ECE 3600 Synchronous Generator & Motor Examples

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Ex. 1

(F08 E2) A 60 Hz, 4-pole, 3-phase synchronous generator supplies 90 kW of power to a 4 kV bus. The synchronous reactance is 50 Ω/phase. The generator emf is 3 kV. Find the following.

a) The power angle, δ.

givens $f := 60 \cdot Hz$ $\quad N_{poles} := 4$ $\quad$ Assume Y-connected: Otherwise E_A would be very low. $\quad V_\phi := \dfrac{4 \cdot kV}{\sqrt{3}}$

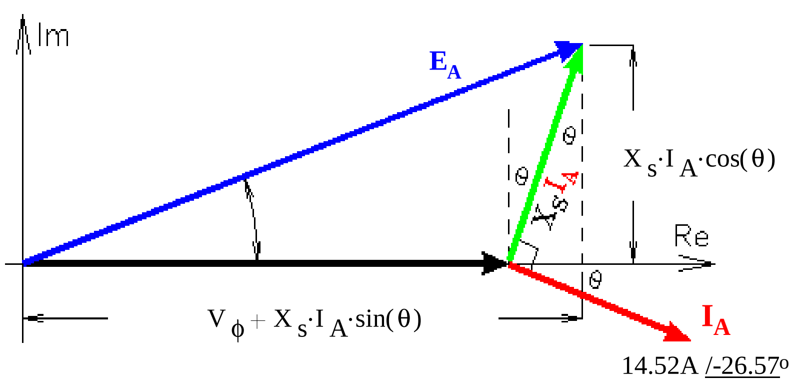
$X_s := 50 \cdot \Omega \qquad E_A := 3 \cdot kV \qquad P_{3\phi} := 90 \cdot kW$

$$P_{1\phi} := \frac{P_{3\phi}}{3} \qquad P_{1\phi} = 30 \cdot kW \quad = \frac{V_\phi \cdot E_A \cdot \sin(\delta)}{X_s} \qquad \delta := \operatorname{asin}\left(\frac{P_{1\phi} \cdot X_s}{V_\phi \cdot E_A}\right) \qquad \delta = 12.5 \cdot deg$$

b) The total reactive power generated.

$$Q_{1\phi} = \frac{V_\phi \cdot E_A \cdot \cos(\delta) - V_\phi^{\,2}}{X_s} \qquad Q_{3\phi} = 3 \cdot \frac{V_\phi \cdot E_A \cdot \cos(\delta) - V_\phi^{\,2}}{X_s} = 85.83 \cdot kVAR$$

c) Find a new magnitude of the generator emf so that $Q := 45 \cdot kVAR$



$$Q_{1\phi} := \frac{Q}{3} \qquad S_{1\phi} := \sqrt{P_{1\phi}^{\,2} + Q_{1\phi}^{\,2}}$$

$$I_A := \frac{S_{1\phi}}{V_\phi} \qquad I_A = 14.52 \cdot A$$

$$\theta := \operatorname{atan}\left(\frac{Q_{1\phi}}{P_{1\phi}}\right) \qquad \theta = 26.57 \cdot deg$$

Notice that θ is positive in the downward direction, contrary to the notes. Also notice how that affects the figure at left. Know what you're doing, don't just use formulas!

by Pythagorean theorem: $E_A := \sqrt{\left(V_\phi + X_s \cdot I_A \cdot \sin(\theta)\right)^2 + \left(X_s \cdot I_A \cdot \cos(\theta)\right)^2} \qquad E_A = 2.713 \cdot kV$

OR $\mathbf{I_A} := I_A \cdot e^{-(j \cdot \theta)} \qquad \mathbf{E_A} := V_\phi + \mathbf{I_A} \cdot (j \cdot X_s) \qquad E_A = \left|\mathbf{E_A}\right| = 2713 \cdot V \qquad \delta = \arg(\mathbf{E_A}) = 13.851 \cdot deg$

d) The shaft speed.

$$n := \frac{7200 \cdot rpm}{N_{poles}} \qquad n = 1800 \cdot rpm \qquad \omega_m := n \cdot \frac{2 \cdot \pi \cdot rad}{rev} \cdot \left(\frac{min}{60 \cdot sec}\right) \qquad \omega_m = 188.496 \cdot \frac{rad}{sec}$$

OR $\quad \omega_m = \dfrac{\left(377 \cdot \dfrac{rad}{sec}\right)}{\left(\dfrac{N_{poles}}{2}\right)} = 188.5 \cdot \dfrac{rad}{sec}$

e) The shaft torque.
Often called the "Prime-mover torque"

$$P_{3\phi} = \omega T \qquad T := \frac{P_{3\phi}}{\omega_m} \qquad T = 477.5 \cdot N \cdot m$$

f) The shaft torque is decreased to half the value found in part e). What is the new P and Q?

