ECE 3600 Notation and Per-Unit notes

Per-Unit values

Power systems generally include many transformers. Each transformer transforms the voltage by the turns ration and inversely, the current. Impedances can also be transformed from one side of the transformer to the other by the turns ratio squared. Because all these transformations can become cumbersome, it is often easier to redefine "per-unit" voltages, currents, impedances, etc. in terms of base values that change across each transformer. This way the transformers disappear as long as you use the per-unit values

Base Values

At least two base values must be specified in order to find all the other base values. Apparent power (or just power) is one of the most common base values since it isn't changed by transformers-- it's the same across the entire system. If the power base is changed all the pu values must also change. The second most common base is the voltage which will, of course, change at each transformer.

Analysis is usually done on a per-phase basis, so divide 3-phase powers by 3 and line voltages by $\sqrt{3}$.

most common starting values

$$S_{base} = \frac{S_{3\phi}}{3} = \frac{{V_{base}}^2}{Z_{base}} = {I_{base}}^2 \cdot Z_{base}$$

$P_{base} = Q_{base} = S_{base}$ don't need to be separately defined

$$V_{base} = \frac{V_L}{\sqrt{3}} = \frac{V_{LL}}{\sqrt{3}} = \frac{S_{base}}{I_{base}} = I_{base} \cdot Z_{base}$$

Other bases

$$I_{base} = \frac{S_{base}}{V_{base}} = \frac{V_{base}}{Z_{base}}$$

This is the base for line current or Y-connected devices

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{{V_{base}}^2}{S_{base}} = \frac{S_{base}}{{I_{base}}^2}$$

Y-connected

$R_{base} = X_{base} = Z_{base}$ don't need to be separately defined

Expressing values as "per-unit". (Multiply by 100% to express as %)

$$S_{pu} = \frac{S}{S_{base}} \qquad P_{pu} = \frac{P}{S_{base}} \qquad Q_{pu} = \frac{Q}{S_{base}}$$

$$V_{pu} = \frac{V}{V_{base}} \qquad I_{pu} = \frac{I}{I_{base}}$$

resistance $R_{pu} = \frac{R}{Z_{base}}$ conductance $G = \frac{1}{R}$ $G_{pu} = \frac{1}{R_{pu}}$

reactance $X_{pu} = \frac{X}{Z_{base}}$ susceptance $B = \frac{1}{X}$ $B_{pu} = \frac{1}{X_{pu}}$

impedance $Z_{pu} = \frac{Z}{Z_{base}}$ admittance $Y = \frac{1}{Z}$ $Y_{pu} = \frac{1}{Z_{pu}}$

The V_{pu}, I_{pu}, Z_{pu} and S_{pu} values are not affected by transformers.

The voltage, current and impedance bases WILL change at each transformer. The power base will NOT.

Power base changes affect all values. This means that Z_{pu}s calculated for one power base cannot be used with Z_{pu}s calculated from another power base. This comes up commonly when a Z_{pu} is given for a piece of equipment based on its own power rating and it's different from the general power base. Z_{pu}s can be changed from one base to another by finding the actual Z and then relating it to the general power base. This is the preferred method since it is very straight forward.

Otherwise: $Z_{pu.2} = Z_{pu.1} \cdot \frac{S_{base.2} \cdot (V_{base.1})^2}{S_{base.1} \cdot (V_{base.2})^2}$ OR $Z_{pu.2} = Z_{pu.1} \cdot \frac{S_{base.2}}{S_{base.1}}$ SAME V_{base}s

Note:

If you specify a 3-phase power base: $S_{3\phi base}$ most of the relations above will have to be modified.

However, $\frac{S_{3\phi}}{S_{3\phi base}} = \frac{S_{1\phi}}{S_{1\phi base}} = S_{pu}$ result in the same value pu (balanced system).

