In this chapter we discuss single-phase fractional horsepower motors and a variety of small machines that are used for special and general purpose applications. These machines have either unusual operating characteristics or construction details not found on the common type of machines discussed so far. For example, permanent magnet fields, brushless dc motors, stepper motors, printed circuit board armatures, and so on. Many of these devices are employed together with electronic circuitry to provide precise speed control and positioning control. We will present unique features for the most common variety of small electric motors.

9.1 Fractional-Horsepower Machines

Production of Torque

In the three-phase induction motor, the three-phase distributed armature winding sets up a rotating magnetic field which is fairly constant in magnitude and rotates at synchronous speed. In single-phase induction motors we have only a simple field winding excited with alternating current; therefore, it does not have a true revolving field, and hence it is not inherently self-starting. Various methods have been devised to initiate rotation of the squirrel cage, and the particular method employed to start the motor will designate the specific type.
First let us examine the behavior of the magnetic field as set up by an ac current in the single-phase field winding. With reference to Figure 9.1, we have current flowing in the field winding. If this current is sinusoidal, then, neglecting saturation effects of the magnetic iron circuit, the flux through the armature will vary sinusoidally with time. The magnetic field created is as shown at the particular instant in time; it will reverse during the next half-cycle of the ac supply voltage cycle. Since the flux is pulsating it will induce currents in the rotor bars, which in turn will create a rotor flux which by Lenz’s law opposes that of the main field. From this the current direction in the rotor bars can be determined, as shown in Figure 9.1, as well as the torque created between the field and rotor currents. It is apparent that the clockwise torque produced is counteracted by the counterclockwise torque, hence no motion results. Since the field is pulsating, the torque is pulsating, although no net torque is produced over a full cycle of the ac supply frequency.

However, any pulsating field can be resolved into two components, equal in magnitude but oppositely rotating vectors, as shown in Figure 9.2a. The maximum value of the component fields equals one-half of φ_{max}. As can be observed, the resultant field of φ₁ and φ₂ in Figure 9.2a as they rotate at an angular velocity dictated by the supply frequency, must always lie on the vertical axis. The resultant value of these two vectors at any instantaneous time equals the value of the magnetic field as it actually exists. A physical interpretation of the two oppositely rotating field components is as predicted in Figure 9.2b. Each component field slides around the air gap in opposite directions and equal velocities; their instantaneous sum represents the instantaneous resultant field, which changes between ±φ_{max}. This method of field analysis is commonly known as the double-revolving field theory. Each field component acts independently on the rotor and in a fashion similar to that of the rotating field in a three-phase induction motor, except that here there are two, one tending to rotate the rotor clockwise, the other tending to rotate it counterclockwise.

Considering the clockwise flux component by itself, it would produce the torque-speed characteristic labeled T_{cw} in Figure 9.3, while the counterclockwise flux component produces the torque T_{ccw}. Observe that at standstill (s = 1.0) the two torque components produced are equal, but directed oppositely. Although the net torque produced at standstill is zero (T_{cw} = T_{ccw}), if the rotor were to be advanced in either direction, a net torque will result, and the motor will continue to rotate in the direction in which it has been started. For example, if we assume that the rotor is in some way started in the clockwise direction, the torque T_{cw} will exceed T_{ccw} immediately and the rotor will accelerate in the direction of T_{cw}. Steady-state speed will be reached near synchronous speed at a slip dictated by the load. It is interesting to note that at this slip speed, T_{cw} predominates over T_{ccw}, which is fairly small but exists nevertheless. Also, the rotor operates at a small slip value as far as T_{cw} is concerned, but the slip is nearly 2 with regard to T_{ccw}. This implies induced rotor currents due to T_{ccw} which are at double the line frequency. These rotor currents do not produce any significant countertorque because of their high frequency, since the rotor reactance is many times its value at slip frequency.

In the three-phase induction machine the rotating field strength does not vary appreciably as it rotates, and the field is said to be circular, as shown in Figure 9.4. The field set up by a single-phase motor on the other hand is usually elliptical in shape. To see why this occurs we have to examine the rotor field more closely.

