

**Requirements of the power system:**

1. The power generation meets the demand,  $P_{\text{generated}} = P_{\text{desired}}$ . Always true: the net  $P = 0$  and net  $Q = 0$ .
2. Bus voltages are within limits.
3. Generators operate within their real and reactive power limits.
4. Transformers and transmission lines are not overloaded.

Sensors placed around the network can let operators know if these requirements are being met. The sensors, the telemetry and the display of this information is called the SCADA (**S**upervisory **C**ontrol and **D**ata **A**cquisition) system.

**Dispatch Center**

This is where the operators of the power grid work to meet these requirements. They have a number of responsibilities:

1. Predict power requirements in advance, using calendar and weather information in conjunction with prior experiences.
2. Make plans in advance to meet the predicted power needs while staying within the safe limits of the system and minimizing costs. The operators may give instructions to in-house power plants, buy from independent power producers (IPPs), or buy from neighboring power companies. They also have to account for wind and solar sources over which they have little control and operate according to regulations set by NERC and WECC.
3. Process requests from other departments within the company to schedule maintenance and replacements of important pieces of equipment. If critical equipment needs to be shut down, it should be done at a time and in a way that minimizes the effects on customers.
4. Monitor the system in real time and make adjustments as necessary to maintain balance. They especially watch the system frequency and voltage levels.
5. Address problems, outages and malfunctions as they come up, all while staying within regulatory requirements.

All of the items listed here require more than just the SCADA system. Operators also need a way to predict the effects of changes, that is, they need to know how the power will flow from sources to users and what the resulting voltages will be. In a highly interconnected system, this "power - flow problem" is not easy to solve.

**The Power - Flow Problem** (sometimes called Load - Flow problem)

To predict how the system will respond to different situations, you have to solve a complex problem involving many sources, transformers, transmission lines and loads, typically using nodal analysis, where the buses are the nodes. This is a steady-state analysis.

A few assumptions and simplifications make the problem more tractable:

1. Assume a balanced 3-phase system so you can work with just one phase (per phase).
2. Work with per-unit values with a single  $S_{\text{base}}$  and  $V_{\text{base}}$ s such that the transformers become simple impedances.
3. Assume at least one bus is connected to the larger power grid and that this bus can supply whatever  $P$  and  $Q$  are needed to make up for whatever slack there may be locally. This is called the "Slack bus" or "Swing bus". The voltage phase angle of this bus is taken to be the  $0^\circ$  reference.
4. Positive  $P$ ,  $Q$  and  $I$  are *into* the local system. Negative goes the other way. Thus, power generation results in positive  $P$  and  $I$  and loads result in negative  $P$  and  $I$ . Same for  $Q$ s that would normally be considered positive.

**Knowns** Assumptions of what you *will* know and some bus "types": (A **bus** is substation or generation site.)

5. **Generator bus** At buses where generators are connected and holding the voltage constant you will know the power,  $P$ , and the voltage magnitude,  $V$ . This type of bus is called a "Generator bus", a "Voltage controlled bus" or simply a "PV bus".

$P$  is positive unless loads are also connected to this bus, then load requirements greater than the generator output could make  $P$  negative.

6. **Load bus** At buses where only loads are connected you will know the power,  $P_{\text{load}}$ , and the reactive power,  $Q_{\text{load}}$ , consumed by the load. This type of bus is called a "Load bus" or simply a "PQ bus". Power out of the system is considered negative, so:  $P = -P_{\text{load}}$ ,  $Q = -Q_{\text{load}}$ .