Similarly $V_{Lbase} = \sqrt{3}\cdot V_{base}$ can also be specified and $\frac{V_L}{V_{Lase}} = \frac{V_{LN}}{V_{base}} = V_{pu}$

Beyond this these other bases are not usually helpful and must be used differently in per-phase calculations.

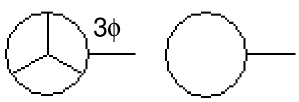
Trick: $Z_{base} = \frac{V_{Lbase}^{\;2}}{S_{3\phi base}}$

One-Line Diagrams

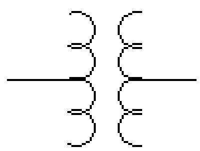
Full 3-phase diagrams can be very cumbersome. In a balanced system you only need to consider one phase (per-phase analysis). Anything not Y-connected can be converted to a Y-equivalent.
In a balanced system neutral current is zero, so in one-line diagrams, even the neutral connections are omitted.

$S_{1\phi} = \frac{S_{3\phi}}{3}$ $\qquad P_{1\phi} = \frac{P_{3\phi}}{3}$ $\qquad V_{LN} = \frac{V_L}{\sqrt{3}} = \frac{V_{LL}}{\sqrt{3}}$ $\qquad Z_Y = \frac{Z_\Delta}{3}$

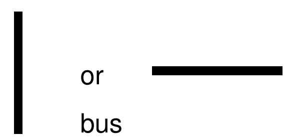
Important symbols



generator or motor

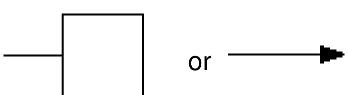


transformer

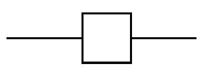


All items connected to one bus to have same voltage. Like a circuit node.

transmission line



Load:



liquid (oil) filled

air

Circuit breakers

manual powered

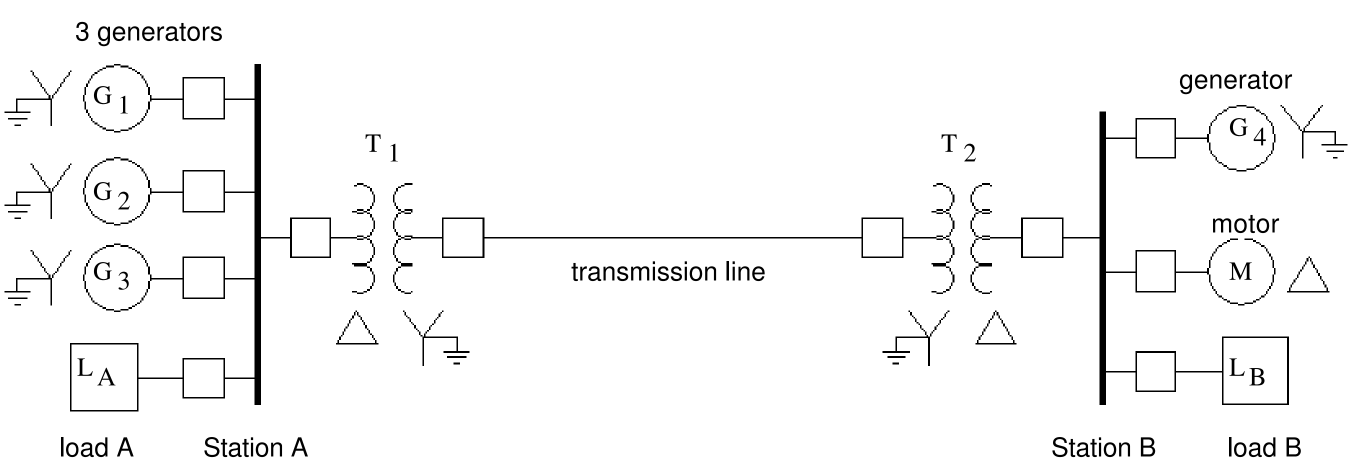
Switches

Connection symbols:

delta wye wye with ground

Can also include resistors, inductors, capacitors and impedances

Example

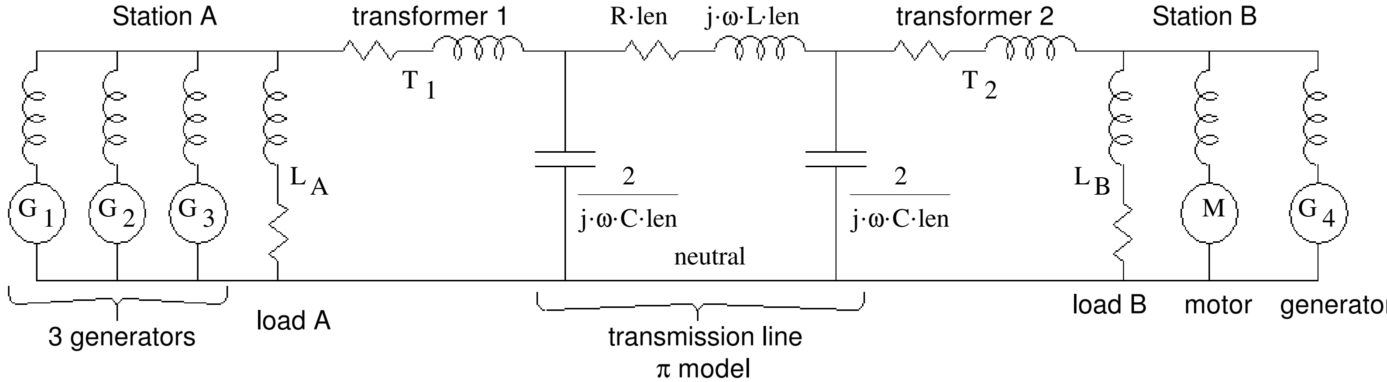


Impedance Diagrams

Component values are per-unit (pu).

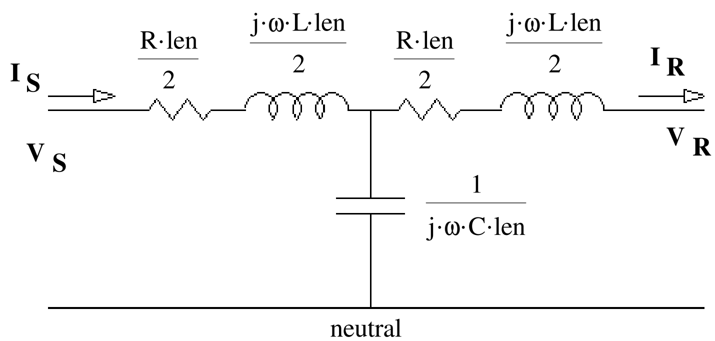
If you didn' t use pu values the they would you would have transform impedances across the transformers.

Same system



A T model of the transmission line may be easier to work with

T model:

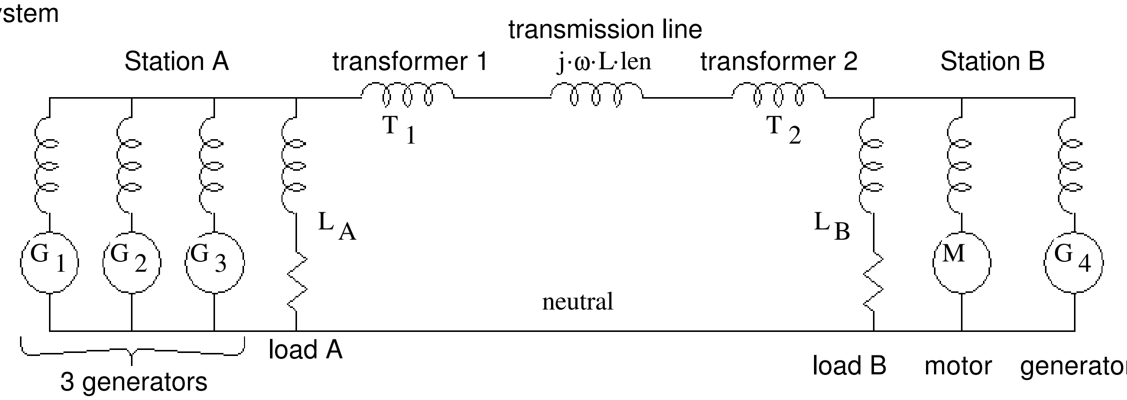


Reactance Diagrams

Ignore the line capacitances, and all resistors but those in the loads

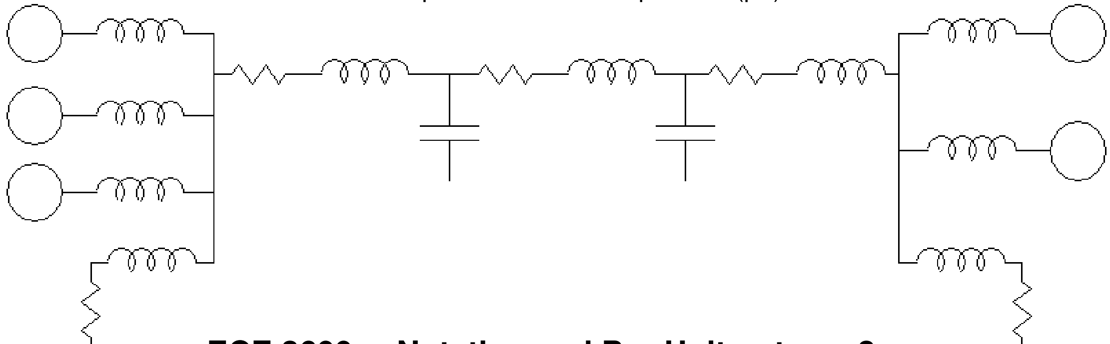
Component values are per-unit (pu).

Same system

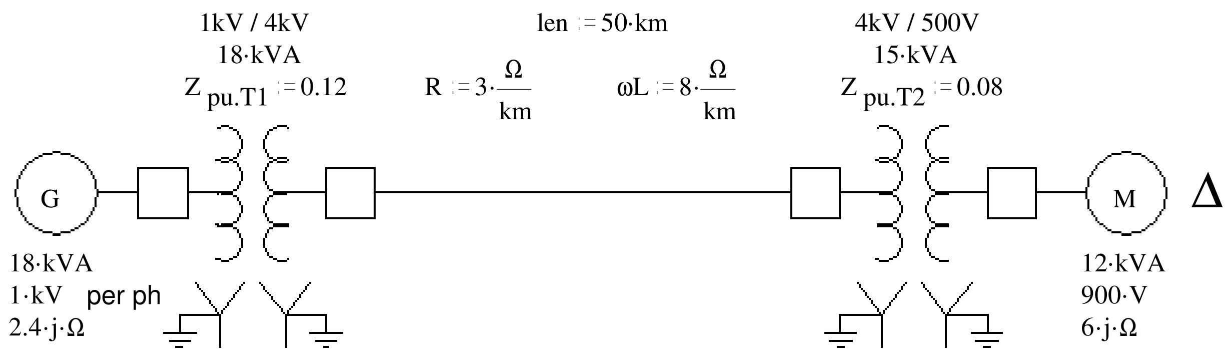


One-Line Impedance Diagrams

Component values are per-unit (pu).



