Ex. 1 (F08 E2) A 60 Hz, 4-pole, 3-phase, Y connected synchronous generator supplies 90 kW of power to a 4 kV bus. The synchronous reactance is 50 Ω/phase. The generator emf is 3 kV. Find the following.

a) The power angle, \( \delta \).

\[
P_{1\phi} = \frac{P_{3\phi}}{3} \quad P_{1\phi} = 30 \text{ kW} = \frac{V \phi E_A \sin(\delta)}{X_s} \quad \delta = \sin \left( \frac{P_{1\phi} X_s}{V \phi E_A} \right) \quad \delta = 12.5 \text{ deg}
\]

b) The total reactive power generated.

\[
Q_{1\phi} = \frac{V \phi E_A \cos(\delta)}{X_s} - \frac{V^2}{X_s} \quad Q_{3\phi} = 3 \cdot \frac{V \phi E_A \cos(\delta) - V^2}{X_s} = 85.83 \text{ kVAR}
\]

c) Find a new magnitude of the generator emf so that \( Q = 45 \text{ kVAR} \).

\[
Q_{1\phi} = \frac{Q}{3} \quad S_{1\phi} = \sqrt{P_{1\phi}^2 + Q_{1\phi}^2} \quad I_A = 14.52 \text{ A} \quad \theta = \arctan \left( \frac{Q_{1\phi}}{P_{1\phi}} \right) \quad \theta = 26.57 \text{ deg}
\]

by Pythagorean theorem: \( E_A = \sqrt{\left( V \phi + X_s I_A \sin(\theta) \right)^2 + \left( X_s I_A \cos(\theta) \right)^2} \quad E_A = 2.713 \text{ kV} \)

Alternatively, by complex numbers: \( \Delta V = I_A X_s e^{j(90 \text{ deg} - \theta)} \quad E_A = V \phi + \Delta V \quad |E_A| = 2.713 \text{ kV} \quad \delta = \arg(E_A) = 13.851 \text{ deg} \)

d) The shaft speed.

\[
n := \frac{7200 \text{ rpm}}{N_{\text{poles}}} \quad n = 1800 \text{ rpm} \quad \omega_m := n \cdot \frac{2 \pi \text{ rad}}{\text{rev}} \cdot \left( \frac{\text{min}}{60 \text{ sec}} \right) \quad \omega_m = 188.496 \text{ rad/ sec} \quad \text{OR} \quad \omega_m = \frac{2 \cdot \left( 377 \text{ rad/ sec} \right)}{N_{\text{poles}}} = 188.5 \text{ rad/ sec} \]

e) The shaft torque.

Often called the "Prime-mover torque"

\[
P_{3\phi} = \omega \cdot \tau \quad \tau := \frac{P_{3\phi}}{\omega_m} \quad \tau = 477.5 \text{ N-m}
\]

f) The shaft torque is decreased to half the value found in part e). What is the new \( P \) and \( Q \)?

\[
P'_{1\phi} = \frac{1}{2} \cdot P_{1\phi} \quad P'_{1\phi} = 15 \text{ kW} \quad \delta := \sin \left( \frac{P'_{1\phi} X_s}{V \phi E_A} \right) \quad \delta = 6.87 \text{ deg}
\]

\[
Q_{3\phi} = 3 \cdot \frac{V \phi E_A \cos(\delta) - V^2}{X_s} = 53.23 \text{ kVAR}
\]
Ex. 2

A 60 Hz, 2-pole, Y-connected, 3-phase synchronous generator supplies 15 MW of power to a 18 kV bus. The synchronous reactance is 6 Ω/phase. The magnitude of the generator emf equals the magnitude of the bus voltage.

givens \( V_\phi = \frac{18 \cdot 10^3 V}{\sqrt{3}} \) \( V_\phi = 10.392 \cdot kV \) \( X_s := 6 \Omega \) \( P_{1\phi} := \frac{15-MW}{3} \) \( P_{1\phi} = 5 \cdot MW \)

\( E_A := V_\phi \)

Find:

a) The power angle, \( \delta \).

\[ P_{1\phi} = V_\phi I_A \cos(\theta) \]

\[ I_A := \frac{P_{1\phi}}{V_\phi \cos(\theta)} \]

\[ I_A = 485.9 \cdot A / 8.06^\circ \]

b) The complex phase current, (Assume the bus voltage phase angle is 0°).