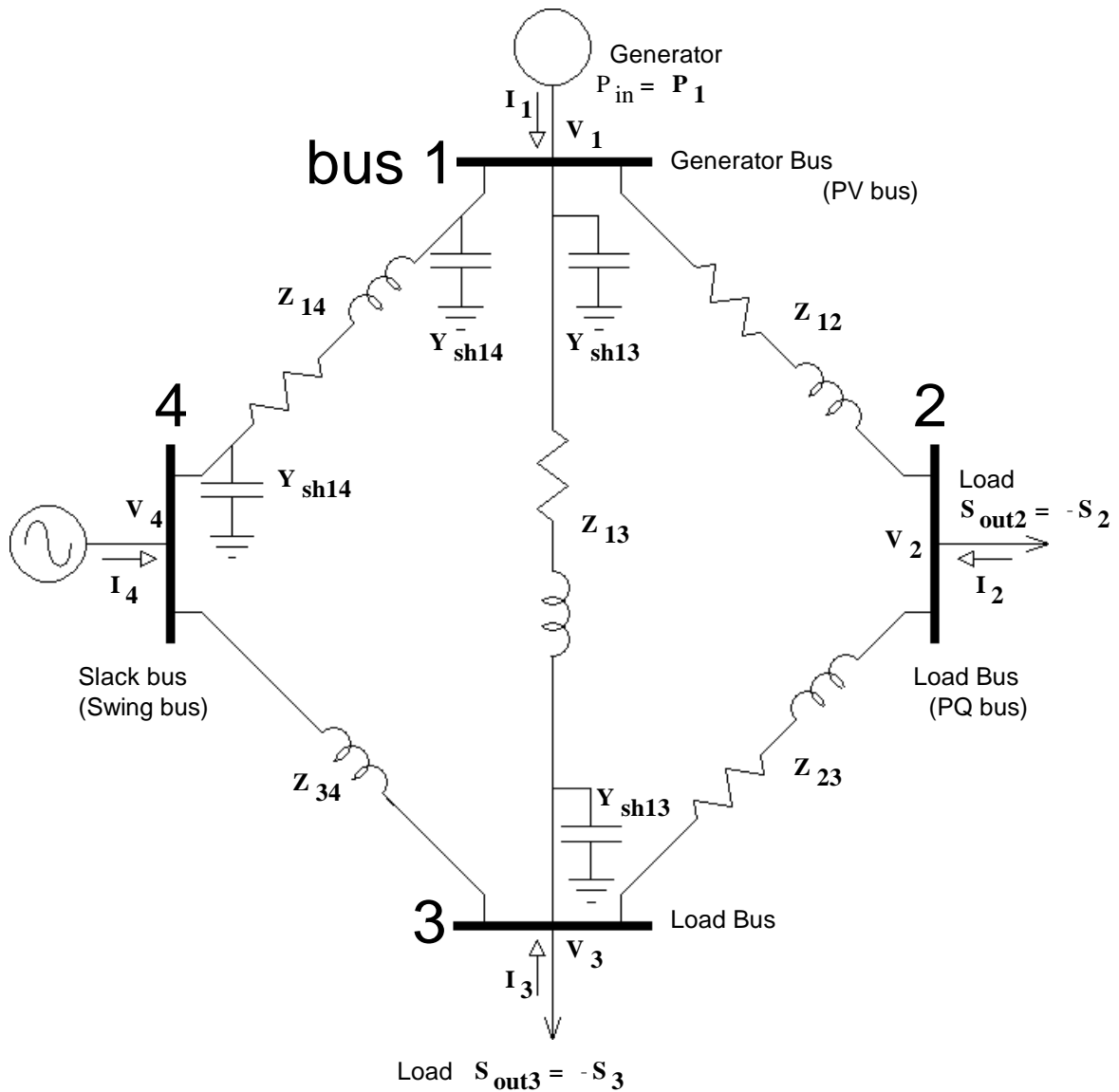
Buses where only transmission lines and transformers are connected are considered a variation of a Load bus or PQ bus, with  $P = 0$  and  $Q = 0$ .

7. **Slack (or Swing) bus** This is the reference bus or a connection to a larger grid. Its voltage is the reference voltage (1 pu) and  $0^\circ$  phase.  $P$  and  $Q$  are unknown and adjusted to make the overall net  $P$  and  $Q$  of the system 0. Also called the Swing bus or "Infinite" bus.
8. All line and transformer characteristics.

# A one-line diagram of a simple, 4-bus, power system or sub-grid.

The actual calculations use admittances ( $Y$ ) rather than impedances.  $Y = \frac{1}{Z}$   
 Each bus is a "node" for nodal analysis.

