A 3-hp dc motor has the following nameplate information: 150 V, 1400 rpm, 18 A, $R_A = 0.8 \Omega$, and $R_F = 300 \Omega$. The field is shunt connected and the 18 A includes the field current. Assume rotational losses are constant.

a) Find the efficiency of the motor at nameplate operation. (Include the field in your calculations)

$$V_T := 150 \text{ V} \quad n_{FL} := 1400 \text{ rpm} \quad I_{FL} := 18 \text{ A} \quad R_A := 0.8 \Omega \quad R_F := 300 \Omega$$

$$P_{out} := \frac{3 \text{- hp}}{745.7 \text{ W}} \quad P_{out} = 2.237 \text{ kW}$$

$$P_{in} := V_T I_{FL} \quad P_{in} = 2.7 \text{ kW}$$

$$\eta = \frac{P_{out}}{P_{in}} = 82.86\%$$

b) Find the rotational losses at nameplate operation.

- Field current: $I_F := \frac{V_T}{R_F} \quad I_F = 0.5 \text{ A}$
- Armature full-load current: $I_{AFL} := I_{FL} - I_F \quad I_{AFL} = 17.5 \text{ A}$
- $E_{AFL} := V_T - I_{AFL} R_A \quad E_{AFL} = 136 \text{ V}$
- $P_{conv} := E_{AFL} I_{FL} \quad P_{conv} = 2.38 \text{ kW}$
- $P_{rot} := P_{conv} - P_{out} \quad P_{rot} = 142.9 \text{ W} \quad P_{rot} = 0.192 \text{ hp}$

either answer

c) Find the required current for a developed power of 1.5 hp with $V_T = 150 \text{ V}$.

$$P_{conv} := 1.5 \text{ hp} \quad P_{conv} = 1.119 \text{ kW} = E_A I_A$$

$$V_T = E_A + I_A R_A = \frac{P_{conv}}{I_A} + I_A R_A$$

Rearrange

$$0 = R_A I_A^2 - V_T I_A + P_{conv}$$

Solving for $I_A = \left[ \frac{1}{2R_A} \left( V_T + \sqrt{V_T^2 - 4R_A P_{conv}} \right) \right] \left( 179.72 \right) \text{ A}$

$$I_A := 7.78 \text{ A}$$

$$I_S := I_A + I_F \quad I_S = 8.28 \text{ A}$$

either answer

d) Find the output power if the developed power is 1.5 hp with $V_T = 150 \text{ V}$.

$$P_{out} := P_{conv} - P_{rot} \quad P_{out} = 975.7 \text{ W} \quad P_{out} = 1.308 \text{ hp}$$


e) Find the shaft speed if the developed power is 1.5 hp with $V_T = 150 \text{ V}$.

$$E_A := \frac{P_{conv}}{I_A} \quad E_A = 143.773 \text{ V} \quad n := \frac{E_A}{E_{AFL}} n_{FL}$$

$$n = 1480 \text{ rpm} \quad \omega := n \cdot \frac{2 \pi \text{ rad}}{60 \text{ sec}} \text{ rev} \quad \omega = 155 \text{ rad sec}^{-1}$$


either answer

f) A deranged Mouse chews through part of the field winding so that the field current drops and the field flux drops to 40% of its former value. Find the shaft speed if the developed power is still 1.5 hp with $V_T = 150 \text{ V}$.

$$E_A := K \phi \omega \quad \text{so...} \quad \frac{n}{n_{FL}} = \frac{\omega}{\omega_{orig}} = \frac{E_A}{\phi_{new}} \quad \frac{E_A}{\phi_{orig}}$$

$$n_{new} := \frac{E_A 100\%}{E_A 40\%} \quad n_{new} = 3700 \text{ rpm}$$

either answer

g) Find the load torque if the developed power is still 1.5 hp with $V_T = 150 \text{ V}$.

$$\tau := \frac{P_{out}}{n_{new} \cdot \frac{2 \pi \text{ rad}}{60 \text{ sec}}} \text{ rev} \quad \tau = 2.518 \text{ N m}$$
Ex.2 A dc motor has the following nameplate information: 5-hp, 1200 rpm, Armature: 200 V, 22 A, 1 Ω; Field: 200 V, 1 A

a) Find the rotational losses at nameplate operation.

\[ V_T := 200 \text{ V} \quad n_{FL} := 1200 \text{ rpm} \quad I_{FL} := 22 \text{ A} \quad R_A := 1 \Omega \]

\[ P_{outFL} := 5\text{-hp} \cdot \frac{745.7 \text{ W}}{\text{hp}} \]

\[ E_{AFL} := V_T - I_{FL} \cdot R_A \]

\[ E_A := 178 \text{ V} \]

\[ P_{convFL} := E_{AFL} \cdot I_{FL} \]

\[ P_{rotFL} := P_{convFL} - P_{outFL} \]

\[ P_{rotFL} = 187.5 \text{ W} \]

\[ P_{rotFL} = 0.251 \text{ hp} \]

b) The load torque is constant (not dependent on speed) and the rotational losses are proportional to speed, find the shaft speed at half the rated armature voltage and full rated field voltage.