\[ E_A \parallel_\delta \]

\[ \theta := \frac{\delta}{2} \]

\[ \theta = 8.06^\circ \cdot \text{deg lead} \]

\[ V_\phi \]

\[ P_{1\phi} \]

\[ X_s \]

\[ E_A \]

\[ V_\phi \]

\[ I_A \]

\[ \delta \]

\[ \theta \]

\[ I_L := \sqrt{3} \cdot I_A \]

\[ I_L = 76.24 \cdot A \]

c) The magnitude and direction of reactive power.

\[ Q_{3\phi} := 3 \cdot V_\phi I_A \sin(-\theta) \]

Ex. 3

A 60-Hz, three-phase, 6-pole, \( \Delta \)-connected synchronous motor is connected to 480 V and produces 80 hp. The motor draws minimum current with an excitation voltage of \( E_A = 520 \) V per phase. Mechanical losses are 5 hp.

givens \( N_{poles} := 6 \) \( V_\phi := 480 \cdot V \) \( P_{3\phi} := 80-\text{hp} + 5-\text{hp} \) \( E_A := 520 \cdot V \)

\( P_{1\phi} := \frac{85-\text{hp} \cdot 745.7 \cdot \text{W}}{3 \cdot \text{hp}} \) \( P_{1\phi} = 21.1 \cdot \text{kW} \)

Determine the following:

a) The current.

Minimum current implies \( pf := 1 \) so...

\[ I_A := \frac{P_{1\phi}}{V_\phi} \]

\[ I_A = 44.02 \cdot A \]

b) The line current. \( I_L := \sqrt{3} \cdot I_A \)

\[ I_L = 76.24 \cdot A \]

c) The synchronous reactance.

by Pythagorean theorem: \( I_A \cdot X_s = \sqrt{E_A^2 - V_\phi^2} \)

\[ X_s := \frac{\sqrt{E_A^2 - V_\phi^2}}{I_A} \]

\[ X_s = 4.544 \cdot \Omega \]

d) The torque.

\[ \omega_{mech} := \frac{4 \cdot \pi \cdot f}{N_{poles}} \]

\[ \tau_{mech} = \frac{80-\text{hp} \cdot 745.7 \cdot \text{W}}{475 \cdot \text{N} \cdot \text{m}} \]

\[ \delta = \frac{\theta}{2} \]

\[ \delta = 22.62 \cdot \text{deg} \]

e) The rotor power angle.

f) The maximum power this motor could provide at this excitation voltage.

\[ \delta := 90 \cdot \text{deg} \]

\[ P_{3\phi} = 3 \cdot \frac{E_A \cdot V_\phi}{X_s} \sin(\delta) - \frac{1-\text{hp}}{745.7 \cdot \text{W}} - 5-\text{hp} = 216 \cdot \text{hp} \]

Note: The current rating of the motor may be exceeded at this load.
Ex. 4

(F09 Fin) You make the following measurements on a 3-phase, Y-connected, synchronous generator.

\[ P_{3\Phi} = 120 \text{ kW} \quad V_{LL} = 480 \text{ V} \quad I_L = 160 \text{ A} \quad X_s = 1.2 \Omega \]

Unfortunately, you don't know the phase angle of current.

(a) Draw a phasor diagram of one of the two possible interpretations of these numbers.

Find the induced armature voltage \((E_{af})\) and the power angle, \(\delta\).

My first assumption:

\[ V_T = \frac{V_{LL}}{\sqrt{3}} \quad V_T = 277.13 \text{ V} \]

\[ P_{1\Phi} = \frac{P_{3\Phi}}{3} \quad P_{1\Phi} = 40 \text{ kW} \]

\[ \theta := \cos^{-1}(\text{pf}) \quad \theta = 25.563 \text{ deg} \]

\[ I_L := I_L e^{j\theta} \]

\[ E_A := V_T + I_L (jX_s) \quad E_A = |E_A| = 260.28 \text{ V} \quad \delta = \arg(E_A) = 41.718 \text{ deg} \]

(b) Draw a phasor diagram of other possible interpretation of these numbers.

Find the induced armature voltage \((E_{af})\) and the power angle, \(\delta\).

My second assumption:

\[ \theta := \cos^{-1}(\text{pf}) \quad \theta = 25.563 \text{ deg} \]

\[ I_L := I_L e^{j\theta} \]

\[ E_A := V_T + I_L (jX_s) \quad E_A = |E_A| = 399.48 \text{ V} \quad \delta = \arg(E_A) = 25.695 \text{ deg} \]

c) A traveling carnival uses a combination of this generator and the local power company to run its load, mainly induction motors. When the generator is connected to the carnival's power distribution network, it supplies half of the required power, but the current from the power company only decreases by about 30%. Which of the calculations above is most likely correct?

Assumption in a)

Give me the reasoning behind your answer (no calculations required).