But watch out for - signs in the matrix, below



Admittance Matrix

Nodal analysis:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

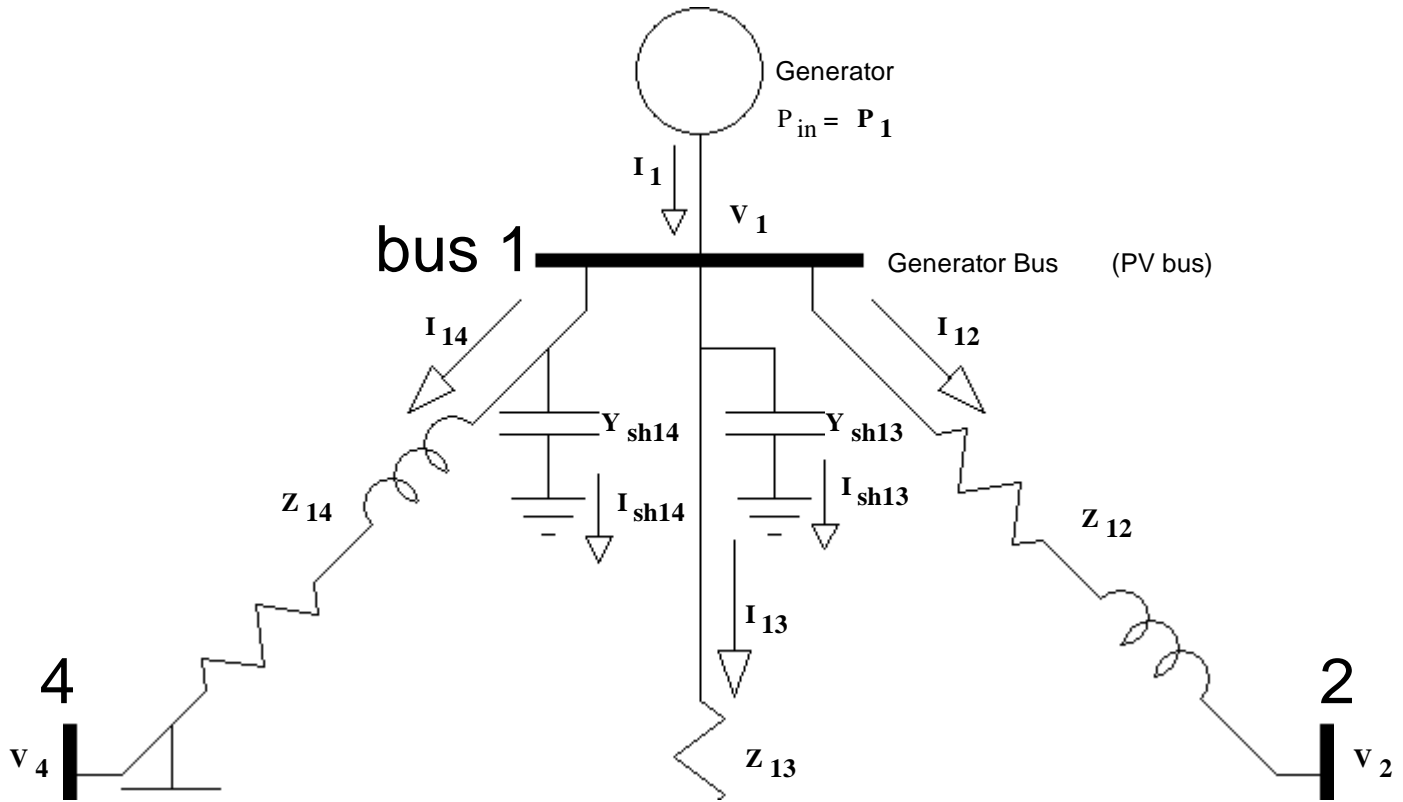
This is a symmetric matrix,  $Y_{12} = \frac{-1}{Z_{12}} = Y_{21}$ ,  $Y_{13} = \frac{-1}{Z_{13}} = Y_{31}$ , ...  $Y_{43} = \frac{-1}{Z_{14}} = Y_{34}$   
 The negative signs will be explained on the next page

No direct connection between buses 2 and 4 means that  $Y_{24} = 0 = Y_{42}$

Finding the admittance matrix is covered next and in section 10.3 (p.498) in your textbook.