Ex.1 Find the current and voltage at the load



primary	primary

$$S_{base.T1} := \frac{18\cdot kVA}{3} \qquad V_{base.T1} := 1\cdot kV$$

$$I_{base.T1} := \frac{S_{base.T1}}{V_{base.T1}} \qquad Z_{base.T1} := \frac{V_{base.T1}}{I_{base.T1}}$$

$$I_{base.T1} = 6\cdot A \qquad Z_{base.T1} = 166.667\cdot\Omega$$

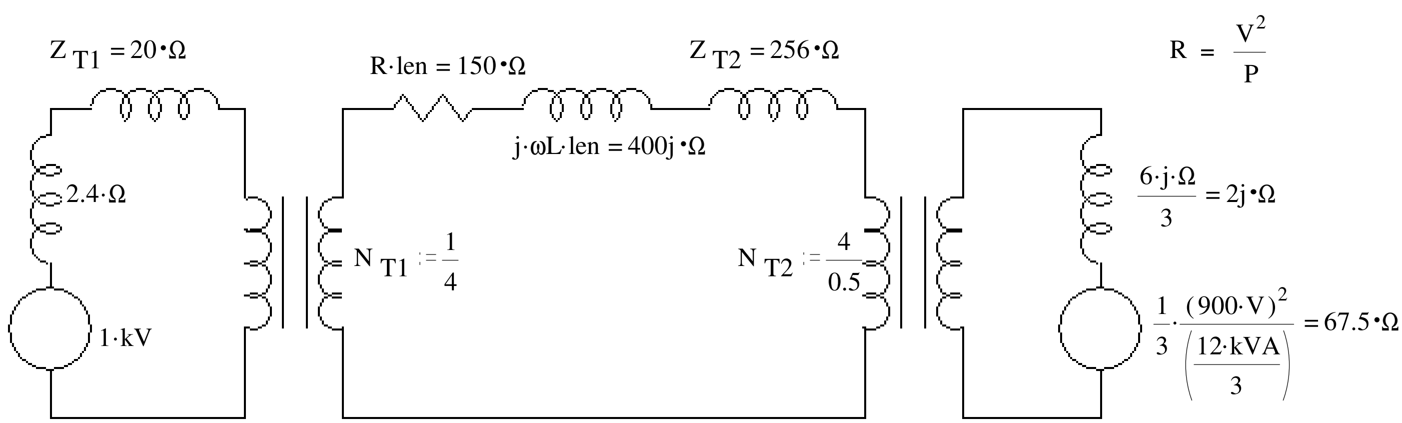
$$Z_{T1} := 0.12\cdot Z_{base.T1}$$

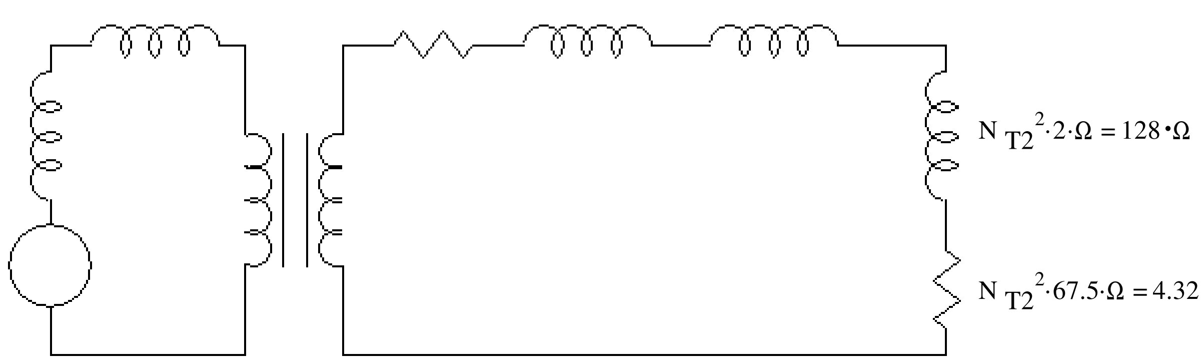
$$S_{base.T2} := \frac{15\cdot kVA}{3} \qquad V_{base.T2} := 4\cdot kV$$

