ECE 3600 Direct Current (DC) Motors

A Stolp 11/9/20

Armature

The rotating part (rotor)

Field (Excitation)

Provided by the stationary part of the motor (Stator)

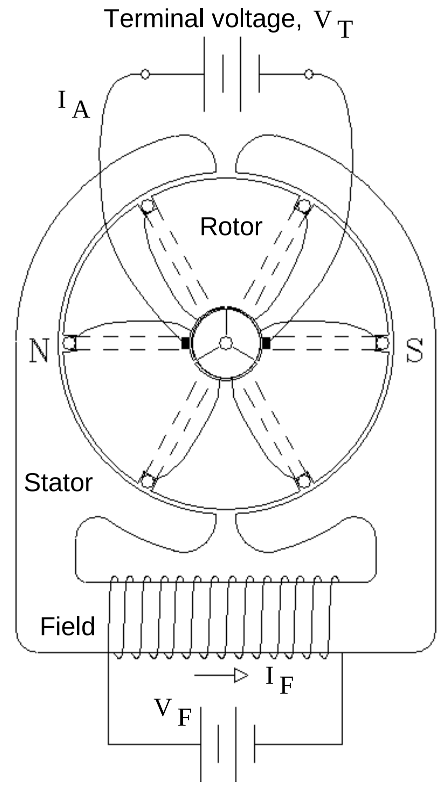
Permanent Magnet

Winding
- Separately excited
- Parallel with terminal voltage source (Shunt excited)
- Series with terminal voltage source (Series excited)

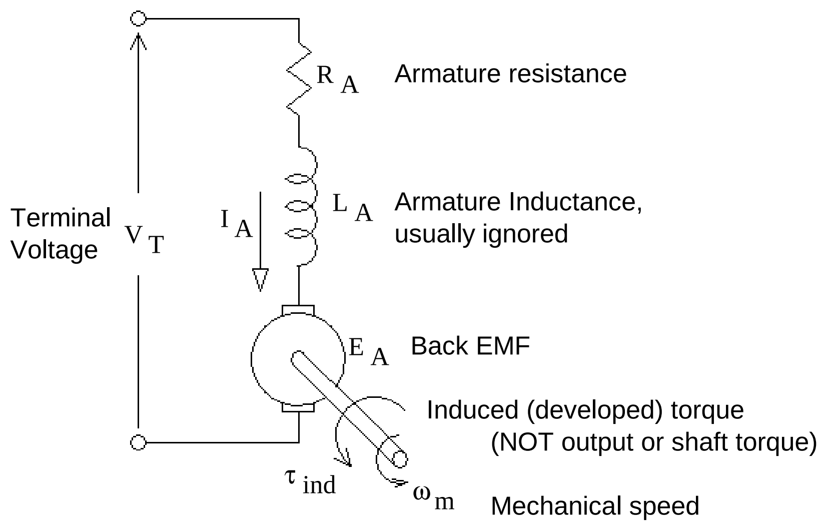
Commutator and commutation

Rotary contacts and brushes which keep switching the current direction in the armature so that the motor torque is always in the same direction.

Explained and visualized in class



Electrical Model



Important relationships

$E_A = K\cdot\phi\cdot\omega_m$

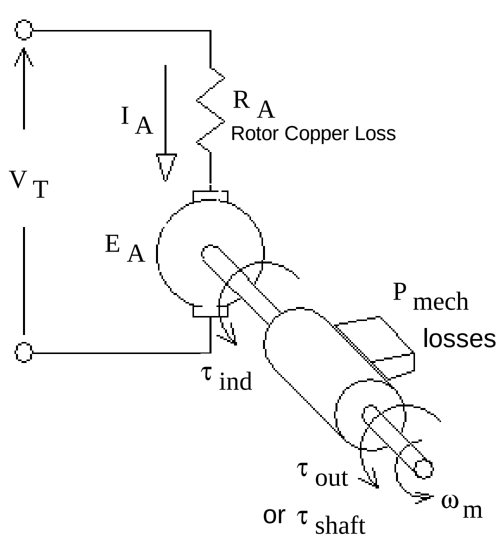
$\tau_{ind} = K\cdot\phi\cdot I_A$

$$\frac{\omega_2}{\omega_1} = \frac{E_{A2}}{E_{A1}}\cdot\frac{\phi_1}{\phi_2} = \frac{n_2}{n_1}$$

$$\frac{\tau_{ind2}}{\tau_{ind1}}\cdot\frac{\phi_1}{\phi_2} = \frac{I_{A2}}{I_{A1}}$$

Simplified Model we will use

$V_T = I_A\cdot R_A + E_A$



shaft drives a load not pictured

<u>If the field is constant</u>

$$\frac{\omega_2}{\omega_1} = \frac{E_{A2}}{E_{A1}} = \frac{n_2}{n_1}$$

$$\frac{\tau_{ind2}}{\tau_{ind1}} = \frac{I_{A2}}{I_{A1}}$$

$P_{conv} = E_A\cdot I_A = \tau_{ind}\cdot\omega_m$

$I_A = \dfrac{P_{conv}}{E_A}$ $\qquad$ $E_A = \dfrac{P_{conv}}{I_A}$

These are often substituted in other eq.