Looking just at bus 1:

From the matrix:  $I_1 = V_1 \cdot Y_{11} + V_2 \cdot Y_{12} + V_3 \cdot Y_{13} + V_4 \cdot Y_{14}$



From

the diagram:  $I_1 = I_{sh13} + I_{sh14} + I_{12} + I_{13} + I_{14}$

$$\begin{aligned} \text{and: } I_1 &= V_1 \cdot Y_{sh13} + V_1 \cdot Y_{sh14} + (V_1 - V_2) \cdot \frac{1}{Z_{12}} + (V_1 - V_3) \cdot \frac{1}{Z_{13}} + (V_1 - V_4) \cdot \frac{1}{Z_{14}} \\ &= V_1 \cdot (Y_{sh13} + Y_{sh14}) + \frac{1}{Z_{12}} \cdot V_1 - \frac{1}{Z_{12}} \cdot V_2 + \frac{1}{Z_{13}} \cdot V_1 - \frac{1}{Z_{13}} \cdot V_3 + \frac{1}{Z_{14}} \cdot V_1 - \frac{1}{Z_{14}} \cdot V_4 \\ &= V_1 \cdot \left( Y_{sh13} + Y_{sh14} + \frac{1}{Z_{12}} + \frac{1}{Z_{13}} + \frac{1}{Z_{14}} \right) - \frac{1}{Z_{12}} \cdot V_2 - \frac{1}{Z_{13}} \cdot V_3 - \frac{1}{Z_{14}} \cdot V_4 \\ &= V_1 \cdot \left( Y_{sh13} + Y_{sh14} + \frac{1}{Z_{12}} + \frac{1}{Z_{13}} + \frac{1}{Z_{14}} \right) + V_2 \cdot \left( -\frac{1}{Z_{12}} \right) + V_3 \cdot \left( -\frac{1}{Z_{13}} \right) + V_4 \cdot \left( -\frac{1}{Z_{14}} \right) \end{aligned}$$

From matrix:  $I_1 = V_1 \cdot Y_{11} \quad \text{and} \dots \quad + V_2 \cdot Y_{12} + V_3 \cdot Y_{13} + V_4 \cdot Y_{14}$

$$\begin{aligned} Y_{11} &= \left( Y_{sh13} + Y_{sh14} + \frac{1}{Z_{12}} + \frac{1}{Z_{13}} + \frac{1}{Z_{14}} \right) \\ &= \sum \text{ of all admittances connected to bus 1} \\ Y_{12} &= \left( -\frac{1}{Z_{12}} \right) = - \text{ admittance 12} \\ Y_{13} &= \left( -\frac{1}{Z_{13}} \right) \\ Y_{14} &= \left( -\frac{1}{Z_{14}} \right) \end{aligned}$$

So, even though it's called the admittance matrix,  $Y_{mn} \neq \frac{1}{Z_{mn}}$  EASY mistake to make!

## ECE 3600 Power Flow notes p4

Start with the voltages you know from the slack bus and any generator buses. Guess the rest to start.

Solve this with a computer:

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \\ \mathbf{I}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} & \mathbf{Y}_{13} & \mathbf{Y}_{14} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} & \mathbf{Y}_{23} & \mathbf{Y}_{24} \\ \mathbf{Y}_{31} & \mathbf{Y}_{32} & \mathbf{Y}_{33} & \mathbf{Y}_{34} \\ \mathbf{Y}_{41} & \mathbf{Y}_{42} & \mathbf{Y}_{43} & \mathbf{Y}_{44} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \\ \mathbf{V}_4 \end{bmatrix}$$

To get all the unknown  $\mathbf{I}$ s and  $\mathbf{V}$ s.

