ECE 3600 Induction Motor Examples

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Ex. 1 A 480-V. six-pole, 60-Hz, Δ-connected, 3-phase induction motor is rated at 60 hp.
Its equivalent circuit components are

$R_1 := 0.480\cdot\Omega$ $\quad R_2 := 0.6\cdot\Omega$ $\quad R_C := \infty\cdot\Omega$

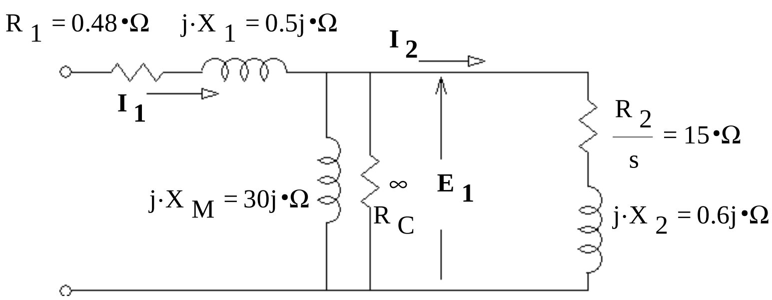
$X_1 := 0.50\cdot\Omega$ $\quad X_2 := 0.60\cdot\Omega$ $\quad X_M := 30\cdot\Omega$

$P_{mech} := 600\cdot W$ $\quad P_{misc} := 150\cdot W$ $\quad P_{core} := 200\cdot W$

$N_{poles} := 6$ $\quad V_\phi := 480\cdot V$ $\quad V_\phi = 480\cdot V$

For a slip of 0.04 $\quad s := 4\cdot\%$, find:

a) The line current (magnitude)



$$\mathbf{Z_{E1}} := \frac{1}{\dfrac{1}{j\cdot X_M} + \dfrac{1}{R_C} + \dfrac{1}{\dfrac{R_2}{s} + j\cdot X_2}}$$

$$\mathbf{Z_{E1}} = 11.624 + 6.286j \cdot\Omega$$

$$\mathbf{Z_{eq}} := R_1 + j\cdot X_1 + \mathbf{Z_{E1}} \qquad \mathbf{Z_{eq}} = 12.104 + 6.786j \cdot\Omega$$

$$\mathbf{I_1} := \frac{V_\phi}{\mathbf{Z_{eq}}} \qquad \mathbf{I_1} = 30.171 - 16.916j \cdot A$$

$$|\mathbf{I_1}| = 34.59\cdot A \qquad \arg(\mathbf{I_1}) = -29.278\cdot deg$$

Δ-connected $\quad I_L = |\mathbf{I_1}|\cdot\sqrt{3} = 59.91\cdot A$

b) The stator copper losses $\quad P_{SCL} := 3\cdot\left[(|\mathbf{I_1}|)^2\cdot R_1\right]$ $\qquad P_{SCL} = 1.723\cdot kW$

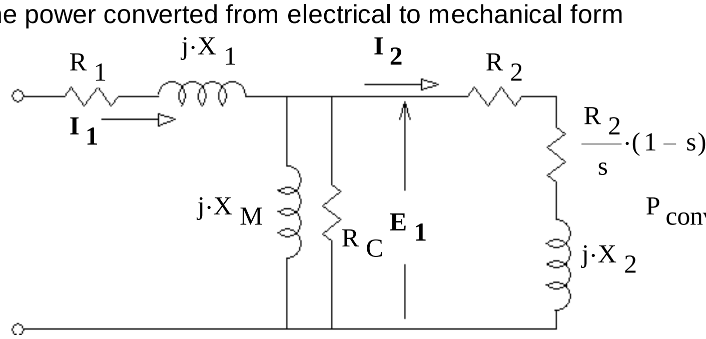
c) The air-gap power P_{AG}

$$\mathbf{E_1} := \mathbf{I_1}\cdot\mathbf{Z_{E1}} \qquad \mathbf{E_1} = 457.06 - 6.966j \cdot V$$

$$\mathbf{I_2} := \frac{\mathbf{E_1}}{j\cdot X_2 + \dfrac{R_2}{s}} \qquad P_{AG} := 3\cdot\left[(|\mathbf{I_2}|)^2\cdot\frac{R_2}{s}\right] \qquad P_{AG} = 41.72\cdot kW$$