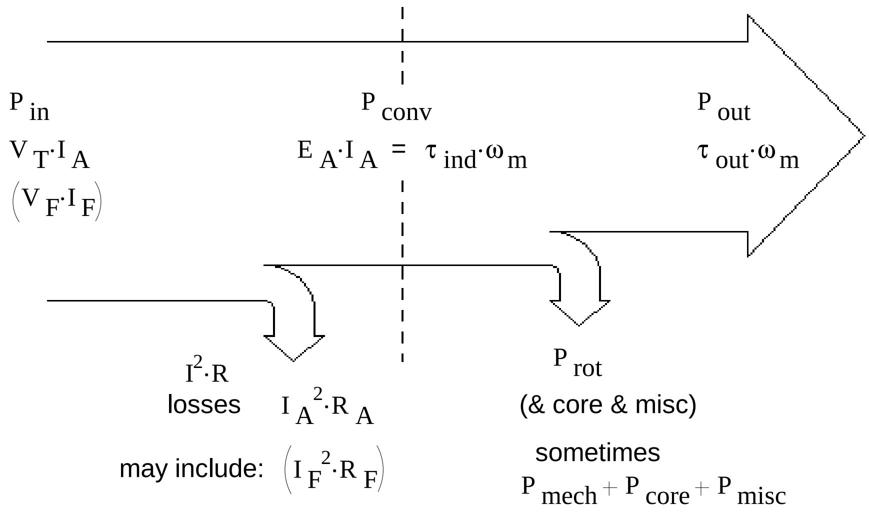
$V_T = I_A\cdot R_A + E_A$ becomes:

$V_T = I_A\cdot R_A + \dfrac{P_{conv}}{I_A}$ OR $0 = I_A^2 - \dfrac{V_T}{R_A}\cdot I_A + \dfrac{P_{conv}}{R_A}$

OR

$V_T = \dfrac{P_{conv}}{E_A}\cdot R_A + E_A$ OR $0 = E_A^2 - V_T\cdot E_A + P_{conv}\cdot R_A$

Powers



$1 \cdot hp = 745.7 \cdot W$

Nameplate Operation

The Nameplate gives the rated Voltage, Current(s), Speed and output Power (often as horsepower, hp). $1 \cdot hp = 745.7 \cdot W$
This is considered full-load operation.

Motor Constant, K

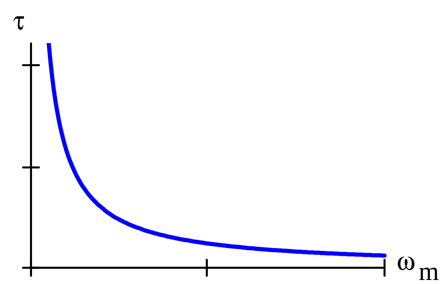
Our book defines the motor constant such that it does not include the field flux, ϕ. Often the motor constant is defined differently, as $K\phi$, but just called K, which is a function of the field current. $K\phi$ (or K) is most easily found by operating the motor as a generator with no load, then $V_T = E_A$.

Spin Direction: Reverse the leads to either of the windings and the motor will run in the opposite direction.

Mechanical Loads, Losses and Torque - Speed Curves

And how the different loads translate into motor calculations.

<u>Constant power</u> $P_{cp} = \tau_{cp} \cdot \omega_m$ $\qquad I_{Acp} = \dfrac{P_{cp}}{E_{Ac}}$ $\qquad E_{Acp} = \dfrac{P_{cp}}{I_{Acp}}$



No load acts like this

If an output or loss power is constant for all speeds, then the torque is inversely proportional to the speed.

This is highly unlikely in real life.

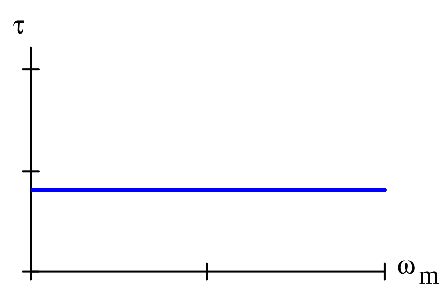
But is still useful to calculate your motor's speed if the load power is specified.