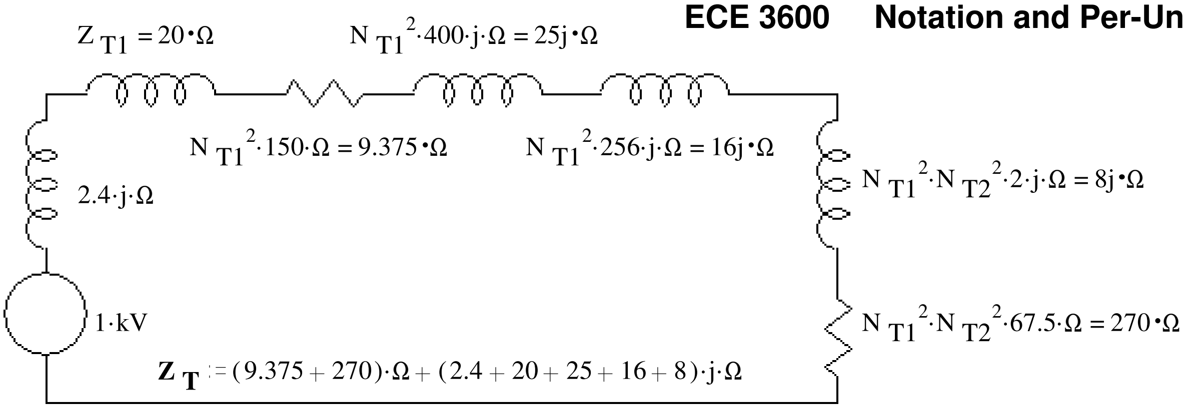
$$I_{base.T2} := \frac{S_{base.T2}}{V_{base.T2}} \qquad Z_{base.T2} := \frac{V_{base.T2}}{I_{base.T2}}$$

$$I_{base.T2} = 1.25\cdot A \qquad Z_{base.T2} = 3.2\cdot 10^{3}\cdot\Omega$$

$$Z_{T2} := 0.08\cdot Z_{base.T2}$$







$\mathbf{I_1} := \dfrac{1\cdot kV}{\mathbf{Z_T}}$ $\quad \mathbf{I_1} = 3.36 - 0.859j \;\cdot A$ $\quad \mathbf{I_1}\cdot N_{T1}\cdot N_{T2} = 6.72 - 1.717j \;\cdot A$

$\dfrac{\mathbf{I_1}\cdot(270 + 8\cdot j)\cdot\Omega}{N_{T1}\cdot N_{T2}} = 457.029 - 102.485j \;\cdot V$ $\quad \left|\dfrac{\mathbf{I_1}\cdot(270 + 8\cdot j)\cdot\Omega}{N_{T1}\cdot N_{T2}}\right|\cdot\sqrt{3} = 811.3\cdot V$

Motor volts per winding

Motor: $\sqrt{2^2 + 67.5^2}\cdot\Omega = 67.53\cdot\Omega$ $\quad I_M := \dfrac{468.4\cdot V}{67.5\cdot\Omega}\cdot\dfrac{1}{\sqrt{3}}$ $\quad I_M = 4.01\cdot A$

Motor current per winding

Per-unit method

Choose $S_{base} := \dfrac{15\cdot kVA}{3}$ Because transformer T2 limits the power transfer

Generator, G $\quad V_{base.G} := 1\cdot kV$

$I_{base.G} := \dfrac{S_{base}}{V_{base.G}}$ $\quad Z_{base.G} := \dfrac{V_{base.G}}{I_{base.G}}$ $\quad Z_{Gpu} := \dfrac{2.4\cdot j\cdot\Omega}{Z_{base.G}}$ $\quad Z_{T1pu} := \dfrac{20\cdot j\cdot\Omega}{Z_{base.G}}$

$I_{base.G} = 5\cdot A$ $\quad Z_{base.G} = 200\cdot\Omega$ $\quad Z_{Gpu} = 0.012j\cdot pu$ $\quad Z_{T1pu} = 0.1j\cdot pu$