$$P_{AG} = 55.95\cdot hp$$

d) The power converted from electrical to mechanical form



$$P_{conv} = 3\cdot\left[(|\mathbf{I_2}|)^2\cdot\frac{R_2}{s}\cdot(1-s)\right] = (1-s)\cdot P_{AG}$$

$$P_{conv} := (1-s)\cdot P_{AG} \qquad P_{conv} = 40.05\cdot kW$$

$$P_{conv} = 53.71\cdot hp$$

e) The motor speed in revolutions per minute and radians per second

$$n_{sync} := \frac{7200\cdot rpm}{N_{poles}} \qquad n_{sync} = 1200\cdot rpm \qquad \omega_{sync} := n_{sync}\cdot\left(2\cdot\pi\cdot\frac{rad}{rev}\right)\cdot\left(\frac{min}{60\cdot sec}\right) \qquad \omega_{sync} = 125.7\cdot\frac{rad}{sec}$$

$$n_m := (1-s)\cdot n_{sync} \qquad n_m = 1152\cdot rpm \qquad \omega_m := (1-s)\cdot\omega_{sync} \qquad \omega_m = 120.6\cdot\frac{rad}{sec}$$

f) The induced torque $\tau_{ind} := \dfrac{P_{conv}}{\omega_m}$ OR: $\tau_{ind} := \dfrac{P_{AG}}{\omega_{sync}}$ (easier) $\tau_{ind} = 332 \cdot N \cdot m$

g) The load torque τ_{load}

Use P_{core} here. Lump it in with the mechanical losses, P_{misc} and P_{mech}. Read the last 2 paragraphs on p.302.

$$P_{out} := P_{conv} - P_{core} - P_{mech} - P_{misc} \qquad P_{out} = 39.105 \cdot kW$$

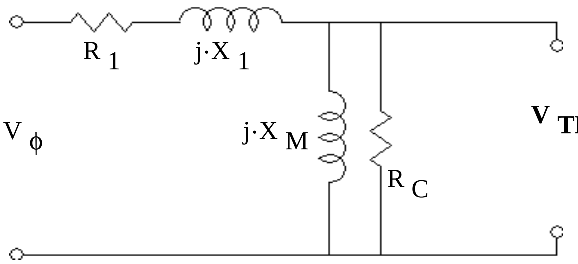
$$\tau_{load} := \frac{P_{out}}{\omega_m} \qquad \tau_{load} = 324.152 \cdot N \cdot m$$

h) The overall machine efficiency $\eta := \dfrac{P_{out}}{3 \cdot V_{\phi} \cdot \mathrm{Re}(\mathbf{I_1})}$ $\eta = 90.006 \cdot \%$

Ex. 2 For the motor in Example 1,

a) what is the slip at the pullout torque?

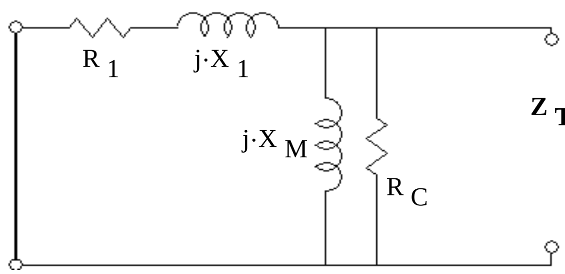
Create a Thevenin equivalent of the model with ($R_2/s + X_2$) as the load.



$$\mathbf{V_{TH}} := V_{\phi} \cdot \frac{j \cdot X_M}{R_1 + j \cdot X_1 + j \cdot X_M}$$

$$\mathbf{V_{TH}} = 472.014 + 7.428j \cdot V$$

$$\left|\mathbf{V_{TH}}\right| = 472.073 \cdot V$$



$$\mathbf{Z_{TH}} := \frac{1}{\dfrac{1}{R_1 + j \cdot X_1} + \dfrac{1}{j \cdot X_M}}$$

$$\mathbf{Z_{TH}} = 0.464 + 0.499j \cdot \Omega$$

$$\left|\mathbf{Z_{TH}}\right| = 0.6817 \cdot \Omega \qquad \arg\left(\mathbf{Z_{TH}}\right) = 47.07 \cdot deg$$

At max P_{AG} $\dfrac{R_2}{s} = \left|\mathbf{Z_{TH}} + j \cdot X_2\right| = 1.193 \cdot \Omega$ $s := \dfrac{R_2}{\left|\mathbf{Z_{TH}} + j \cdot X_2\right|}$ $s = 50.287 \cdot \%$

b) What is the pullout torque of this motor?

