$

$$P_{SCL} = 3 \cdot I_{\phi}^2 \cdot R_1$$

Find  $P_{rot} = P_{core} + P_{FW} + P_{misc}$   
 $P_{FW}$  = Power lost to friction and windage

And approximate  $X_1 + X_M \approx \frac{V_{\phi}}{I_{\phi}}$

**Locked-Rotor Test**

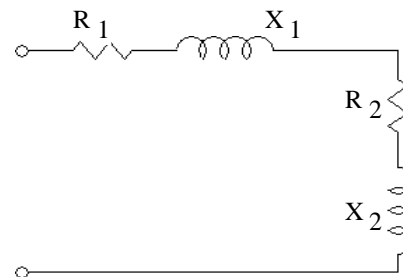
Shaft is not allowed to turn

This test is often performed at 10 or 15 Hz ( $f_{test}$ ) rather than the normal 60 Hz ( $f_{rated}$ ).

The motor is subjected to a very short test at rated current,

Measure:  $P_{3\phi}$ ,  $I_{\phi}$ , and  $V_{\phi}$  to find  $Z_{LR}$

After simplification, see Fig. 7-34



Determine  $Z_{LR} = (R_1 + R_2) + X'_{LR} \cdot j$  which yields  $R_2$  since  $R_1$  is known from the DC test.

$$(X_1 + X_2) \approx \frac{f_{rated}}{f_{test}} \cdot X'_{LR}$$

separate into  $X_1$  and  $X_2$  using the table below (or Figure 7-35 in text)

Rotor Design	Split of $X_1 + X_2$	
	$X_1$	$X_2$
Wound rotor	$0.5 \cdot (X_1 + X_2)$	$0.5 \cdot (X_1 + X_2)$
Design A	$0.5 \cdot (X_1 + X_2)$	$0.5 \cdot (X_1 + X_2)$
Design B	$0.4 \cdot (X_1 + X_2)$	$0.6 \cdot (X_1 + X_2)$
Design C	$0.3 \cdot (X_1 + X_2)$	$0.7 \cdot (X_1 + X_2)$
Design D	$0.5 \cdot (X_1 + X_2)$	$0.5 \cdot (X_1 + X_2)$

Then use  $X_1$  and the value of  $X_1 + X_M$  from the no-load test to find  $X_M$ .