ECE 3600 DC Motor Examples

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Ex.1 A 3-hp dc motor has the following nameplate information: 150 V, 1400 rpm, 18 A, $R_A = 0.8\ \Omega$, and $R_F = 300\ \Omega$. The field is shunt connected and the 18 A <u>includes</u> the field current. Assume rotational losses are constant.

a) Find the efficiency of the motor at nameplate operation. (Include the field in your calculations) $1\cdot hp = 745.7\cdot W$

$V_T := 150\cdot V$ $\quad n_{FL} := 1400\cdot rpm$ $\quad I_{FL} := 18\cdot A$ $\quad R_A := 0.8\cdot\Omega$ $\quad R_F := 300\cdot\Omega$

$P_{out} := 3\cdot hp\cdot\frac{745.7\cdot W}{hp}$ $\quad P_{out} = 2.237\cdot kW$

$P_{in} := V_T\cdot I_{FL}$ $\quad P_{in} = 2.7\cdot kW$ $\quad \eta = \frac{P_{out}}{P_{in}} = 82.86\cdot\%$

b) Find the rotational losses at nameplate operation.

field current: $I_F := \frac{V_T}{R_F}$ $\quad I_F = 0.5\cdot A$

armature full-load current: $I_{AFL} := I_{FL} - I_F$ $\quad I_{AFL} = 17.5\cdot A$

$E_{AFL} := V_T - I_{AFL}\cdot R_A$ $\quad E_{AFL} = 136\cdot V$

$P_{conv} := E_{AFL}\cdot I_{AFL}$ $\quad P_{conv} = 2.38\cdot kW$

$P_{rot} := P_{conv} - P_{out}$ $\quad P_{rot} = 142.9\cdot W$ $\quad P_{rot} = 0.192\cdot hp$

either answer

c) Find the required current for a developed power of 1.5 hp with $V_T = 150$ V.

$P_{conv} := 1.5\cdot hp$ $\quad P_{conv} = 1.119\cdot kW = E_A\cdot I_A$

$V_T = E_A + I_A\cdot R_A = \frac{P_{conv}}{I_A} + I_A\cdot R_A$ $\quad$ Rearrange $\quad 0 = R_A\cdot I_A^2 - V_T\cdot I_A + P_{conv}$

Solving for

$$I_A = \begin{bmatrix} \frac{1}{(2\cdot R_A)}\cdot\left(V_T + \sqrt{V_T^2 - 4\cdot R_A\cdot P_{conv}}\right) \\ \frac{1}{(2\cdot R_A)}\cdot\left(V_T - \sqrt{V_T^2 - 4\cdot R_A\cdot P_{conv}}\right) \end{bmatrix} = \begin{pmatrix} 179.72 \\ 7.78 \end{pmatrix}\cdot A$$

$I_A := 7.78\cdot A$

$I_S := I_A + I_F$

$I_S = 8.28\cdot A$

d) Find the output power if the developed power is 1.5 hp with $V_T = 150$ V.

$P_{out} := P_{conv} - P_{rot}$ $\quad P_{out} = 975.7\cdot W$ $\quad P_{out} = 1.308\cdot hp$

e) Find the shaft speed if the developed power is 1.5 hp with $V_T = 150$ V.

$E_A := \frac{P_{conv}}{I_A}$ $\quad E_A = 143.773\cdot V$ $\quad n := \frac{E_A}{E_{AFL}}\cdot n_{FL}$ $\quad n = 1480\cdot rpm$ $\quad$ either answer

$\omega := n\cdot\frac{2\cdot\pi\cdot rad}{60\cdot\frac{sec}{min}\cdot rev}$ $\quad \omega = 155\cdot\frac{rad}{sec}$

f) A deranged Mouse chews through part of the field winding so that the field current drops and the field flux drops to 40% of its former value. Find the shaft speed if the developed power is still 1.5 hp with $V_T = 150$ V.

$E_A = K\cdot\phi\cdot\omega$ $\quad$ so... $\quad \frac{n}{n_{FL}} = \frac{\omega}{\omega_{orig}} = \frac{\left(\frac{E_A}{\phi_{new}}\right)}{\left(\frac{E_A}{\phi_{orig}}\right)} = \frac{E_A}{E_A}\cdot\frac{\phi_{orig}}{\phi_{new}}$ $\quad n_{new} := \frac{E_A}{E_A}\cdot\frac{100\cdot\%}{40\cdot\%}\cdot n$ $\quad n_{new} = 3700\cdot rpm$ $\quad$ either answer

g) Find the load torque if the developed power is still 1.5 hp with $V_T = 150$ V.

