#### **Armature**

The rotating part (rotor)

## Field (Excitation)

Provided by the stationary part of the motor (Stator)

Permanent Magnet

Winding

Separately excited

Parallel with terminal voltage source (Shunt excited)

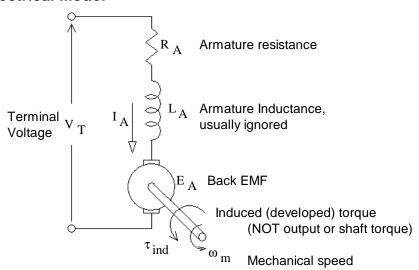
Series with terminal voltage source (Series excited)

#### **Commutator and commutation**

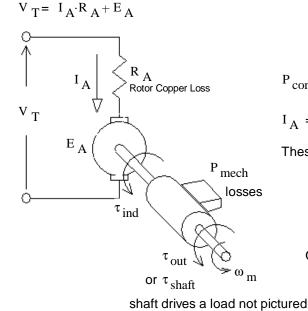
Rotary contacts and brushes which keep switching the current direction in the armature so that the motor torque is always in the same direction.

Explained and visualized in class

## **Electrical Model**



#### Simplified Model we will use



$$P_{conv} = E_A \cdot I_A = \tau_{ind} \cdot \omega_m$$

$$I_A = \frac{P_{conv}}{E_A}$$
  $E_A = \frac{P_{conv}}{I_A}$ 

These are often substituted in other eq.

$$V_T = I_A \cdot R_A + E_A$$
 becomes:

$$V_{T} = I_{A} \cdot R_{A} + \frac{P_{conv}}{I_{A}}$$
 OR  $0 = I_{A}^{2} - \frac{V_{T}}{R_{A}} \cdot I_{A} + \frac{P_{conv}}{R_{A}}$ 

$$V_{T} = \frac{P_{conv}}{E_{A}} \cdot R_{A} + E_{A} \qquad OR \qquad 0 = E_{A}^{2} - V_{T} \cdot E_{A} + P_{conv} \cdot R_{A}$$

Rotor
Stator

Important relationships

$$E_A = K \cdot \phi \cdot \omega$$

$$\tau_{ind} = K \cdot \phi \cdot I_A$$

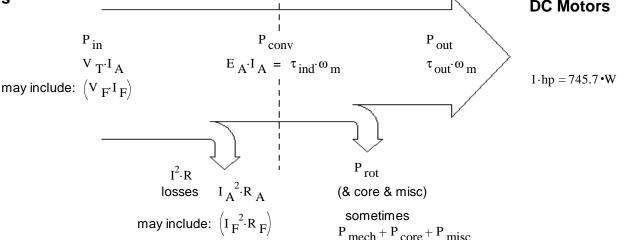
#### If the field is constant

$$\frac{\omega_2}{\omega_1} = \frac{E_{A2}}{E_{A1}} = \frac{n_2}{n_1}$$

$$\frac{\tau_{ind2}}{\tau_{ind1}} = \frac{I_{A2}}{I_{A1}}$$

If field decreases, speed can increase to an unsafe level





# **Mechanical Loads and losses**

$$P = \tau \cdot \omega_m$$

Constant power

If an output or loss power is constant for all speeds, then the torque is inversely proportional to the speed.

This is highly unlikely in real life.

Proportional to speed

If a power is proportional to speed, then the torque is constant with speed.

Conversely, if a torque is constant for all speeds, the power is proportional to speed.

A bit more likely in real life.

Torque proportional to speed

Power is proportional to the square of the speed

Approximates more real-life loads, torque required to turn a load usually increases with speed.

## **Nameplate Operation**

The Nameplate gives the rated Voltage, Current(s), Speed and output Power (often as horsepower, hp).  $1 \cdot hp = 745.7 \cdot W$  This is considered full-load operation.

## Motor Constant, K

Our book defines the motor constant such that it does not include the field flux,  $\phi$ . Often the motor constant is defined differently, as  $K\phi$ , but just called K, which is a function of the field current.  $K\phi$  (or K) is most easily found by operating the motor as a generator with no load, then  $V_T = E_A$ .