When the rotor is rotating, voltages are induced in the rotor conductors which are in phase with the stator field. Since these voltages are speed dependent they are referred to as speed EMFs, as opposed to transformer EMFs, which are produced by transformer action. Both are, of course, produced by a changing flux, the speed EMF as a result of relative motion between the field and conductor, the transformer EMF as a result of a pulsating field.

**FIGURE 9.1.** Torque produced in squirrel cage of single-phase induction motor having a simple field winding.

**FIGURE 9.2.** Pulsating field resolved into two oppositely rotating fields.
FIGURE 9.3. Torque-speed characteristic of a single-phase squirrel-cage induction motor.


Therefore, since rotor-induced currents flow in the rotor bars, which represent almost entirely a reactive impedance, these rotor currents will lag the rotor-induced voltage by nearly 90°. In turn, the field created by the rotor currents is also displaced by 90° and is known as a cross field, as indicated in Figure 9.5. Thus the stator sets up a pulsating field while the rotating rotor sets up a second field which pulsates 90° behind the stator field in space and time. When the rotor rotates at nearly synchronous speed, these two fields will virtually be equal and will combine to produce a nearly circular field. Upon loading the single-phase induction motor its speed will drop, thereby reducing the induced EMF in the rotor. This in turn reduces the cross field and the resulting field becomes elliptical, as shown in Figure 9.4b. Further decreasing the speed until finally, standstill is reached, the resulting field will only pulsate in value along the stator axis and does not rotate.

FIGURE 9.5. Cross field \( \phi_c \) created by rotor rotation.

Thus, once started, the single-phase motor, having a simple winding as explained, will continue to run in the direction in which it is started. Non-self-starting is not a desirable feature in practice, and modifications are introduced to obtain the torque required to start. To accomplish this a quadrature flux component in time and space with the stator flux must be provided at standstill. Auxiliary windings normally placed on the stator have proved effective in developing this starting torque. The method employed to accomplish this will now be described.

**Split-Phase Induction Motors**

One of the most widely used types of single-phase motors is the split-phase induction motor. Its service includes a wide variety of applications such as used for refrigerators, washing machines, portable hoists, many small machine tools, blowers, fans, centrifugal pumps, and many others. The essential parts of the split-phase motor are shown in Figure 9.6a. It shows the auxiliary winding, also called the starting winding, in space quadrature (i.e., 90 electrical degrees displacement) with the main stator winding. The rotor is normally the normal squirrel-cage type. The two stator windings are connected in parallel to the ac supply voltage. A phase displacement between the winding currents is obtained by adjusting the winding impedances, either by inserting a resistor in series with the starting winding or as is generally the practice, by using a smaller-gauge wire for the starting winding. A phase displacement between the currents of 30° can be achieved at the instant of starting. A typical phasor diagram for this motor at startup is illustrated in Figure 9.6b.

When the motor has come up to about 70 to 75% of synchronous speed, the starting winding may be opened by a centrifugal switch, and the motor will continue to operate as a single-phase motor. At the point where the starting winding is disconnected, the motor develops nearly as much torque with the main winding as with both windings connected.
as can be observed from the typical torque-speed characteristic for this type of motor (see Figure 9.7).

The starting winding is designed to take the minimum starting current for the required torque. The locked rotor starting current is typically in the range 5 to 7 times rated current, while the starting torque is about 1.5 to 2 times rated torque. The high starting current as such is not objectionable, since once started it drops off almost instantly. The major disadvantages are the relatively low starting torque and the high slip at which it operates when heavily loaded. As you can appreciate from the earlier discussion, when the speed drops significantly (of course, not to the extent that the centrifugal switch operates), the induced EMF is reduced. This results in an elliptical or pulsating torque, which makes this motor somewhat noisy. It is precisely for this reason that the split-phase motor is operated and employed where the drive loads themselves are noisy.