$\tau := \frac{P_{out}}{n_{new}\cdot\left(\frac{2\cdot\pi\cdot rad}{60\cdot\frac{sec}{min}\cdot rev}\right)}$ $\quad \tau = 2.518\cdot N\cdot m$

Ex.2 A dc motor has the following nameplate information: 5-hp, 1200 rpm, Armature: 200 V, 22 A, 1 Ω; Field: 200 V, 1 A

a) Find the rotational losses at nameplate operation.

$V_T := 200\cdot V$ $\quad n_{FL} := 1200\cdot rpm$ $\quad I_{FL} := 22\cdot A$ $\quad R_A := 1\cdot\Omega$

$P_{outFL} := 5\cdot hp\cdot\frac{745.7\cdot W}{hp}$ $\quad P_{outFL} = 3.728\cdot kW$

$E_{AFL} := V_T - I_{FL}\cdot R_A$ $\quad E_{AFL} = 178\cdot V$

$P_{convFL} := E_{AFL}\cdot I_{FL}$ $\quad P_{convFL} = 3.916\cdot kW$

$P_{rotFL} := P_{convFL} - P_{outFL}$ $\quad P_{rotFL} = 187.5\cdot W$ $\quad P_{rotFL} = 0.251\cdot hp$

b) The load torque is constant (not dependent on speed) and the rotational losses are proportional to speed, find the shaft speed at half the rated armature voltage and full rated field voltage.

$P = \tau\cdot\omega_m$ so, if a power is proportional to speed, then the torque is constant.

OR, conversely, if the torque is constant, the power is proportional to speed.

In this case, ALL the power converted is proportional to speed and ALL the the induced torque is constant. $\quad P_{conv} = \frac{n_{new}}{n_{FL}}\cdot P_{convFL}$

One way: $\tau_{ind} = K\cdot\phi\cdot I_A$ so, if τ_{ind} and the field current are constant, then I_A is constant. $\quad I_A := I_{FL}$

$V_T := \frac{200\cdot V}{2}$ $\quad E_A := V_T - I_A\cdot R_A$ $\quad E_A = 78\cdot V$ $\quad n_{new} = \frac{E_A}{E_{AFL}}\cdot n_{FL} = 525.8\cdot rpm$

Another solution: recognize that: $E_A = \frac{n_{new}}{n_{FL}}\cdot E_{AFL}$ because the field is constant

$$V_T = E_A + I_A\cdot R_A = E_A + \frac{P_{conv}}{E_A}\cdot R_A = E_A + \frac{\frac{n_{new}}{n_{FL}}\cdot P_{convFL}}{\frac{n_{new}}{n_{FL}}\cdot E_{AFL}}\cdot R_A = E_A + \frac{P_{convFL}}{E_{AFL}}\cdot R_A$$

$E_A := \frac{200\cdot V}{2} - \frac{P_{convFL}}{E_{AFL}}\cdot R_A$ $\quad E_A = 78\cdot V$ $\quad n_{new} = \frac{E_A}{E_{AFL}}\cdot n_{FL} = 525.8\cdot rpm$

c) The torque is constant (like part b)) , find the shaft speed at half the rated voltage for both the armature and field. You may assume that the flux is proportional to the field current. $\quad \phi_{100} = \frac{\phi_{200}}{2}$

One way: $\tau_{ind} = K\cdot\phi\cdot I_A$ so, if τ_{ind} is constant and the field current (and flux) is halved, then I_A is constant at twice the value it used to be. $\quad I_A := 2\cdot I_{FL}$ $\quad I_A = 44\cdot A$

$V_T := \frac{200\cdot V}{2}$ $\quad E_A := V_T - I_A\cdot R_A$

$E_A = 56\cdot V = K\cdot\phi_{100}\cdot\omega_m$ $\quad n_{new} = \frac{\left(\frac{E_A}{\phi_{100}}\right)}{\left(\frac{E_{AFL}}{\phi_{200}}\right)}\cdot n_{FL} = \frac{2\cdot E_A}{E_{AFL}}\cdot n_{FL} = 755.1\cdot rpm$

Another solution: recognize that: $E_A = K\cdot\phi\cdot\omega_m$ is halved because the flux is halved $\quad E_A = \frac{1}{2}\cdot\frac{E_{AFL}}{n_{FL}}\cdot n_{new}$

$$V_T = E_A + I_A\cdot R_A = E_A + \frac{P_{conv}}{E_A}\cdot R_A = E_A + \frac{\frac{n_{new}}{n_{FL}}\cdot P_{convFL}}{\frac{1}{2}\cdot\frac{n_{new}}{n_{FL}}\cdot E_{AFL}}\cdot R_A = E_A + 2\cdot\frac{P_{convFL}}{E_{AFL}}\cdot R_A$$

$E_A := \frac{200\cdot V}{2} - 2\cdot\frac{P_{convFL}}{E_{AFL}}\cdot R_A$ $\quad E_A = 56\cdot V$ $\quad n_{new} = \frac{2\cdot E_A}{E_{AFL}}\cdot n_{FL} = 755.1\cdot rpm$

d) If the load torque and rotational loss torque are both proportional to speed (most like real life), find the shaft speed when the motor armature is hooked to half the rated voltage. The field is at rated voltage.