**Spin Direction:** Reverse the leads to either of the windings and the motor will run in the opposite direction.

Torque - Speed curves

$$V_T = I_A \cdot R_A + E_A$$

becomes: 
$$V_T = \frac{\tau_{ind}}{K \cdot \phi} \cdot R_A + K \cdot \phi \cdot \omega_m$$
 
$$\tau_{ind} = \left( V_T - K \cdot \phi \cdot \omega_m \right) \cdot \frac{K \cdot \phi}{R_A}$$

At 0 speed (locked rotor): 
$$V_T = \frac{\tau_{ind}}{K \cdot \phi} \cdot R_A$$
  $\tau_{ind} = (V_T) \cdot \frac{K \cdot \phi}{R_A}$ 

At max speed (no induced torque):  $V_T = K \cdot \phi \cdot \omega_m$ 

$$0 = \left( V_T - K \cdot \phi \cdot \omega_m \right) \cdot \frac{K \cdot \phi}{R_A}$$

**p2** 

#### **Output Torque instead of Induced Torque**

If lost torque is proportional to speed:

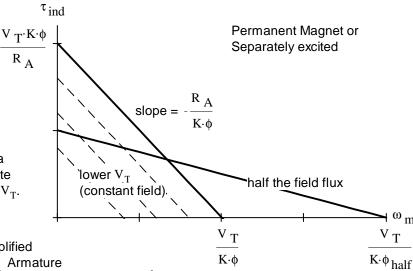
$$\tau_{shaft} = \tau_{ind} - fric \cdot \omega_{m} \qquad V_{T} = \frac{\tau_{shaft} + fric \cdot \omega_{m}}{K \cdot \phi} \cdot R_{A} + K \cdot \phi \cdot \omega_{m} \qquad = \frac{\tau_{shaft}}{K \cdot \phi} \cdot R_{A} + \left(\frac{fric \cdot R_{A}}{K \cdot \phi} + K \cdot \phi\right) \cdot \omega_{m}$$

DC Motors p2

#### **Permanent Magnet**

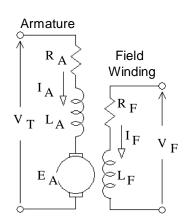
Permanent-Magnet motors are typically small. They can be quite powerful for their size, especially if made with rare-earth magnets. These motors are common in children's toys and servo systems. Some electric cars use Large permanent-magnet DC motors.

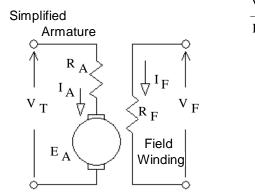
The characteristics are like the separately excited motor with a constant flux. Since the flux can't be changed, the motor constant times flux,  $K\phi$ , is usually simplified to just a different motor constant, usually also called K, which includes the constant flux.



#### **Separately Excited**

The field flux is by current flowing through a field winding, which is supplied by a separate power source or at a different voltage than  $V_{\rm T}$ .

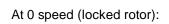


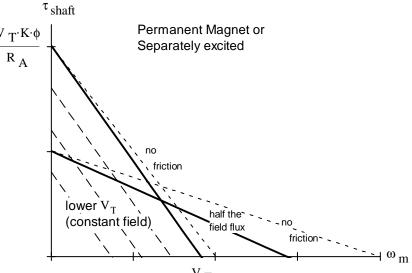


Output Torque instead of Induced Torque, If lost torque is proportional to speed:

$$\tau_{\text{shaft}} = \tau_{\text{ind}} - \text{fric} \cdot \omega_{\text{m}}$$

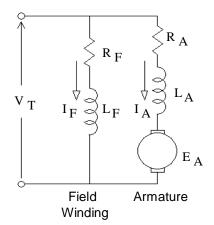
$$V_{T} = \frac{\tau_{shaft} + \operatorname{fric} \cdot \omega_{m}}{K \cdot \phi} \cdot R_{A} + K \cdot \phi \cdot \omega_{m} = \frac{\tau_{shaft}}{K \cdot \phi} \cdot R_{A} + \left(\frac{\operatorname{fric} \cdot R_{A}}{K \cdot \phi} + K \cdot \phi\right) \cdot \omega_{m}$$





At max speed (no induced torque):  $\omega_m = \frac{\sqrt{T}}{\frac{\text{fric} \cdot R}{A} + K \cdot e}$ 

Field winding is connected in parallel with the armature to the same terminal voltage source, V<sub>T</sub>.