Transmission line,TL

$V_{base.TL} := \dfrac{V_{base.G}}{N_{T1}}$ $\quad I_{base.TL} := \dfrac{S_{base}}{V_{base.TL}}$ $\quad Z_{base.TL} := \dfrac{V_{base.TL}}{I_{base.TL}}$ $\quad \mathbf{Z_{TLpu}} := \dfrac{150\cdot\Omega + 400\cdot j\cdot\Omega}{Z_{base.TL}}$

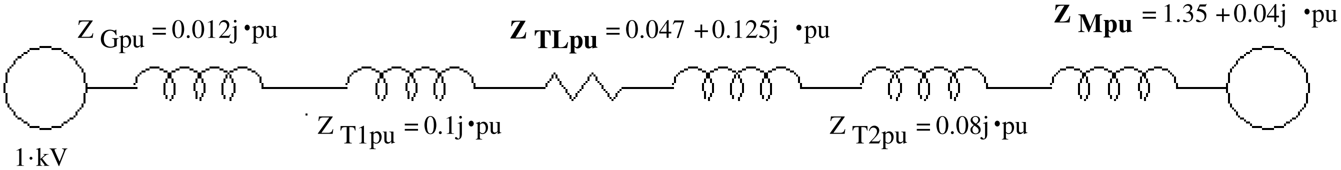
$V_{base.TL} = 4\cdot kV$ $\quad I_{base.TL} = 1.25\cdot A$ $\quad Z_{base.TL} = 3.2\cdot 10^3 \;\cdot\Omega$ $\quad \mathbf{Z_{TLpu}} = 0.047 + 0.125j \;\cdot pu$

$Z_{T2pu} := \dfrac{256\cdot j\cdot\Omega}{Z_{base.TL}}$ $\quad Z_{T2pu} = 0.08j\cdot pu$

Motor,M

$V_{base.M} := \dfrac{V_{base.TL}}{N_{T2}}$ $\quad I_{base.M} := \dfrac{S_{base}}{V_{base.M}}$ $\quad Z_{base.M} := \dfrac{V_{base.M}}{I_{base.M}}$ $\quad \mathbf{Z_{Mpu}} := \dfrac{67.5\cdot\Omega + 2\cdot j\cdot\Omega}{Z_{base.M}}$

$V_{base.M} = 500\cdot V$ $\quad I_{base.M} = 10\cdot A$ $\quad Z_{base.M} = 50\cdot\Omega$ $\quad \mathbf{Z_{Mpu}} = 1.35 + 0.04j \;\cdot pu$



$\mathbf{Z}_{Tpu} := Z_{Gpu} + Z_{T1pu} + \mathbf{Z}_{TLpu} + Z_{T2pu} + \mathbf{Z}_{Mpu}$ $\quad$ $\mathbf{Z}_{Tpu} = 1.397 + 0.357j \cdot pu$

$\mathbf{I}_{pu} := \dfrac{1 \cdot pu}{\mathbf{Z}_{Tpu}}$ $\quad$ $\mathbf{I}_{pu} = 0.672 - 0.172j \cdot pu$

$\mathbf{I}_{M} := \mathbf{I}_{pu} \cdot I_{base.M}$ $\quad$ $\mathbf{I}_{M} = 6.72 - 1.717j \cdot A$ $\quad$ $|\mathbf{I}_{M}| = 6.936 \cdot A$

Motor current per winding $\dfrac{|\mathbf{I}_{M}|}{\sqrt{3}} = 4.004 \cdot A$

$\mathbf{V}_{Mpu} := \mathbf{I}_{pu} \cdot \mathbf{Z}_{Mpu}$ $\quad$ $\mathbf{V}_{Mpu} = 0.914 - 0.205j$

$\mathbf{V}_{M} := \mathbf{V}_{Mpu} \cdot V_{base.M}$ $\quad$ $\mathbf{V}_{M} = 457.029 - 102.485j \cdot V$ $\quad$ $|\mathbf{V}_{M}| = 468.379 \cdot V$

Motor volts per winding $|\mathbf{V}_{M}| \cdot \sqrt{3} = 811.256 \cdot V$

What if the transformers were wired differently?