<u>Power is Proportional to speed</u> <u>Torque is Constant</u>

$$\frac{\omega_2}{\omega_1} = \frac{P_2}{P_1} = \frac{n_2}{n_1}$$

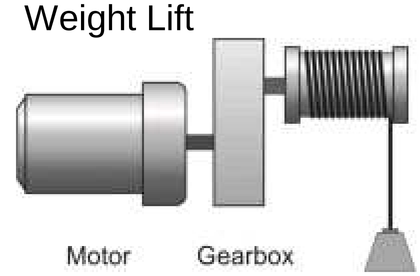
Note: P may be only the fraction of the total power, the fraction that is proportional to speed.

If a power is proportional to speed, then the torque is constant with speed.



Conversely, if a torque is constant for all speeds, the power is proportional to speed.

Weight Lift



Example

A bit more likely in real life.

$\tau_{ind} = K \cdot \phi \cdot I_A$

If a torque is constant for all speeds, $\phi \cdot I_A$ is constant.

And , I_A is constant if ϕ is constant.

$\tau_c = K \cdot \phi \cdot I_{Ac}$

$P_c = \tau_c \cdot \omega_m$

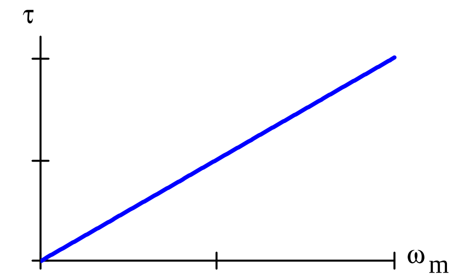
Power is proportional to the Square of the speed, Torque is Proportional to speed

Torque is proportional to speed AND, I_A is proportional to speed

$$\frac{\omega_2}{\omega_1} = \frac{\tau_2}{\tau_1} = \frac{\phi_2 \cdot I_{A2}}{\phi_1 \cdot I_{A1}} = \frac{n_2}{n_1} = \frac{E_{A2}}{E_{A1}} \cdot \frac{\phi_1}{\phi_2}$$

Note: τ is only the torque that is proportional to speed.

AND, if ALL torque is proportional to speed, AND flux (ϕ) is a constant, $\frac{n_2}{n_1} = \frac{V_{T2}}{V_{T1}}$



Simple Friction

Power is proportional to the square of the speed.

$$\frac{\omega_2^{\,2}}{\omega_1^{\,2}} = \frac{P_2}{P_1} = \frac{n_2^{\,2}}{n_1^{\,2}}$$

Note: P is only the power that is proportional to speed^2.

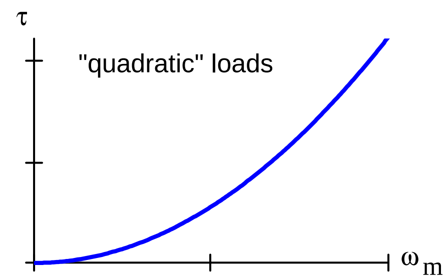
$$\tau_p = K \cdot \phi \cdot I_{Ap} = K_{pload} \cdot \omega_m$$

$$P_p = \tau_p \cdot \omega_m = K_{pload} \cdot \omega_m^{\,2}$$

Models simple dynamic friction.
Approximates more real-life loads, torque required to turn a load usually increases with speed.

Torque is proportional to the Square of the speed

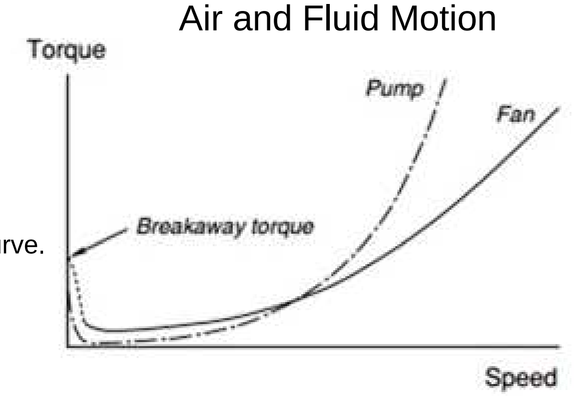
When an object moves though the air or a fluid, the drag force on the object is usually characterized by a drag coefficient (C_D) which relates the drag force to the velocity squared. This drag coefficient is then adjusted for friction drag, pressure drag, laminar flow and turbulent flow. In short, it's complicated, but a torque can be proportional to the rotational speed squared, at least over some region of operation, especially when air or fluid motion is involved. For us, that means that true load torque-speed curves often curve upward.