If flux is proportional to field current

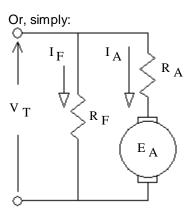
$$\phi = c \cdot I_F = c \cdot \frac{V_T}{R_F}$$
  $c = \text{the "core constant"}$ 

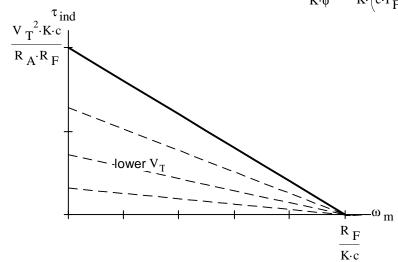
At 0 speed (locked rotor):  $V_T = \frac{\tau_{ind}}{K \cdot \phi} \cdot R_A$ 

so: 
$$\tau_{ind} = (V_T) \cdot \frac{K \cdot \phi}{R_A} = (V_T) \cdot \frac{K \cdot \left(c \cdot \frac{V_T}{R_F}\right)}{R_A} = \frac{V_T^2 \cdot K \cdot c}{R_A \cdot R_F}$$

At max speed (no induced torque):  $V_T = K \cdot \phi \cdot \omega_m$ 

$$\omega_{\rm m} = \frac{V_{\rm T}}{K \cdot \phi} = \frac{V_{\rm T}}{K \cdot (c \cdot I_{\rm F})} = \frac{R_{\rm F}}{K \cdot c}$$





$$\tau_{\text{shaft}} = \tau_{\text{ind}} - \text{fric} \cdot \omega_{\text{m}}$$

$$V_{T} = \frac{\tau_{shaft} + fric \cdot \omega_{m}}{K \cdot \phi} \cdot R_{A} + K \cdot \phi \cdot \omega_{m}$$

Output Torque instead of Induced Torque, If lost torque is proportional to speed: 
$$\tau_{shaft} = \tau_{ind} - \operatorname{fric} \cdot \omega_m \qquad V_T = \frac{\tau_{shaft} + \operatorname{fric} \cdot \omega_m}{K \cdot \phi} \cdot R_A + K \cdot \phi \cdot \omega_m \qquad = \frac{\tau_{shaft}}{K \cdot \phi} \cdot R_A + \left(\frac{\operatorname{fric} \cdot R_A}{K \cdot \phi} + K \cdot \phi\right) \cdot \omega_m$$

At max speed (no shaft torque): 
$$\omega_m = \frac{V_T}{\frac{\text{fric} \cdot R_A}{K \cdot \left(c \cdot \frac{V_T}{R_F}\right)} + K \cdot \left(c \cdot \frac{V_T}{R_F}\right)}$$

$$= \frac{V_T}{\frac{\text{fric} \cdot R_A \cdot R_F}{K \cdot c \cdot V_T} + K \cdot c \cdot \frac{V_T}{R_T}}$$

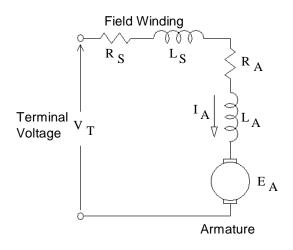
$$= \frac{1}{V_T^2 \cdot \frac{\text{fric} \cdot R_A \cdot R_F}{K \cdot c} + \frac{K \cdot c}{R_F}}$$

$$\frac{V_{T}^{2} \cdot K \cdot c}{R_{A} \cdot R_{F}}$$

$$\frac{V_{T}^{2} \cdot K \cdot c}{R_{A} \cdot R_{F}}$$

$$\frac{1}{V_{T}^{2} \cdot \frac{\text{fric} \cdot R_{A} \cdot R_{F}}{K \cdot c} + \frac{K \cdot c}{R_{F}}}$$

Field winding is connected in series with the armature and is designed with much thicker windings, so it can handle much larger current and has much less resistance. The resistance of the field winding is now called R<sub>S</sub>. Since R<sub>S</sub> is in series with R<sub>A</sub>, they are often combined into R<sub>AS</sub>.