Effective turns ratios $\dfrac{N_{T1}}{\sqrt{3}} = 0.144$ $\quad$ $N_{T2} \cdot \sqrt{3} = 13.856$

Transmission line,TL

$V_{base.TL} := \dfrac{V_{base.G}}{\dfrac{N_{T1}}{\sqrt{3}}}$ $\quad$ $I_{base.TL} := \dfrac{S_{base}}{V_{base.TL}}$ $\quad$ $Z_{base.TL} := \dfrac{V_{base.TL}}{I_{base.TL}}$ $\quad$ $\mathbf{Z}_{TLpu} := \dfrac{150 \cdot \Omega + 400 \cdot j \cdot \Omega}{Z_{base.TL}}$

$V_{base.TL} = 6.928 \cdot kV$ $\quad$ $I_{base.TL} = 0.722 \cdot A$ $\quad$ $Z_{base.TL} = 9.6 \cdot 10^{3} \cdot \Omega$ $\quad$ $\mathbf{Z}_{TLpu} = 0.016 + 0.042j \cdot pu$

$Z_{T2pu} := \dfrac{256 \cdot j \cdot \Omega}{Z_{base.TL}}$ $\quad$ $Z_{T2pu} = 0.027j \cdot pu$

Motor,M

$V_{base.M} := \dfrac{V_{base.TL}}{N_{T2} \cdot \sqrt{3}}$ $\quad$ $I_{base.M} := \dfrac{S_{base}}{V_{base.M}}$ $\quad$ $Z_{base.M} := \dfrac{V_{base.M}}{I_{base.M}}$ $\quad$ $\mathbf{Z}_{Mpu} := \dfrac{67.5 \cdot \Omega + 2 \cdot j \cdot \Omega}{Z_{base.M}}$

$V_{base.M} = 500 \cdot V$ $\quad$ $I_{base.M} = 10 \cdot A$ $\quad$ $Z_{base.M} = 50 \cdot \Omega$ $\quad$ $\mathbf{Z}_{Mpu} = 1.35 + 0.04j \cdot pu$

$\mathbf{Z}_{Tpu} := Z_{Gpu} + Z_{T1pu} + \mathbf{Z}_{TLpu} + Z_{T2pu} + \mathbf{Z}_{Mpu}$ $\quad$ $\mathbf{Z}_{Tpu} = 1.366 + 0.22j \cdot pu$

$\mathbf{I}_{pu} := \dfrac{1 \cdot pu}{\mathbf{Z}_{Tpu}}$ $\quad$ $\mathbf{I}_{pu} = 0.714 - 0.115j \cdot pu$

$\mathbf{I}_{M} := \mathbf{I}_{pu} \cdot I_{base.M}$ $\quad$ $\mathbf{I}_{M} = 7.137 - 1.151j \cdot A$ $\quad$ $|\mathbf{I}_{M}| = 7.229 \cdot A$

Motor current per winding $\dfrac{|\mathbf{I}_{M}|}{\sqrt{3}} = 4.174 \cdot A$

$\mathbf{V}_{Mpu} := \mathbf{I}_{pu} \cdot \mathbf{Z}_{Mpu}$ $\quad$ $\mathbf{V}_{Mpu} = 0.968 - 0.127j$

$\mathbf{V}_{M} := \mathbf{V}_{Mpu} \cdot V_{base.M}$ $\quad$ $\mathbf{V}_{M} = 484.042 - 63.451j \cdot V$ $\quad$ $|\mathbf{V}_{M}| = 488.183 \cdot V$

Motor volts per winding $|\mathbf{V}_{M}| \cdot \sqrt{3} = 845.558 \cdot V$