\[ P = \tau \cdot \omega_m \quad \text{so, if a power is proportional to speed, then the torque is constant.} \]

\[ OR, \text{ conversely, if the torque is constant, the power is proportional to speed.} \]

In this case, ALL the power converted is proportional to speed and ALL the the induced torque is constant.

\[ P_{conv} = \frac{n_{new}}{n_{FL}} \cdot P_{convFL} \]

One way:

\[ \tau_{ind} = K \cdot \phi \cdot I_A \quad \text{so, if } \tau_{ind} \text{ and the field current are constant, then } I_A \text{ is constant.} \]

\[ I_A := I_{FL} \]

Another solution: recognize that:

\[ E_A = \frac{n_{new}}{n_{FL}} \cdot E_{AFL} \]

because the field is constant

\[ V_T := \frac{200 \text{ V}}{2} \]

\[ E_A := V_T - I_A \cdot R_A \]

\[ E_A = 78 \text{ V} \]

\[ n_{new} = \frac{E_A}{E_{AFL}} \cdot n_{FL} = 525.8 \text{ rpm} \]

Another solution: recognize that:

\[ E_A = \frac{n_{new}}{n_{FL}} \cdot E_{AFL} \]

\[ E_A := \frac{200 \text{ V}}{2} - \frac{P_{convFL}}{E_{AFL}} \cdot R_A \]

\[ E_A = 56 \text{ V} \]

\[ n_{new} = \frac{E_A}{E_{AFL}} \cdot n_{FL} = 525.8 \text{ rpm} \]

c) The torque is constant (like part b)) , find the shaft speed at half the rated voltage for both the armature and field. You may assume that the flux is proportional to the field current.

\[ \phi_{100} = \frac{\phi_{200}}{2} \]

One way:

\[ \tau_{ind} = K \cdot \phi \cdot I_A \quad \text{so, if } \tau_{ind} \text{ is constant and the field current (and flux) is halved, then } I_A \text{ is constant at twice the value it used to be.} \]

\[ I_A := 2 \cdot I_{FL} \quad I_A = 44 \cdot A \]

Another solution: recognize that:

\[ E_A = \frac{1}{2} \cdot \frac{E_{AFL}}{n_{new}} \]

\[ V_T := \frac{200 \text{ V}}{2} \]

\[ E_A := V_T - I_A \cdot R_A \]

\[ E_A = 56 \cdot V \]

\[ n_{new} = \frac{\phi_{100}}{\phi_{200}} \cdot n_{FL} = 755.1 \text{ rpm} \]

Another solution: recognize that:

\[ E_A = \frac{1}{2} \cdot \frac{E_{AFL}}{n_{new}} \]

\[ V_T := \frac{200 \text{ V}}{2} - 2 \cdot \frac{P_{convFL}}{E_{AFL}} \cdot R_A \]

\[ E_A = 56 \cdot V \]

\[ n_{new} = \frac{2 \cdot E_A}{E_{AFL}} \cdot n_{FL} = 755.1 \text{ rpm} \]
d) If the load torque and rotational loss torque are both proportional to speed (most like real life), find the shaft speed when the motor armature is hooked to half the rated voltage. The field is at rated voltage.

\[ \tau_{\text{ind}} = \frac{n_{\text{new}}}{n_{\text{FL}}} \tau_{\text{indFL}} \quad P = \tau \cdot \omega_m \] which leads to:

\[ P_{\text{conv}} = \left( \frac{n_{\text{new}}}{n_{\text{FL}}} \right)^2 P_{\text{convFL}} \]

One way: \( \tau_{\text{ind}} = K \phi I_A \) so, if \( \tau_{\text{ind}} \) is proportional to speed and the field current is constant, then \( I_A \) is also proportional to speed. \( I_A = \frac{n_{\text{new}}}{n_{\text{FL}}} I_{\text{FL}} \)

\[ V_T = E_A + I_A R_A = E_A + \frac{n_{\text{new}}}{n_{\text{FL}}} I_{\text{FL}} R_A = E_A + \frac{n_{\text{new}}}{n_{\text{FL}}} \left( E_{\text{AFL}} + I_{\text{FL}} R_A \right) = \frac{n_{\text{new}}}{n_{\text{FL}}} \left( 200 \cdot V \right) \]

\[ n_{\text{new}} = \frac{100 \cdot V}{200 \cdot V} n_{\text{FL}} = 600 \text{ rpm} \]

Another solution: \( E_A = \frac{n_{\text{new}}}{n_{\text{FL}}} E_{\text{AFL}} \) because the field is constant

\[ V_T = E_A + I_A R_A = E_A + \frac{P_{\text{conv}}}{E_A} R_A = E_A + \frac{n_{\text{new}} E_{\text{AFL}}}{n_{\text{FL}}} R_A = \left( \frac{n_{\text{new}}}{n_{\text{FL}}} \right) \left( E_{\text{AFL}} + \frac{P_{\text{convFL}}}{E_{\text{AFL}}} R_A \right) \]

\[ = \frac{n_{\text{new}}}{n_{\text{FL}}} \left( V_T \right) \]

same as above

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e) If the load torque is proportional to speed and rotational loss torque is constant, find the shaft speed when the motor armature is hooked to half the rated voltage. The field is at rated voltage.