The induction motors represent a lagging pf load, they use lots of VARs. If the local generator were supplying those VARs, then the current would go down by about half and quite possibly more. The small reduction in current implies that the generator also consumes VARs (creates negative VARs). That is condition a).

d) What do you change at the generator to reduce the current flow from the power company?

Tell me what you adjust and if you turn it up or down.

Turn up the field current.
Ex. 5

(F09 E2, p4) An industrial plant is powered from a 480-V, 3-phase bus and currently draws 60 kW at a power factor of 0.8 lagging. A new mill is to be added at the plant. This mill requires a shaft torque of 600 Nm at 1200 rpm. Your job is to specify a motor which will run the mill and correct the plant power factor at the same time. Be sure to specify the type of motor including the number of poles. Tell me how the motor should be connected to the bus (This is an arbitrary decision here, but it will affect many of your other answers). Specify its minimum hp, voltage, and current ratings. Tell me what the back emf should be. You may assume the synchronous reactance is 1 Ω/phase and that losses are negligible.

\[ P_{3\phi} := 60 \text{ kW} \]
\[ P_{1\phi} := \frac{P_{3\phi}}{3} \]
\[ P_{1\phi} = 20 \text{ kW} \]
\[ 1 \text{-hp} = 745.7 \text{ W} \]

\[ P_{1\phi} = 25 \text{ kVA} \]
\[ Q_{1\phi} := \sqrt{S_{1\phi}^2 - P_{1\phi}^2} \]
\[ Q_{1\phi} = 15 \text{ kVAR} \]

\[ N_{\text{poles}} := \frac{7200}{1200} \]
\[ N_{\text{poles}} = 6 \]
\[ \omega_{\text{mech}} := \frac{4 \pi f}{N_{\text{poles}}} \]
\[ \omega_{\text{mech}} = 125.7 \text{ rad/sec} \]

\[ \omega_{\text{mech}} = 6 \text{ rad/sec} \]

\[ \tau_{\text{mech}} := 600 \text{ Nm} \]
\[ \tau_{\text{mech}} = 75.398 \text{ kW} \]
\[ \tau_{\text{mech}} = \frac{1 \text{-hp}}{745.7 \text{ W}} = 101.111 \text{ hp} = \text{min hp rating} \]

\[ P_{m1\phi} := \frac{\tau_{\text{mech}} \omega_{\text{mech}}}{3} \]
\[ P_{m1\phi} = 25.133 \text{ kW} \]

To fix the plant pf,
\[ Q_{m1\phi} := -Q_{1\phi} \]
\[ Q_{m1\phi} = -15 \text{ kVAR} \]

If you select Y-connected
\[ V_{\phi} := \sqrt[3]{480 \text{ V}} \]
\[ V_{\phi} = 277.1 \text{ V} = \text{min voltage rating} \]

Current per phase:
\[ I := \sqrt{\frac{P_{m1\phi}^2 + Q_{m1\phi}^2}{V_{\phi}}} \]
\[ I = 105.61 \text{ A} = \text{min current rating} \]

Phase angle of the current:
\[ \theta := \text{atan} \left( \frac{-Q_{m1\phi}}{P_{m1\phi}} \right) \]
\[ \theta = 30.83 \text{ deg} \]

\[ X_s := 1 \text{ } \Omega \]
\[ \Delta V := 1 \text{ } e^{j\theta} \cdot X_s \cdot j \]
\[ \Delta V = -54.127 + 90.69j \text{ } \cdot \text{V} \]

motor back emf
\[ E_A := V_{\phi} - \Delta V \]
\[ E_A = 331.255 - 90.69j \text{ } \cdot \text{V} \]
\[ |E_A| = 343.44 \text{ } \cdot \text{V} = \text{required motor emf} \]

If you select Δ-connected
\[ V_{\phi} := 480 \text{ V} \]
\[ V_{\phi} = 480 \text{ V} = \text{min voltage rating} \]

Current per phase:
\[ I := \sqrt{\frac{P_{m1\phi}^2 + Q_{m1\phi}^2}{V_{\phi}}} \]
\[ I = 60.98 \text{ A} = \text{min current rating} \]

\[ X_s := 1 \text{ } \Omega \]
\[ \Delta V := 1 \text{ } e^{j\theta} \cdot X_s \cdot j \]
\[ \Delta V = -31.25 + 52.36j \text{ } \cdot \text{V} \]

motor back emf
\[ E_A := V_{\phi} - \Delta V \]
\[ E_A = 511.25 - 52.36j \text{ } \cdot \text{V} \]
\[ |E_A| = 513.92 \text{ } \cdot \text{V} = \text{required motor emf} \]