Notice that you still don't actually have the powers, even though we're solving the "power-flow" problem

Combine with power calculations:

Complex "power"  $\mathbf{S} = \mathbf{V} \cdot \overline{\mathbf{I}}$  =  $P + jQ$  if  $P$  and  $Q$  are to be found, say at slack bus

$$\overline{\mathbf{I}} = \frac{P + jQ}{\mathbf{V}} \quad \text{OR} \quad \mathbf{V} = \frac{P + jQ}{\overline{\mathbf{I}}} \quad \text{to get new } P \text{ \& } Q \text{ values for iterative solutions}$$

$$\mathbf{I} = \overline{\left( \frac{P + jQ}{\mathbf{V}} \right)} = \frac{P - jQ}{\overline{\mathbf{V}}} \quad \text{if } P \text{ and } Q \text{ are known}$$

For a load bus:  $\mathbf{I}_2 = \mathbf{V}_1 \cdot \mathbf{Y}_{21} + \mathbf{V}_2 \cdot \mathbf{Y}_{22} + \mathbf{V}_3 \cdot \mathbf{Y}_{23} + \mathbf{V}_4 \cdot \mathbf{Y}_{24} = \frac{P_2 - jQ_2}{\overline{\mathbf{V}_2}}$

To get a new value of  $\mathbf{V}_2$  for iterative solutions

$$\mathbf{V}_2 = \frac{1}{\mathbf{Y}_{22}} \cdot \left[ \frac{P_2 - jQ_2}{\overline{\mathbf{V}_2}} - (\mathbf{V}_1 \cdot \mathbf{Y}_{21} + \mathbf{V}_3 \cdot \mathbf{Y}_{23} + \mathbf{V}_4 \cdot \mathbf{Y}_{24}) \right]$$

For a generator bus:

To get value of  $Q_1 = -\text{Im} \left[ \overline{\mathbf{V}_1} \cdot (\mathbf{V}_1 \cdot \mathbf{Y}_{11} + \mathbf{V}_2 \cdot \mathbf{Y}_{12} + \mathbf{V}_3 \cdot \mathbf{Y}_{13} + \mathbf{V}_4 \cdot \mathbf{Y}_{14}) \right]$

To get a new value of  $\mathbf{V}_1$  for iterative solutions

$$\mathbf{V}_1 = \frac{1}{\mathbf{Y}_{11}} \cdot \left[ \frac{P_1 - jQ_1}{\overline{\mathbf{V}_1}} - (\mathbf{V}_2 \cdot \mathbf{Y}_{12} + \mathbf{V}_3 \cdot \mathbf{Y}_{13} + \mathbf{V}_4 \cdot \mathbf{Y}_{14}) \right]$$

Generator busses are handled a little differently, see section 11.2, p. 530 in textbook.

These problems are solved by computers using iterative, numerical methods, like the Newton-Raphson or the Gauss-Siedel method. They may require a starting guess and may not always converge to a solution.

Once you have  $\mathbf{I}$ s and  $\mathbf{V}$ s,

The line currents can be found from:  $\mathbf{I}_{L12} = \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_{\text{series12}}} = (\mathbf{V}_1 - \mathbf{V}_2) \cdot \mathbf{Y}_{\text{series12}}$