Unlike the three-phase induction motor, which may start in either direction, the split-phase motor is factory connected, and as such its direction of rotation is fixed (counterclockwise when viewed from the opposite end of the shaft extension). To reverse the direction of rotation it is necessary to reverse the connection to the starting winding. Again in contrast to its three-phase counterpart, this reversal (plugging) cannot be done under running conditions, since the split-phase motor torque will be much less than the torque developed by the single main winding, and rotation will not reverse.

In the event that the centrifugal switch contacts fuse, the starting winding will be permanently connected in the circuit during normal operation. Although in principle this does not affect motor operation significantly, it must be realized that this winding is designed for intermittent operation (for startup only). Therefore, when permanently connected in the circuit due to switch failure, it will quickly heat up and raise the motor temperature excessively and eventually burn out the windings.

As Figure 9.6b shows, the starting winding current \( I_s \) lags the supply voltage by about 15°; the running or main winding current \( I_m \) lags the voltage by about 45°. Although the currents are not equal, their quadrature or in-phase components with the voltage are nearly the same. This is illustrated in Example 9.1.

**EXAMPLE 9.1**

A 1/4-hp 120-V split-phase motor draws at the instant of starting a current of 4 A in its starting winding, while the main winding current takes 5.8 A, lagging the supply voltage by 15° and 45°, respectively. At startup, determine:
a. the line current and power factor, and
b. the in-phase components of the currents with the supply voltage.

**Solution**

\[
I_{\text{start}} = 4.2 - j1.04 \text{ A}
\]
\[
I_{\text{main}} = 5.8 - j4.10 \text{ A}
\]
\[
I_{\text{line}} = I_{\text{start}} + I_{\text{main}} = 7.96 - j5.14 = 9.48 \angle -33^\circ \text{ A}
\]

Power factor: \( \cos(-33^\circ) = 0.84 \) lagging

b. From the calculated results in part (a) we see that the in-phase components of currents with the line voltage, being the real parts of the respective currents, are 3.86 A and 4.10 A for the starting winding and main winding current, respectively. As shown, these components are practically equal at the instant of starting.

**Capacitor-Start Motors**

In the split-phase motor the phase shift between stator currents was accomplished by adjusting the impedances of the windings by making the starting winding a relatively higher resistance. This resulted in a phase shift of nearly 30°. Since the developed torque of any split-phase motor is proportional to the pole flux produced and the rotor current, it is also dependent on the angle between the winding currents. This implies that if a capacitor is connected in series with the starting winding, the starting torque will increase. This is indeed the case. By proper selection of the capacitor value the current in the starting winding will lead the voltage across it and a greater displacement between winding currents is obtained. This results in a significantly greater starting torque than that obtained in split-phase motors, as Example 9.2 will illustrate. Typical starting torques may be in the range of four times rated torque.

Figure 9.8 shows the capacitor start motor and its corresponding phasor diagram, indicating a typical displacement between winding currents of about 80°. The value of capacitor needed to accomplish this is typically 135 \( \mu \text{F} \) for a 1/4-hp motor and 175 \( \mu \text{F} \) for a 1/3-hp motor. Since they are rated for ac line voltages their size is about 1 1/2 in. in diameter and 3 1/2 in. long. Contrary to the split-phase motor discussed, the capacitor-start motor under running conditions is reversible. If temporarily disconnected from the supply line, its speed will drop, allowing the centrifugal switch to close. The lead connections to the starting winding are reversed during this interval and the motor reconnected to the supply once the centrifugal switch closes. The resulting rotating field will now rotate opposite to the direction the motor rotates.

Since the current displacement between windings is much larger in this motor than in the split-phase motor, the torque being proportional to this will be much larger and exceed the torque produced by the rotor. Therefore, the motor will slow down, stop, and then reverse its direction. Once up to about 75 to 85% of synchronous speed, the centrifugal switch opens and the motor will reach a speed as dictated by the load.

**FIGURE 9.8.** Capacitor-start induction motor: (a) circuit diagram; (b) phasor diagram; (c) torque-speed characteristic.

Because of their higher starting torques, capacitor-start motors are used in applications where not only higher starting torques are required, but also where reversible motors are needed. Applications of capacitor motors are in washing machines, belt fans and blowers, dryers, pumps, and compressors.