\[ \tau_{\text{load}} = \frac{n_{\text{new}}}{n_{\text{FL}}} \tau_{\text{loadFL}} \quad \tau_{\text{loss}} = \tau_{\text{lossFL}} = \frac{P_{\text{rotFL}}}{\omega_{mFL}} \]

\[ \tau_{\text{ind}} = \frac{n_{\text{new}}}{n_{\text{FL}}} \tau_{\text{loadFL}} + \tau_{\text{lossFL}} = K \phi I_A \]

\[ I_A = \frac{n_{\text{new}}}{n_{\text{FL}}} \left( \frac{\tau_{\text{loadFL}}}{K \phi} + \frac{\tau_{\text{lossFL}}}{K \phi} \right) = \frac{n_{\text{new}}}{n_{\text{FL}}} I_{\text{AloadFL}} + I_{\text{AlossFL}} \]

\[ I_{\text{AloadFL}} = \frac{P_{\text{outFL}}}{E_{\text{AFL}}} \quad I_{\text{AloadFL}} = 20.947 \cdot \text{A} \]

\[ I_{\text{AlossFL}} = \frac{P_{\text{rotFL}}}{E_{\text{AFL}}} \]

\[ I_{\text{AlossFL}} = 1.053 \cdot \text{A} \]

Note: \( I_{\text{AloadFL}} + I_{\text{AlossFL}} = 22 \cdot \text{A} = I_{\text{AFL}} \) exactly as it should be
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\[ V_T = E_A + I_A R_A = \frac{n_{\text{new}}}{n_{\text{FL}}} E_{\text{AFL}} + \left( \frac{n_{\text{new}}}{n_{\text{FL}}} I_{\text{loadFL}} + I_{\text{lossFL}} \right) R_A \]

\[ V_T - I_{\text{lossFL}} R_A = \frac{n_{\text{new}}}{n_{\text{FL}}} \left( E_{\text{AFL}} + I_{\text{loadFL}} R_A \right) \]

\[ n_{\text{new}} = \frac{V_T - I_{\text{lossFL}} R_A}{E_{\text{AFL}} + I_{\text{loadFL}} R_A} \cdot n_{\text{FL}} = 596.8 \, \text{rpm} \]

**Ex.3** An unknown, permanent-magnet dc motor is tested at two different loads. In each case the armature voltage is: 24 V.

Load 1: \( I_{A1} = 10 \, \text{A} \) \( n_1 = 163 \, \text{rpm} \)
Load 2: \( I_{A2} = 30 \, \text{A} \) \( n_2 = 127 \, \text{rpm} \)

a) Find the parameters of this motor.

Load 1: \( I_{A1} = 10 \, \text{A} \) \( n_1 = 163 \, \text{rpm} \)

\[ \omega_1 = n_1 \cdot \frac{2 \cdot \pi \, \text{rad}}{60 \, \text{sec} \, \text{rev}} \]

\[ \omega_1 = 17.069 \frac{\text{rad}}{\text{sec}} \]

\[ V_{T1} = 24 \, \text{V} = I_{A1} R_A + K_\phi \omega_1 \]

Load 2: \( I_{A2} = 30 \, \text{A} \) \( n_2 = 127 \, \text{rpm} \)

\[ \omega_2 = n_2 \cdot \frac{2 \cdot \pi \, \text{rad}}{60 \, \text{sec} \, \text{rev}} \]

\[ \omega_2 = 13.299 \frac{\text{rad}}{\text{sec}} \]

\[ V_{T2} = 24 \, \text{V} = I_{A2} R_A + K_\phi \omega_2 \]

Solve:

\[ V_{T1} = 24 \, \text{V} = I_{A1} R_A + K_\phi \omega_1 \]

substitute in for \( R_A \)

\[ V_{T1} = 24 \, \text{V} = I_{A1} \left( \frac{24 \, \text{V} - K_\phi \omega_2}{I_{A2}} \right) + K_\phi \omega_1 \]

\[ \frac{I_{A1}}{I_{A2}} \cdot 24 \, \text{V} - \frac{I_{A1}}{I_{A2}} \cdot K_\phi \omega_2 + K_\phi \omega_1 = \frac{I_{A1}}{I_{A2}} \cdot 24 \, \text{V} + K_\phi \left( \omega_1 - \frac{I_{A1}}{I_{A2}} \omega_2 \right) \]

\[ 24 \, \text{V} \cdot \frac{I_{A1}}{I_{A2}} - 24 \, \text{V} \]

\[ K_\phi = 1.266 \, \text{V} \cdot \text{sec} \]

\[ R_A = \frac{24 \, \text{V} - K_\phi \omega_2}{I_{A2}} \]

b) The rotational loss torque is proportional to speed.

Find the parameters of this motor.

Notice that the induced torque is NOT part of the calculation above. Therefore it doesn't matter how it is split between the loss and the load. The calculations are exactly the same.