$$\tau_{ind} = \frac{n_{new}}{n_{FL}} \cdot \tau_{indFL} \qquad P = \tau \cdot \omega_m \qquad \text{which leads to:} \qquad P_{conv} = \left(\frac{n_{new}}{n_{FL}}\right)^2 \cdot P_{convFL}$$

One way: $\tau_{ind} = K \cdot \phi \cdot I_A$ so, if τ_{ind} is proportional to speed and the field current is constant,

then I_A is also proportional to speed. $I_A := \frac{n_{new}}{n_{FL}} \cdot I_{FL}$

$$V_T = E_A + I_A \cdot R_A = E_A + \frac{n_{new}}{n_{FL}} \cdot I_{FL} \cdot R_A = E_A + \frac{n_{new}}{n_{FL}} \cdot I_{FL} \cdot R_A = \frac{n_{new}}{n_{FL}} \cdot E_{AFL} + \frac{n_{new}}{n_{FL}} \cdot I_{FL} \cdot R_A$$

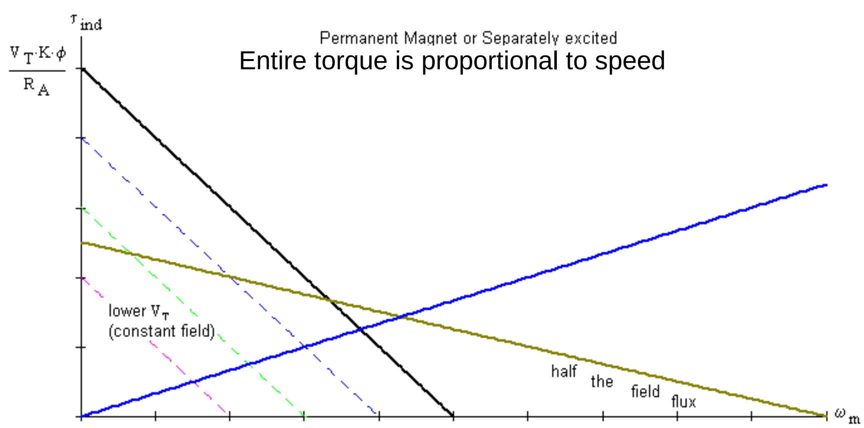
$$= \frac{n_{new}}{n_{FL}} \cdot \underbrace{\left(E_{AFL} + I_{FL} \cdot R_A\right)}_{V_{TFL}} = \frac{n_{new}}{n_{FL}} \cdot (200 \cdot V) \qquad n_{new} = \frac{100 \cdot V}{200 \cdot V} \cdot n_{FL} = 600 \cdot rpm$$

Another solution: $E_A = \frac{n_{new}}{n_{FL}} \cdot E_{AFL}$ because the field is constant

$$V_T = E_A + I_A \cdot R_A = E_A + \frac{P_{conv}}{E_A} \cdot R_A = E_A + \frac{\left(\frac{n_{new}}{n_{FL}}\right)^2 \cdot P_{convFL}}{\frac{n_{new}}{n_{FL}} \cdot E_{AFL}} \cdot R_A = \frac{n_{new}}{n_{FL}} \cdot E_{AFL} + \frac{\frac{n_{new}}{n_{FL}} \cdot P_{convFL}}{E_{AFL}} \cdot R_A$$

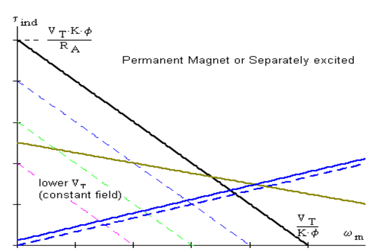
$$= \frac{n_{new}}{n_{FL}} \cdot \underbrace{\left(E_{AFL} + \frac{P_{convFL}}{E_{AFL}} \cdot R_A\right)}_{V_{TFL}}$$

same as above



Speed is proportional to V_T $\frac{V_T}{K \cdot \phi}$

e) If the load torque is proportional to speed and rotational loss torque is constant, find the shaft speed when the motor armature is hooked to half the rated voltage. The field is at rated voltage.