Simplified Model  $V_T = I_A \cdot R_{AS} + E_A$  $P_{\text{mech}}$ loses  $V_T = I_A \cdot R_{AS} + E_A$ or  $\tau_{\, shaft}$  $V_T = I_A \cdot R_{AS} + K \cdot \phi \omega_m$ 

If flux is proportional to field current  $\phi = c \cdot I_A$ 

$$I_{A} = \frac{\tau_{ind}}{K \cdot \phi} = \frac{\tau_{ind}}{K \cdot (c \cdot I_{A})}$$

$$I_{A}^{2} = \frac{\tau_{ind}}{K \cdot c} \quad \text{so: } I_{A} = \sqrt{\frac{\tau_{ind}}{K \cdot c}}$$

$$V_T = I_A \cdot R_{AS} + K \cdot (c \cdot I_A) \cdot \omega_m$$

$$V_T = \ I_{A} \cdot \left( R_{AS} + K \cdot c \cdot \omega_m \right)$$

$$V_{T} = \sqrt{\frac{\tau_{ind}}{\kappa \cdot c}} \left( R_{AS} + \kappa \cdot c \cdot \omega_{m} \right) = \sqrt{\frac{\tau_{ind}}{\kappa \cdot c}} \cdot R_{AS} + \kappa \cdot c \cdot I_{A} \cdot \omega_{m}$$

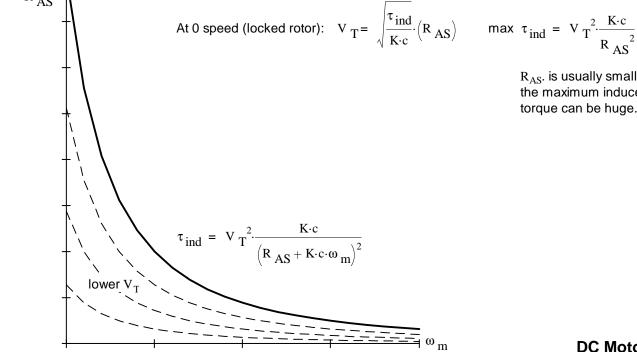
$$\omega_{m} = \frac{V_{T}}{\sqrt{K \cdot c} \cdot \sqrt{\tau_{ind}}} - \frac{R_{AS}}{K \cdot c}$$

Series Excited Torque - Speed curve

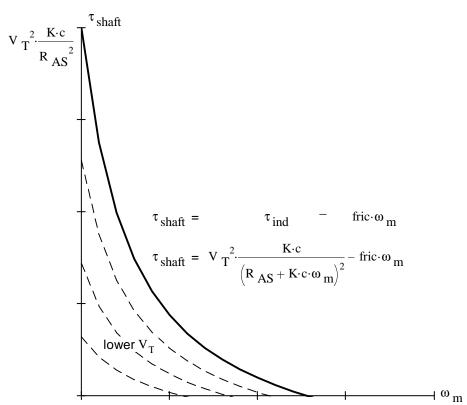
Max speed (no induced torque) is undefined

$$\text{max } \tau_{ind} = V_T^2 \cdot \frac{K \cdot c}{R_{AS}^2}$$

R<sub>AS</sub>. is usually small, so the maximum induced torque can be huge.