**EXAMPLE 9.2**

A capacitor is added to the starting winding of the motor in Example 9.1, with the result that its current now leads the voltage by 40°. The main winding remains as is.

a. With this added capacitor, determine at the instant of starting the line current and the power factor.

b. Compare the results with those calculated in Example 9.1.

**Solution**

a. From Example 9.1, \( Z_{\text{start}} = 120/40^\circ = 30/15^\circ = 28.98 + j7.76 \). With the added capacitor, the starting winding impedance becomes
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\[ Z_{\text{start}} = 28.989 - j24.32 = 37.83 \Omega - 40^\circ \]

Hence,

\[ I_{\text{start}} = 3.17 \times 40 = 2.43 + j2.04 \text{ A} \]
\[ I_{\text{max}} = 5.8 \times 45 = 4.10 - j4.10 \text{ A} \]
\[ I_{\text{line}} = I_{\text{start}} + I_{\text{max}} = 6.53 - j2.06 = 6.85 \Omega - 18^\circ \text{ A} \]

Power factor = \[ \cos(-18^\circ) = 0.95 \text{ lagging} \]

Note that the value of the capacitor added is 82.7 μF.

b. The line current has been reduced from 9.48 A to 6.85 A and the power factor improved. The motor starting torque, being proportional to the sine of the angle between the winding currents, has also been increased and becomes maximum with minimum starting current. It can be shown that the starting torque of the motor with added capacitor, compared to that without, increases by a factor of

\[ \frac{T_c}{T} = \frac{\sin[40^\circ - (-45^\circ)]}{\sin[45^\circ - (-15^\circ)]} \times \frac{3.17}{4.0} = 1.58 \]

where the subscript \( c \) indicates the developed torque with the added capacitor, and \( T \) that without.

**Capacitor-Run Motors**

In this motor, the capacitor is in series with the auxiliary winding and is not switched out after starting. It remains in the circuit, and therefore the centrifugal switch is not needed. Since the capacitor is in the circuit continuously, it must be an ac paper oil type. The power factor is improved, as is the efficiency, but the capacitor value (on the order of 50 μF) is a compromise between best starting torque and running torque values, hence the starting torque is compromised.

**Capacitor-Start Capacitor-Run Motors**

The capacitor-start motor described has a high starting torque, but average running torque. For many applications this does not present a serious limitation. In cases where high starting torques and high running torques are required, best results will be obtained if a large value of capacitance is used at startup which is then gradually decreased as the speed increases. In practice, two capacitors are used for starting and one is cut out of the circuit by a centrifugal switch once a certain speed is reached, usually at about 75% of full speed. The starting or intermittent capacitor is of fairly high capacity (usually on the order of 10 times the value of the running capacitor, for example, a 0.5 hp motor has \( C_t = 300 \mu F, C_{run} = 40 \mu F \)), which remains in the circuit. Figure 9.9 illustrates the connection diagrams for the capacitor-start capacitor-run motor, showing two methods generally encountered.

![Figure 9.9. Capacitor-start capacitor-run motors: (a) two-value capacitor motor; (b) capacitor motor with autotransformer.](image)

The first method, as shown in Figure 9.9a, uses an electrolytic capacitor in the starting circuit, which is not built to be left in the circuit continuously, since its leakage is too high. Being oil-filled, the second capacitor remains in the circuit; it has little leakage and therefore is suitable for continuous operation.

The second circuit (see Figure 9.9b) uses an autotransformer but only one oil-filled high-voltage capacitor. This method utilizes the transformer principle of reflected impedance from secondary to the primary. As we have discussed, this is proportional to the square of the secondary-to-primary turns ratio. For example, an autotransformer with 180 turns tapped at the 30-turn point would reflect an 8-μF running capacitor to the primary as

\[ \left( \frac{180}{30} \right)^2 \times 8 \mu F = 288 \mu F \]

representing an increase of about 36 times. Thus a running oil-filled capacitor may be used for starting purposes as well, thereby eliminating one capacitor in lieu of the autotransformer, which is of comparable cost. Care must be exercised to ensure that the capacitor can withstand the stepped-up voltage, which is 180/30 = 6 times the rated voltage at startup. For instance, a 120-V motor would have a capacitor voltage at the instant of starting of 720 V. Typically, a 1000-V ac rating capacitor is required.