Air and Fluid Motion

$$\frac{\omega_2^{\,2}}{\omega_1^{\,2}} = \frac{\tau_2}{\tau_1} = \frac{\phi_2 \cdot I_{A2}}{\phi_1 \cdot I_{A1}} = \frac{n_2^{\,2}}{n_1^{\,2}} = \frac{E_{A2}^{\,2}}{E_{A1}^{\,2}} \cdot \left(\frac{\phi_1}{\phi_2}\right)^2$$

Also notice the "Breakaway torque" in the last torque-speed curve. That is due to the static, sticky friction (stiction). Almost all real loads have some stiction.



Torque–speed characteristics for fan- and pump-type loads

$$\tau_s = K \cdot \phi \cdot I_{As} = K_{sload} \cdot \omega_m^{\,2}$$

$$P_s = \tau_s \cdot \omega_m = K_{sload} \cdot \omega_m^{\,3}$$

A combination of the last three cases

Plus some Stiction Most likely in real life.

Superposition, separate causes and add results.

In these cases the power (P), torque (τ) and even the armature current and EMF (I_A and E_A) may be split into multiple pieces, each with its own characteristics.

$$P_{conv} = P_{cp} + P_c + P_p + P_s$$

$$\tau_{ind} = \tau_{cp} + \tau_c + \tau_p + \tau_s$$

$$I_A = I_{Acp} + I_{Ac} + I_{Ap} + I_{As}$$

$$E_A = E_{Acp} + E_{Ac} + E_{Ap} + E_{As}$$

The parts are usual expressed in proportionalities to some known point of operation, like that shown on the nameplate.

Then use the proportionalities to substitute into the basic motor equation: $V_T = I_A \cdot R_A + E_A$

Torque - Speed curves

$V_T = I_A \cdot R_A + E_A$ replace: $I_A = \dfrac{\tau_{ind}}{K \cdot \phi}$ $E_A = K \cdot \phi \cdot \omega_m$

to get: $V_T = \dfrac{\tau_{ind}}{K \cdot \phi} \cdot R_A + K \cdot \phi \cdot \omega_m$ $\tau_{ind} = \left(V_T - K \cdot \phi \cdot \omega_m\right) \cdot \dfrac{K \cdot \phi}{R_A}$

At 0 speed (locked rotor): $V_T = \dfrac{\tau_{ind}}{K \cdot \phi} \cdot R_A$ $\tau_{ind} = \left(V_T\right) \cdot \dfrac{K \cdot \phi}{R_A}$

At max speed (no induced torque): $V_T = K \cdot \phi \cdot \omega_{max}$ $0 = \left(V_T - K \cdot \phi \cdot \omega_{max}\right) \cdot \dfrac{K \cdot \phi}{R_A}$

Output Torque instead of Induced Torque
If lost torque is proportional to speed:

$\tau_{shaft} = \tau_{ind} - fric \cdot \omega_m$ $V_T = \dfrac{\tau_{shaft} + fric \cdot \omega_m}{K \cdot \phi} \cdot R_A + K \cdot \phi \cdot \omega_m$ $= \dfrac{\tau_{shaft}}{K \cdot \phi} \cdot R_A + \left(\dfrac{fric \cdot R_A}{K \cdot \phi} + K \cdot \phi\right) \cdot \omega_m$

DC Motor Types and Characteristics

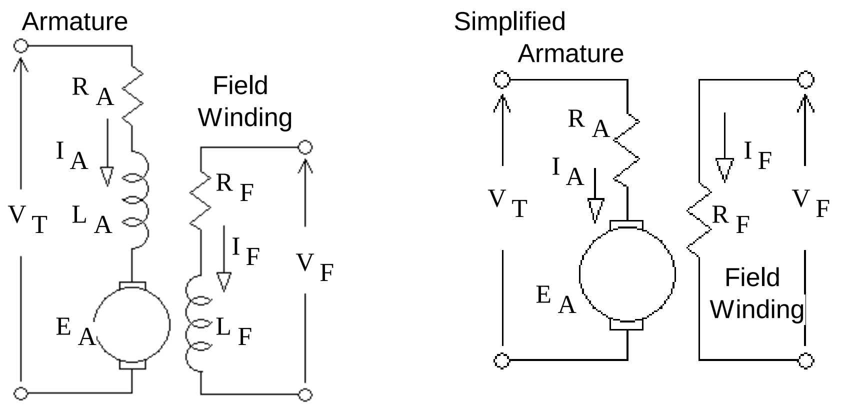
Permanent Magnet

Permanent-Magnet motors are typically small. They can be quite powerful for their size, especially if made with rare-earth magnets. These motors are common in children's toys and servo systems. Some electric cars use Large permanent-magnet DC motors.