$n_m := (1 - s) \cdot n_{sync}$ $n_m = 596.553 \cdot rpm$ $\omega_m := n_m \cdot \left(2 \cdot \pi \cdot \dfrac{rad}{rev}\right) \cdot \left(\dfrac{min}{60 \cdot sec}\right)$ $\omega_m = 62.471 \cdot \dfrac{rad}{sec}$

$$\mathbf{I_2} := \frac{\mathbf{V_{TH}}}{\mathbf{Z_{TH}} + j \cdot X_2 + \dfrac{R_2}{s}}$$

$$P_{AG} := 3 \cdot \left[\left(\left|\mathbf{I_2}\right|\right)^2 \cdot \frac{R_2}{s}\right] \qquad P_{AG} = 201.686 \cdot kW$$

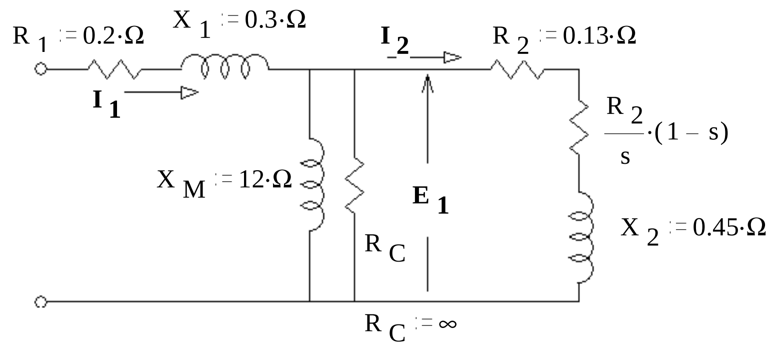
$$P_{conv} := (1 - s) \cdot P_{AG} \qquad P_{conv} = 100.264 \cdot kW$$

$$\tau_{max} := \frac{P_{conv}}{\omega_m} \qquad \tau_{max} = 1605 \cdot N \cdot m$$

Ex. 3 A 3-phase, Y-connected, induction motor has the following equivalent circuit components:

currently running at $n := 3500 \cdot rpm$

current air-gap power $P_{AG} := 7.5 \cdot kW$



a) Find $|\mathbf{I_2}|$

This has to be a 2-pole motor, so $n_{sync} := 3600 \cdot rpm$ and $s := \frac{3600 - 3500}{3600}$ $s = 2.778 \cdot \%$

$\frac{P_{AG}}{3} = I_2^2 \cdot \frac{R_2}{s}$ $I_2 := \sqrt{\frac{P_{AG}}{3} \cdot \frac{s}{R_2}}$ $I_2 = 23.113 \cdot A$

b) Find the rotor copper losses $P_{RCL} := 3 \cdot I_2^2 \cdot R_2$ $P_{RCL} = 208.333 \cdot W$

c) The output shaft torque is $\tau_{load} := 19 \cdot N \cdot m$

Find the output power $P_{out} := \tau_{load} \cdot n \cdot \frac{2 \cdot \pi \cdot rad}{rev} \cdot \frac{min}{60 \cdot sec}$ $P_{out} = 6.964 \cdot kW$

d) Find the mechanical power losses (all lumped together).

$P_{mech} := P_{AG} - P_{RCL} - P_{out}$ $P_{mech} = 327.803 \cdot W$

e) Find the line current. Note: Don't try any shortcuts here. You need to do your math with full complex numbers.

$|\mathbf{I_L}| = ?$ I advise you to assume the phase angle of $\mathbf{I_2}$ is 0^o.