$$P'_{3\phi} := \frac{1}{2} \cdot P_{3\phi} \qquad P'_{3\phi} = 45 \cdot kW \qquad \delta := \operatorname{asin}\left(\frac{P'_{3\phi} \cdot X_s}{3 \cdot V_\phi \cdot E_A}\right) \qquad \delta = 6.87 \cdot deg$$

$$Q_{3\phi} = 3 \cdot \frac{V_\phi \cdot E_A \cdot \cos(\delta) - V_\phi^{\,2}}{X_s} = 53.23 \cdot kVAR$$

Ex. 2

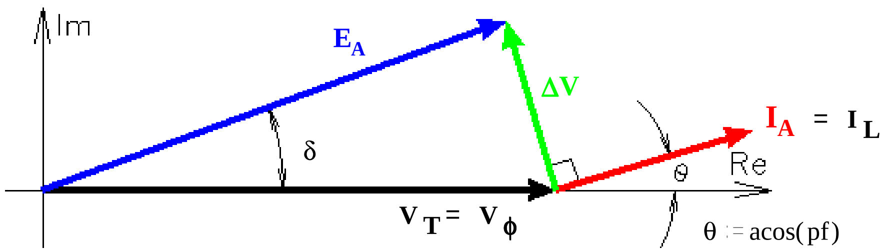
(F09 Fin) You make the following measurements on a 3-phase, Y-connected, synchronous generator.

$P_{3\phi} := 120 \cdot kW$ $\quad V_{LL} := 480 \cdot V$ $\quad I_L := 160 \cdot A$ $\quad X_s := 1.2 \cdot \Omega$

Unfortunately, you don't know the phase angle of current.

a) Draw a phasor diagram of one of the two possible interpretations of these numbers.
Find the induced armature voltage (E_A) and the power angle, δ. $\quad E_A = ?$ $\quad \delta = ?$

My first assumption: I_L leads V_T



$V_T := \frac{V_{LL}}{\sqrt{3}}$ $\quad V_T = 277.13 \cdot V = V_\phi$

$P_{1\phi} := \frac{P_{3\phi}}{3}$ $\quad P_{1\phi} = 40 \cdot kW$

$pf := \frac{P_{1\phi}}{I_L \cdot V_T}$ $\quad pf = 0.902$

$\theta := \mathrm{acos}(pf)$

$\theta = 25.563 \cdot deg$

$\mathbf{I_L} := I_L \cdot e^{j \cdot \theta}$

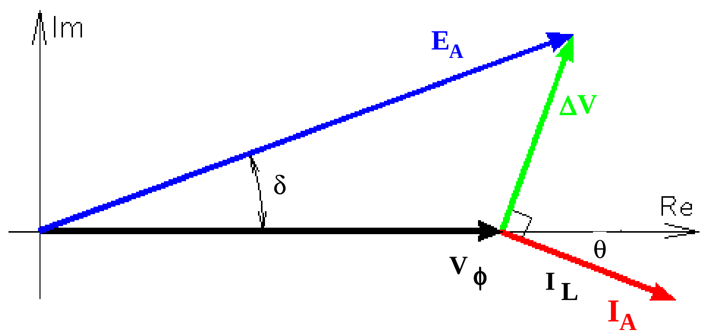
Produces negative Q

$\mathbf{E_A} := V_T + \mathbf{I_L} \cdot (j \cdot X_s)$ $\quad E_A = |\mathbf{E_A}| = 260.28 \cdot V$ $\quad \delta = \arg(\mathbf{E_A}) = 41.718 \cdot deg$

b) Draw a phasor diagram of other possible interpretation of these numbers.
Find the induced armature voltage (E_A) and the power angle, δ. $\quad \delta = ?$

My second assumption:

I_L lags V_T



$\theta := -\mathrm{acos}(pf)$

$\theta = -25.563 \cdot deg$

$\mathbf{I_L} := I_L \cdot e^{j \cdot \theta}$

Produces positive Q

$\mathbf{E_A} := V_T + \mathbf{I_L} \cdot (j \cdot X_s)$ $\quad E_A = |\mathbf{E_A}| = 399.48 \cdot V$ $\quad \delta = \arg(\mathbf{E_A}) = 25.695 \cdot deg$

c) A traveling carnival uses a combination of this generator and the local power company to run its load, mainly induction motors. When the generator is connected to the carnival's power distribution network, it supplies half of the required power, but the current from the power company only decreases by about 30%. Which of the calculations above is most likely correct?

Assumption in a)

Give me the reasoning behind your answer (no calculations required).

The induction motors represent a lagging pf load, they use lots of VARs. If the local generator were supplying those VARs, then the current would go down by about half and quite possibly more. The small reduction in current implies that the generator also consumes VARs (creates negative VARs). That is condition a).

d) What do you change at the generator to reduce the current flow from the power company?
Tell me what you adjust and if you turn it up or down.