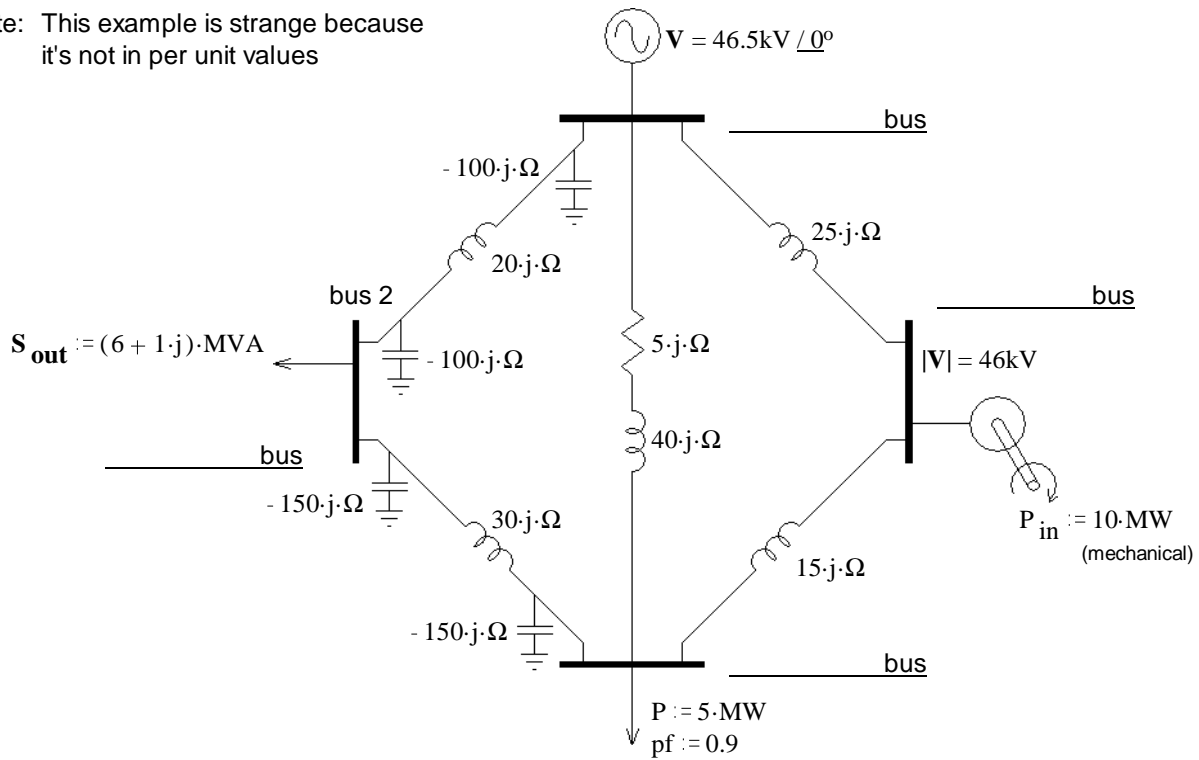
Note: Because all the calculations are done with per-unit values, some issues disappear.

- Transformers become simple impedances, typically reduced to  $X_s$  (leakage reactance) only.
- Voltages as line-to-neutral or line-to-line are the same in pu.
- Powers as one-phase or 3-phase are the same in pu.

Sources: Electric Machinery and Power System Fundamentals, Stephen J, Chapman  
Power Systems Analysis and Design, Glover & Sarma  
First Course on Power Systems, Ned Mohan

Example Consider the power system shown below.

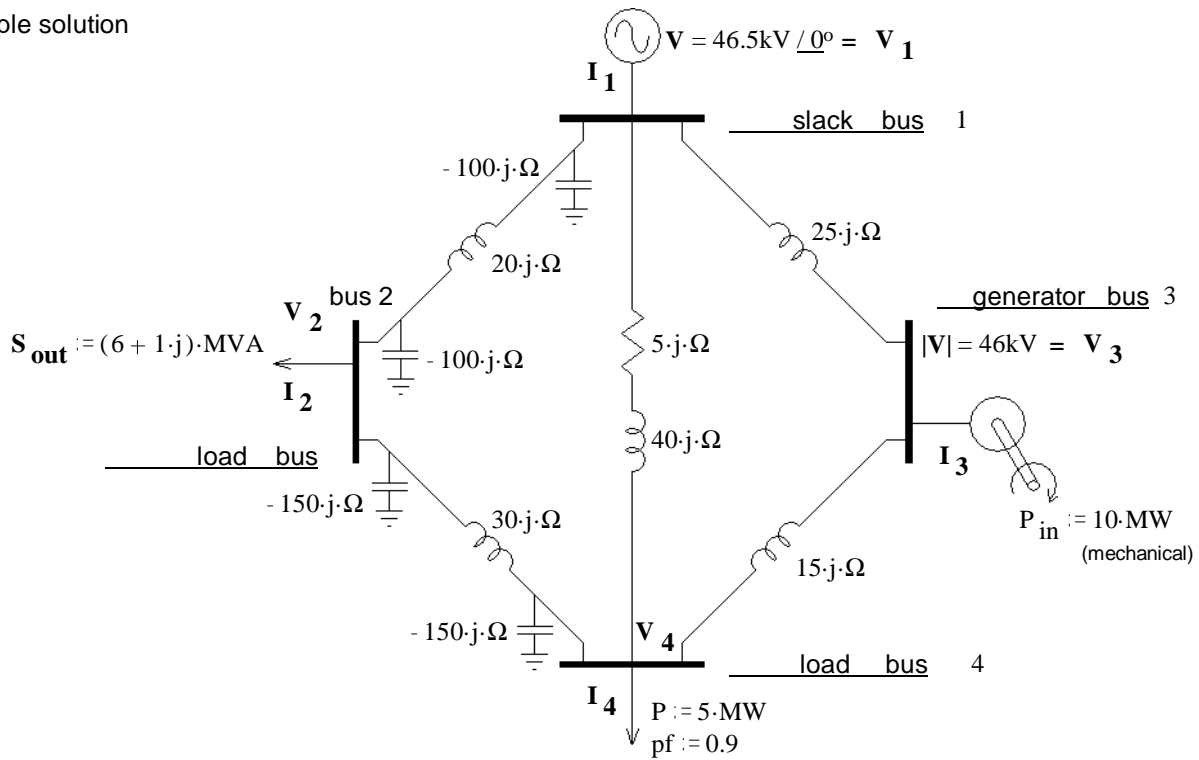
Note: This example is strange because it's not in per unit values