Output Torque instead of Induced Torque, If lost torque is proportional to speed:



# $\tau_{\text{shaft}} = \tau_{\text{ind}} - \text{fric} \cdot \omega_{\text{m}}$

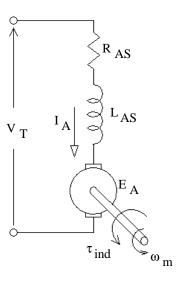
## AC/DC or Universal Motor (Series Excited DC motor)

Because the same current flows through both the field and the armature, the series-excited motor will turn the same direction even if the current flows the opposite direction-- or even if goes back and forth (AC). These motors are common in AC devices because they can provide a lot of power and torque in small, lightweight motor. They are very common in handheld power tools like drills, saws, grinders, weed eaters, hedge trimmers, etc.. They are also found in vacuum cleaners, blenders and food processors. If you look at the motor of an AC device and see brushes and a commutator, then it is a universal motor.

It is easy to vary the speed of these motors by changing the average voltage, usually with a thyristor-based control similar to a light dimmer.

Universal motors tend to be very noisy.

When used with AC supply, the inductance of the windings becomes important.



#### **Compounded Motor**

A motor with both shunt and series field windings. Covered in your book, starting on p. 420.

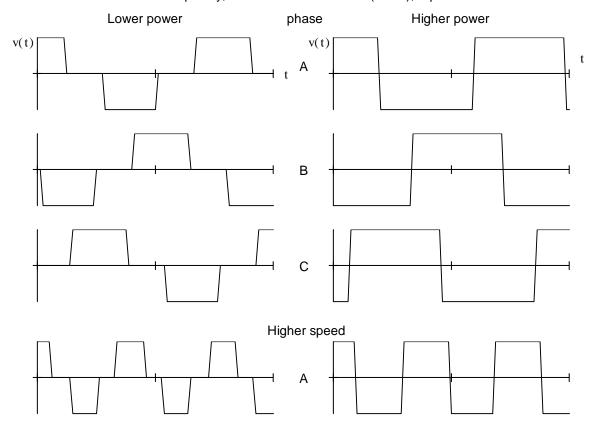
#### **Brushless DC Motors**

Many Brushless DC motors simply replace the commutator with a rotor position sensor (magnetic (Hall effect) optic, inductive, etc.) and power-electronic circuitry which switches the winding current. The field is usually on the rotor (permanent magnet or a winding fed through slip rings) and the armature windings are stationary. These motors are analyzed just like DC motors with brushes. Many DC fans are made this way.

Many Brushless DC motors are actually 3-phase Synchronous or Induction motors driven by power-electronic circuitry which produces variable-frequency, Pulse-Width-Modulated (PWM), 3-phase power to operate the motor. Actually, they don't have to be 3-phase, as long as they have at least 2 phases. 2, 3, 4, and 6 phases are common.

Brushless motors have some important advantages. They are mechanically simpler and more reliable. They can be operated in environments where the sparking between brushes and commutator would be undesirable or unsafe. They are relatively quiet.

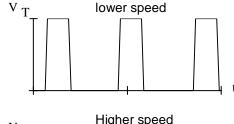
Some sample waveforms are shown on the next page.

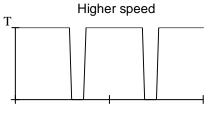


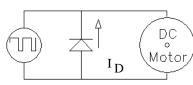
# Pulse Width Modulation (PWM) for speed control

As you can see by the torque - speed curves above, regulating the terminal voltage,  $V_T$ , is a very effective way to control the speed of a DC motor. Unfortunately, it is often an inefficient process. Pulse Width Modulation, shown here, is a very efficient. It is also more linear, especially at low speeds. The torque - speed curves do not show these non-linearities-- due largely to the difference between static and dynamic friction. (Motors are often a bit sticky at startup.)

If you do use PWM to control a motor, it is important to remember that the inductance within the windings will not allow the current to go to zero instantaneously. A diode (called a flyback, flywheel, or freewheel diode when used like this) provides a path for the current still flowing through the motor when a pulse is switched off.