$$\tau_{load} = \frac{n_{new}}{n_{FL}} \cdot \tau_{loadFL} \qquad \tau_{loss} = \tau_{lossFL} = \frac{P_{rotFL}}{\omega_{mFL}}$$

$$\tau_{ind} = \frac{n_{new}}{n_{FL}} \cdot \tau_{loadFL} + \tau_{lossFL} = K \cdot \phi \cdot I_A$$

$$I_A = \frac{n_{new}}{n_{FL}} \cdot \frac{\tau_{loadFL}}{K \cdot \phi} + \frac{\tau_{lossFL}}{K \cdot \phi} = \frac{n_{new}}{n_{FL}} \cdot I_{AloadFL} + I_{AlossFL}$$

the current can be thought of as composed of two parts

$$I_{AloadFL} := \frac{P_{outFL}}{E_{AFL}} \qquad I_{AloadFL} = 20.947 \cdot A \qquad I_{AlossFL} := \frac{P_{rotFL}}{E_{AFL}} \qquad I_{AlossFL} = 1.053 \cdot A$$

Note: $I_{AloadFL} + I_{AlossFL} = 22 \cdot A = I_{AFL}$, exactly as it should be

$$V_T = E_A + I_A \cdot R_A \quad = \frac{n_{new}}{n_{FL}} \cdot E_{AFL} + \left(\frac{n_{new}}{n_{FL}} \cdot I_{AloadFL} + I_{AlossFL} \right) \cdot R_A$$

$$= \frac{n_{new}}{n_{FL}} \cdot E_{AFL} + \frac{n_{new}}{n_{FL}} \cdot I_{AloadFL} \cdot R_A + I_{AlossFL} \cdot R_A$$

$$V_T - I_{AlossFL} \cdot R_A \quad = \frac{n_{new}}{n_{FL}} \cdot \left(E_{AFL} + I_{AloadFL} \cdot R_A \right) \qquad n_{new} = \frac{V_T - I_{AlossFL} \cdot R_A}{E_{AFL} + I_{AloadFL} \cdot R_A} \cdot n_{FL} = 596.8 \cdot rpm$$

Ex.3 An unknown, permanent-magnet dc motor is tested at two different loads. In each case the armature voltage is: 24 V.

$V_T := 24 \cdot V$ Load 1: $I_{A1} := 10 \cdot A$ $n_1 := 163 \cdot rpm$ Load 2: $I_{A2} := 30 \cdot A$ $n_2 := 127 \cdot rpm$

a) Find the parameters of this motor.

Load 1: $I_{A1} := 10 \cdot A$ $n_1 := 163 \cdot rpm$

$$\omega_1 := n_1 \cdot \left(\frac{2 \cdot \pi \cdot rad}{60 \cdot \frac{sec}{min} \cdot rev} \right) \qquad \omega_1 = 17.069 \cdot \frac{rad}{sec}$$

$$V_{T1} = 24 \cdot V = I_{A1} \cdot R_A + K \cdot \phi \cdot \omega_1$$

Load 2: $I_{A2} := 30 \cdot A$ $n_2 := 127 \cdot rpm$

$$\omega_2 := n_2 \cdot \left(\frac{2 \cdot \pi \cdot rad}{60 \cdot \frac{sec}{min} \cdot rev} \right) \qquad \omega_2 = 13.299 \cdot \frac{rad}{sec}$$

$$V_{T2} = 24 \cdot V = I_{A2} \cdot R_A + K \cdot \phi \cdot \omega_2$$ solve for $R_A = \frac{24 \cdot V - K \cdot \phi \cdot \omega_2}{I_{A2}}$

Solve:

$$V_{T1} = 24 \cdot V = I_{A1} \cdot R_A + K \cdot \phi \cdot \omega_1$$ substitute in for R_A

$$V_{T1} = 24 \cdot V = I_{A1} \cdot \left(\frac{24 \cdot V - K \cdot \phi \cdot \omega_2}{I_{A2}} \right) + K \cdot \phi \cdot \omega_1$$

$$= \frac{I_{A1}}{I_{A2}} \cdot 24 \cdot V - \frac{I_{A1}}{I_{A2}} \cdot K \cdot \phi \cdot \omega_2 + K \cdot \phi \cdot \omega_1 \quad = \frac{I_{A1}}{I_{A2}} \cdot 24 \cdot V + K \cdot \phi \cdot \left(\omega_1 - \frac{I_{A1}}{I_{A2}} \cdot \omega_2 \right)$$

$$K \cdot \phi = \frac{24 \cdot V - \frac{I_{A1}}{I_{A2}} \cdot 24 \cdot V}{\left(\omega_1 - \frac{I_{A1}}{I_{A2}} \cdot \omega_2 \right)} = 1.266 \cdot V \cdot sec$$

$$R_A = \frac{24 \cdot V - K \cdot \phi \cdot \omega_2}{I_{A2}} = \frac{24 \cdot V - (1.266 \cdot V \cdot sec) \cdot \omega_2}{I_{A2}} = 0.239 \cdot \Omega$$

b) The rotational loss torque is proportional to speed.
Find the parameters of this motor.

Notice that the induced torque is NOT part of the calculation above. Therefore it doesn't matter how it is split between the loss and the load. The calculations are exactly the same.