- Identify each bus as "slack", "load", or "generator".
- Number the slack bus as "bus 1". I have labeled bus 2. Label the other two on the drawing.
- Show  $V_1$ ,  $V_2$ ,  $V_3$ , and  $V_4$  on the drawing.
- Show  $I_1$ ,  $I_2$ ,  $I_3$ , and  $I_4$  on the drawing and draw arrows to indicate the direction of each.
- Find elements **A**, **B**, **C**, and **D** in the matrix below and any zero elements.

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} - & \mathbf{A} & - & - \\ - & \mathbf{B} & - & - \\ \mathbf{C} & - & - & - \\ \mathbf{D} & - & - & - \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Example solution



- a) Identify each bus as "slack", "load", or "generator".
- b) Number the slack bus as "bus 1". I have labeled bus 2. Label the other two on the drawing.
- c) Show  $V_1$ ,  $V_2$ ,  $V_3$ , and  $V_4$  on the drawing.
- d) Show  $I_1$ ,  $I_2$ ,  $I_3$ , and  $I_4$  on the drawing and draw arrows to indicate the direction of each.
- e) Find elements **A**, **B**, **C**, and **D** in the matrix below and any zero elements.

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} - & \mathbf{A} & - & - \\ - & \mathbf{B} & 0 & - \\ \mathbf{C} & 0 & - & - \\ \mathbf{D} & - & - & - \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$Z_{12} := j \cdot 20 \cdot \Omega \quad \mathbf{A} := -\frac{1}{Z_{12}} \quad \mathbf{A} = 0.05j \cdot \frac{1}{\Omega}$$

$$Z_{21} := j \cdot 20 \cdot \Omega \quad Z_{24} := j \cdot 30 \cdot \Omega \quad Z_{31} := j \cdot 25 \cdot \Omega \quad Z_{41} := (5 + j \cdot 40) \cdot \Omega$$

$$\mathbf{B} := \frac{1}{-100j \cdot \Omega} + \frac{1}{-150j \cdot \Omega} + \frac{1}{Z_{21}} + \frac{1}{Z_{24}} \quad \mathbf{B} = -0.06667j \cdot \frac{1}{\Omega}$$

$$\mathbf{C} := -\frac{1}{Z_{31}} \quad \mathbf{C} = 0.04j \cdot \frac{1}{\Omega} \quad \mathbf{D} := -\frac{1}{Z_{41}} \quad \mathbf{D} = -0.00308 + 0.02462j \cdot \frac{1}{\Omega}$$

**C** and **D** may be reversed, depending on your choice of bus numbers.

No connection between bus 2 & 3, so:  $Y_{23} = 0 = Y_{32}$