Turn up the field current.

A 60 Hz, 2-pole, Y-connected, 3-phase synchronous generator supplies 15 MW of power to a 18 kV bus. The synchronous reactance is 6 Ω/phase. The magnitude of the generator emf equals the magnitude of the bus voltage.

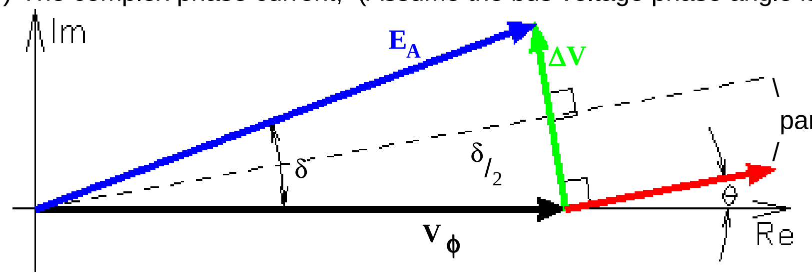
givens $V_\phi := \frac{18\cdot kV}{\sqrt{3}}$ $V_\phi = 10.392\cdot kV$ $X_s := 6\cdot\Omega$ $P_{1\phi} := \frac{15\cdot MW}{3}$ $P_{1\phi} = 5\cdot MW$

$E_A := V_\phi$

Find:

a) The power angle, δ. $P_{1\phi} = \frac{E_A\cdot V_\phi}{X_s}\cdot\sin(\delta)$ $\delta := \mathrm{asin}\left(\frac{P_{1\phi}\cdot X_s}{E_A\cdot V_\phi}\right)$ $\delta = 16.13\cdot deg$

b) The complex phase current, (Assume the bus voltage phase angle is 0º).



$\theta := \frac{\delta}{2}$ $\theta = 8.06\cdot deg$ lead

$P_{1\phi} = V_\phi\cdot I_A\cdot\cos(\theta)$ $I_A := \frac{P_{1\phi}}{V_\phi\cdot\cos(\theta)}$ $I_A = 485.9\cdot A$

c) The magnitude and direction of reactive power.

$Q_{3\phi} := 3\cdot V_\phi\cdot I_A\cdot\sin(-\theta)$ $Q_{3\phi} = -2.125\cdot MVAR$

Since the current leads the voltage, this generator absorbs reactive power (produces -VARs)

Ex. 4

A 60-Hz, three-phase, 6-pole, Δ-connected synchronous motor is connected to 480 V and produces 80 hp. The motor draws minimum current with an excitation voltage of $E_A = 520$ V per phase. Mechanical losses are 5hp .

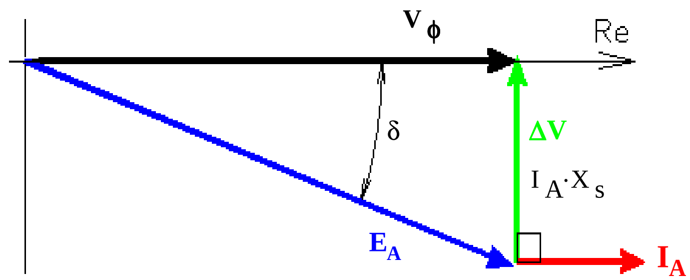
givens $N_{poles} := 6$ $V_\phi := 480\cdot V$ $P_{3\phi} := 80\cdot hp + 5\cdot hp$ $E_A := 520\cdot V$

$P_{1\phi} := \frac{85\cdot hp}{3}\cdot\frac{745.7\cdot W}{hp}$ $P_{1\phi} = 21.1\cdot kW$

Determine the following:

a) The current.

Minimum current implies $pf := 1$ so...



$I_A := \frac{P_{1\phi}}{V_\phi}$ $I_A = 44.02\cdot A$

b) The line current. $I_L := \sqrt{3}\cdot I_A$ $I_L = 76.24\cdot A$

c) The synchronous reactance.

by Pythagorean theorem: $I_A\cdot X_s = \sqrt{E_A{}^2 - V_\phi{}^2}$ $X_s := \frac{\sqrt{E_A{}^2 - V_\phi{}^2}}{I_A}$ $X_s = 4.544\cdot\Omega$

d) The torque.

$\omega_{mech} := \frac{4\cdot\pi\cdot f}{N_{poles}}$ $T_{mech} = \frac{80\cdot hp\cdot\frac{745.7\cdot W}{hp}}{\omega_{mech}} = 475\cdot N\cdot m$

e) The rotor power angle. $\delta = \mathrm{acos}\left(\frac{V_\phi}{E_A}\right) = 22.62\cdot deg$

f) The maximum power this motor could provide at this excitation voltage.