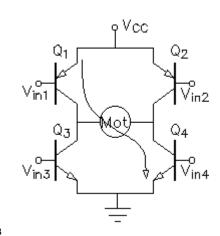






**H-bridge:** Of course, if you want to make the motor turn in both directions you'll need a more complex circuit. Look at the circuit at right, it's has the shape of an H, hence the name. If transistors  $Q_1$  and  $Q_4$  are on, then the current flows as shown, left-to-right through the motor. If transistors  $Q_2$  and  $Q_3$  are on, then the current flows the other way through the motor and the motor will turn in the opposite direction. (The motor here is a permanent-magnet DC motor.) In my circuit, the top two transistors are PNPs, which makes the circuit more efficient. The H-bridge could also be made with all NPNs or with power MOSFET transistors.

An H-bridge requires four inputs, all operated in concert. To turn on  $Q_1$  and  $Q_4$ , as shown,  $V_{in1}$  would have to be low and  $V_{in4}$  would have to be high. At the same time, the other two transistors would have to be off, so  $V_{in2}$  would have to be high and  $V_{in3}$  would have to be low.

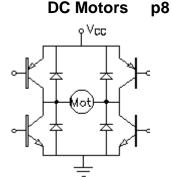


**DC Motors** 

If the control circuit makes a mistake and turns on  $Q_1$  and  $Q_3$  (or  $Q_2$  and  $Q_4$ ) at the same time you'll have a toaster instead of a motor driver, at least for a short while.

The circuit at left requires only two inputs. Transistors  $Q_5$  and  $Q_6$  work as *inverters*, when their inputs are high, their outputs are low and vice-versa. The resistors are known as *pull-up* resistors.

The H-bridge should also include flyback diodes.



# Regenerative Braking

Electric motors are not limited to converting electrical energy to mechanical. They can also convert energy from mechanical to electrical. If that is done for the purpose of mechanical braking, say in an electric car, then it's called Regenerative Braking. It is a way recover kinetic energy when slowing a moving mass, or potential energy of a mass moving from a higher to a lower elevation. Examples: a car coming to a stop at a traffic light or driving down Parley's canyon. This recovered energy can be used to recharge batteries or simply be wasted in resistors. Electric lawnmowers and some electric drills use this technique to stop the moving parts very quickly for safety.

## **Armature Reaction**

This phenomenon is well explained in your book, starting on p.372. One effect is a shift of the neutral plane of the field flux. This is a small twisting of the overall North - South orientation and can increase the sparking at the brushes. If the load and rotation direction are known and constant, a small twist of the brush location (in the same direction) will help mitigate the sparking. Interpoles (shown on p. 379) are a better solution. Please note that Fig. 8-11 on p. 374 is for a generator and that  $\omega$  will be in the opposite direction for a motor. The other effect is an overall flux weakening due to core saturation. A few series windings to shore up the flux at high loads can help with this.

#### **Brush Loss**

Sometimes the voltage drop across the brush-commutator connection is also considered. This voltage drop is usually estimated at about 2V for both brushes, regardless of the armature current. (p. 384 in book.)

# Characterizing an Unknown DC motor

For a motor that can be operated as separately excited and as a generator;

**Motor Constant:** Operate the motor as a generator with no load  $(I_A=0)$ , then  $V_T=E_A$ . Calculate  $K\phi$  from speed and  $E_A$  measurements. You may wish to calculate this at various field currents.

 $\mathbf{R_A}$ : Hook an electrical load to your still-spining generator. Adjust input power to return to no-load speed. Measure  $V_T$  and  $I_A$  and calculate  $R_A$ . You may wish to repeat at several loads and take an average.

Alternatively, measure  $V_T$ ,  $I_A$ , and  $\omega$  at 2 different mechanical loads, solve 2 equations for 2 unknowns,  $K\phi$  and  $R_A$ .

If you can't measure the rotational speed, but can measure the time required to move something a fixed distance and which would be inversely proportional to speed:

$$V_{T1} = I_{A1} \cdot R_A + \frac{K \cdot \phi K_T}{t_1} \qquad V_{T2} = I_{A2} \cdot R_A + \frac{K \cdot \phi K_T}{t_2} \qquad K_T \text{ is just another constant which is found together with } K_{\phi} \text{ as } K_{\phi} K_T.$$