As is the case with the capacitor-start motor, the capacitor-start capacitor-run motor may be damaged for identical reasons if the centrifugal switch fails to operate properly. The primary advantage, then, of a two-value capacitor motor is its high starting torque, good running torque, and quiet operation. Reversing the line leads to one of the windings in the usual manner causes the motor to operate in the opposite direction. It is therefore classed as a reversible-type motor. In operations requiring frequent reversals it is preferred to use a single-value capacitor-run motor using no centrifugal switch.

Single-phase motors operate generally at relatively poor power factors, on the order of 0.5–0.6. Because of energy conservation awareness, considerable attention is being directed toward improving these low power factor devices. Presently, capacitor-start and...
capacitor-run single-phase motors have received the most attention, and as a consequence they now have power factors of up to 0.8.

**Shaded-Pole Motors**

Figure 9.10 illustrates the shaded-pole motor, consisting of a laminated stator core having salient poles and a concentrated main winding. The poles are divided in two parts, the smaller part of which is "shaded"; that is, it contains an auxiliary winding consisting of a single short-circuited turn of copper, called the shading coil. When the main winding is connected to an ac source, the magnetic field will sweep across the pole face from the unshaded to the shaded portion. This, in effect, is equivalent to an actual physical motion of the pole, the result is that the squirrel-cage rotor will rotate in the same direction.

To understand how this sweeping action occurs, let us consider the instant of time when the current flowing in the main winding is increasing most rapidly, as illustrated in Figure 9.11. The main flux \( \phi \) will start to build up in phase with the current. However, the current induced in the shading coil produces an opposing flux, according to Lenz's law. The net result of this is that the flux in the shaded pole portion is less compared to that of the main portion of the pole. When the current in the main winding is at or near its maximum value, the flux does not change appreciably. With an almost constant flux, no voltage is induced in the shading coil and therefore it does not influence the main flux. The result is that the resultant magnetic flux shifts to the center of the pole. A short time later, when the current in the main winding is decreasing at its maximum rate, the flux in the unshaded portion of the pole decreases. However, because of currents induced in the shading coil, it tends to oppose this decrease in flux in the shaded portion of the pole. The result of this action translates into a movement of the magnetic flux axis toward the center of the shaded portion of the pole. Hence, the flux \( \phi_s \) continues to lag behind the flux \( \phi \) during this part of the cycle.

It can similarly be reasoned that at any instant of the current cycle, the flux \( \phi_1 \) lags behind \( \phi \) in time. The net effect of this time and space displacement is to produce a gliding flux across the pole face and consequently in the air gap, which is always directed toward the shaded portion of the pole. Therefore, the direction of rotation of a shaded-pole motor is always from the unshaded part toward the shaded part of the pole.

Simple motors of this type cannot be reversed, but must be assembled so that the rotor shaft extends from the correct end in order to drive the load in the proper direction. There are specially designed shaded-pole motors which are reversible.

Offsetting the simple construction and low cost of this motor are the low starting torque, low overload capacity, and low efficiencies (5 to 35%). These motors are built in sizes ranging from 1/250 hp up to about 1/20 hp. Typical applications of shaded-pole motors are where efficiencies are of "minor concern," such as in toys and fans. Since the applied voltage to the motor greatly affects its speed under load, as the slip increases with reduced voltage, advantage is taken of this fact, particularly when driving fans. Practically, this is generally done by providing line voltage taps to the excitation winding. With fewer turns on this winding, the volts/tturn, as well as the current, is larger than with a full winding. Since the volts/tturn ratio is proportional to the flux, it in turn increases and the motor runs at a greater speed, since a larger torque is developed.

**9.2 Universal (Series) Motors**

Motors that can be used on ac as well as dc sources are called universal motors. All these motors are of the dc series motor type. The direction of the developed torque is determined