$\delta := 90\cdot deg$ $P_{3\phi} = 3\cdot\frac{E_A\cdot V_\phi}{X_s}\cdot\sin(\delta)\cdot\frac{1\cdot hp}{745.7\cdot W} - 5\cdot hp = 216\cdot hp$

Note: The current rating of the motor may be exceeded at this load.

(F09 E2, p4) An industrial plant is powered from a 480-V, 3-phase bus and currently draws 60 kW at a power factor of 0.8 lagging. A new mill is to be added at the plant. This mill requires a shaft torque of 600 Nm at 1200 rpm. Your job is to specify a motor which will run the mill and correct the plant power factor at the same time. Be sure to specify the type of motor including the number of poles. Tell me how the motor should be connected to the bus (This is an arbitrary decision here, but it will affect many of your other answers). Specify its minimum hp, voltage, and current ratings. Tell me what the back emf should be. You may assume the synchronous reactance is 1 Ω/phase and that losses are negligible.

Plant, as is: $P_{3\phi} := 60 \cdot kW$ $\quad P_{1\phi} := \frac{P_{3\phi}}{3}$ $\quad P_{1\phi} = 20 \cdot kW$ $\quad pf := 0.8$

$S_{1\phi} := \frac{P_{1\phi}}{pf}$ $\quad S_{1\phi} = 25 \cdot kVA$ $\quad Q_{1\phi} := \sqrt{S_{1\phi}^2 - P_{1\phi}^2}$ $\quad Q_{1\phi} = 15 \cdot kVAR$

Motor basics

$N_{poles} := \frac{7200}{1200}$ $\quad N_{poles} = 6$ $\quad \omega_{mech} := \frac{4 \cdot \pi \cdot f}{N_{poles}}$ $\quad \omega_{mech} = 125.7 \cdot \frac{rad}{sec}$ $\quad \left(\frac{377}{3}\right)$

Use a 6-pole synchronous motor

$T_{mech} := 600 \cdot N \cdot m$ motor power = $P_{m3\phi} = T_{mech} \cdot \omega_{mech} = 75.398 \cdot kW$ $\quad 1 \cdot hp = 745.7 \cdot W$

$T_{mech} \cdot \omega_{mech} \cdot \frac{1 \cdot hp}{745.7 \cdot W} = 101.111 \cdot hp$ = min hp rating

$P_{m1\phi} := \frac{T_{mech} \cdot \omega_{mech}}{3}$ $\quad P_{m1\phi} = 25.133 \cdot kW$

To fix the plant pf, $Q_{m1\phi} := -Q_{1\phi}$ $\quad Q_{m1\phi} = -15 \cdot kVAR$

Phase angle of the current: $\theta := \operatorname{atan}\left(\frac{-Q_{m1\phi}}{P_{m1\phi}}\right)$ $\quad \theta = 30.83 \cdot deg$

If you select Y-connected

$V_\phi := \frac{480 \cdot V}{\sqrt{3}}$ $\quad V_\phi = 277.1 \cdot V$ = min voltage rating

Current per phase: $I := \frac{\sqrt{P_{m1\phi}^2 + Q_{m1\phi}^2}}{V_\phi}$ $\quad I = 105.61 \cdot A$ = min current rating

$X_s := 1 \cdot \Omega$ $\quad \mathbf{\Delta V} := I \cdot e^{j \cdot \theta} \cdot X_s \cdot j$ $\quad \mathbf{\Delta V} = (-54.127 + 90.69j) \cdot V$

motor back emf $\quad \mathbf{E_A} := V_\phi - \mathbf{\Delta V}$ $\quad \mathbf{E_A} = (331.255 - 90.69j) \cdot V$ $\quad |\mathbf{E_A}| = 343.44 \cdot V$ = required motor emf

$\delta = \arg(\mathbf{E_A}) = -15.311 \cdot deg$ (unneeded)

If you select Δ-connected

$V_\phi := 480 \cdot V$ $\quad V_\phi = 480 \cdot V$ = min voltage rating

Current per phase: $I := \frac{\sqrt{P_{m1\phi}^2 + Q_{m1\phi}^2}}{V_\phi}$ $\quad I = 60.98 \cdot A$ = min current rating

$X_s := 1 \cdot \Omega$ $\quad \mathbf{\Delta V} := I \cdot e^{j \cdot \theta} \cdot X_s \cdot j$ $\quad \mathbf{\Delta V} = (-31.25 + 52.36j) \cdot V$

motor back emf $\quad \mathbf{E_A} := V_\phi - \mathbf{\Delta V}$ $\quad \mathbf{E_A} = (511.25 - 52.36j) \cdot V$ $\quad |\mathbf{E_A}| = 513.92 \cdot V$ = required motor emf

$\delta = \arg(\mathbf{E_A}) = -5.848 \cdot deg$