$\mathbf{E_1} := I_2 \cdot \left(\frac{R_2}{s} + j \cdot X_2\right)$ $\mathbf{E_1} = 108.167 + 10.401j \cdot V$

$\mathbf{I_L} := I_2 + \frac{\mathbf{E_1}}{j \cdot X_M}$ $|\mathbf{I_L}| = 25.617 \cdot A$

$\mathbf{V_\phi} := \mathbf{E_1} + \mathbf{I_L} \cdot (R_1 + j \cdot X_1)$ $|\mathbf{V_\phi}| = 116.7 \cdot V$

NOT asked for

f) The stator copper losses $P_{SCL} := 3 \cdot \left(|\mathbf{I_L}|\right)^2 \cdot R_1$ $P_{SCL} = 393.75 \cdot W$

g) The overall machine efficiency $\eta = \frac{P_{out}}{P_{SCL} + P_{AG}} = 88.22 \cdot \%$

A 480-V. four-pole, 60-Hz, Y-connected, induction motor is rated at 20 hp. Its equivalent circuit components are:

$N_{poles} := 4$ $R_1 := 1 \cdot \Omega$ $R_2 := 0.6 \cdot \Omega$

$X_1 := 2 \cdot \Omega$ $X_2 := 2 \cdot \Omega$ $X_M := 75 \cdot \Omega$

$P_{mech} := 300 \cdot W$ $P_{misc} := 0 \cdot W$ $P_{core} := 0 \cdot W$

For a slip of 0.06, the following values have been calculated for you: $s := 0.06$

$\mathbf{E_1} := (234.208 - 32.217 \cdot j) \cdot V$ $|\mathbf{E_1}| = 236.413 \cdot V$ The line current magnitude: $I_L := 24 \cdot A$

Find the following:

a) The stator copper losses

$P_{SCL} := 3 \cdot \left(I_L^{\,2} \cdot R_1\right)$ $P_{SCL} = 1.728 \cdot kW$

b) The air-gap P_{AG}

$$I_2 := \frac{|\mathbf{E_1}|}{\sqrt{X_2^{\,2} + \left(\frac{R_2}{s}\right)^2}} \qquad P_{AG} := 3 \cdot \left(I_2^{\,2} \cdot \frac{R_2}{s}\right) \qquad P_{AG} = 16.122 \cdot kW$$

c) The power converted from electrical to mechanical form

$P_{conv} := (1 - s) \cdot P_{AG}$ $P_{conv} = 15.155 \cdot kW$

d) The motor speed in revolutions per minute and radians per second

$$n_{sync} := \frac{7200 \cdot rpm}{N_{poles}} \qquad n_{sync} = 1800 \cdot rpm \qquad \omega_{sync} := n_{sync} \cdot \left(2 \cdot \pi \cdot \frac{rad}{rev}\right) \cdot \left(\frac{min}{60 \cdot sec}\right)$$

$\omega_{sync} = 188.496 \cdot \frac{rad}{sec}$ OR $\frac{377}{2} = 188.5 \frac{rad}{sec}$

$$n_m := (1 - s) \cdot n_{sync} \qquad n_m = 1692 \cdot rpm \qquad \omega_m := n_m \cdot \left(2 \cdot \pi \cdot \frac{rad}{rev}\right) \cdot \left(\frac{min}{60 \cdot sec}\right) \qquad \omega_m = 177.186 \cdot \frac{rad}{sec}$$

e) The induced torque τ_{ind}

$\tau_{ind} := \frac{P_{conv}}{\omega_m}$ OR: $\tau_{ind} := \frac{P_{AG}}{\omega_{sync}}$ $\tau_{ind} = 85.533 \cdot N \cdot m$

f) The load torque τ_{load}

$P_{out} := P_{conv} - P_{core} - P_{mech} - P_{misc}$ $P_{out} = 14.855 \cdot kW$

$P_{out} = 19.921 \cdot hp$

$\tau_{load} := \frac{P_{out}}{\omega_m}$ $\tau_{load} = 83.839 \cdot N \cdot m$

g) The overall machine efficiency $\eta = \frac{P_{out}}{P_{SCL} + P_{AG}} = 83.22 \cdot \%$

h) Is this motor running close to it's rated output?

Yes, $P_{out} = 19.921 \cdot hp$ rating is 20 hp