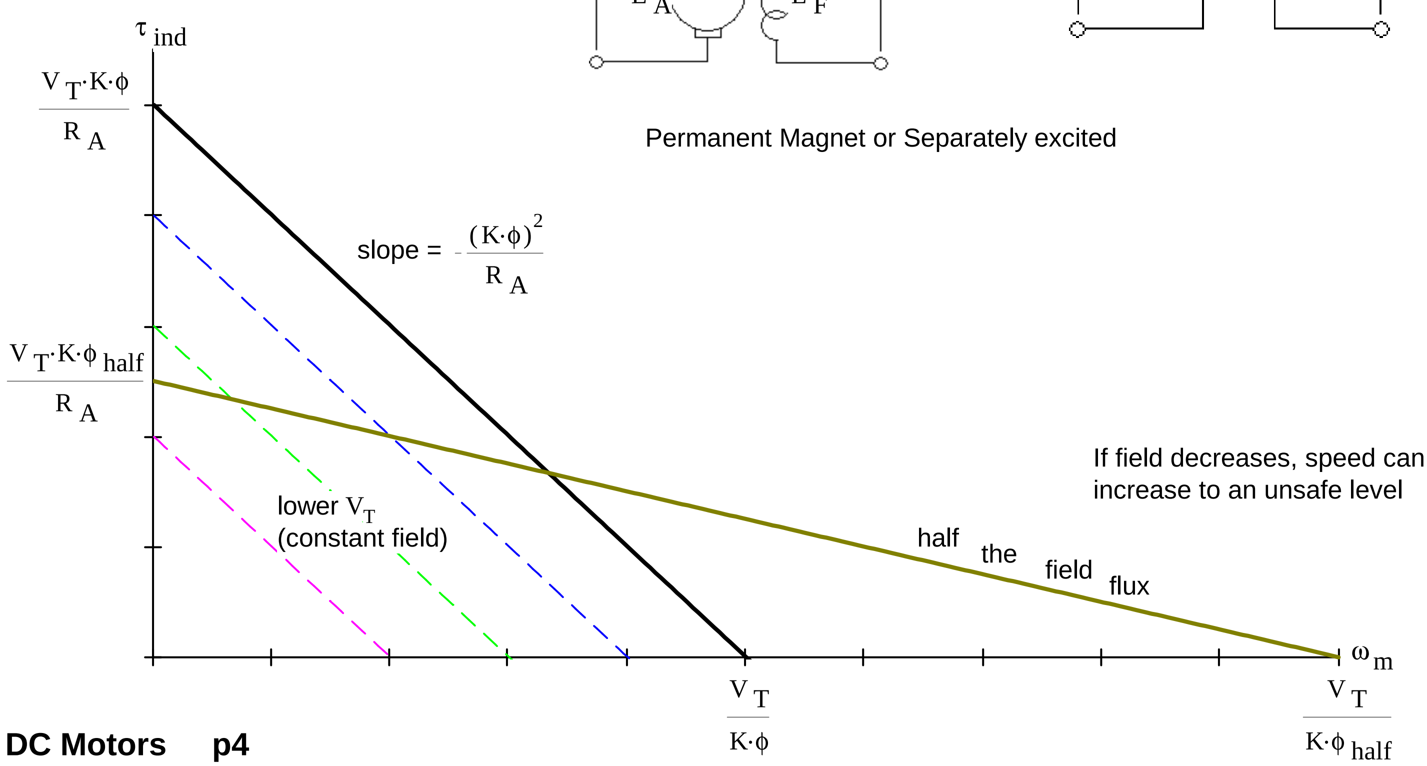
The characteristics are like the separately excited motor with a constant flux. Since the flux can't be changed, the motor constant times flux, $K\phi$, is usually simplified to just a different motor constant, usually also called K, which includes the constant flux.

Separately Excited

The field flux comes from current flowing through a field winding, which is supplied by a separate power source or at a different voltage than V_T.



Permanent Magnet or Separately excited



Output Torque instead of Induced Torque, If lost torque is proportional to speed:

$$\tau_{shaft} = \tau_{ind} - fric\cdot\omega_m \qquad V_T = \frac{\tau_{shaft} + fric\cdot\omega_m}{K\cdot\phi}\cdot R_A + K\cdot\phi\cdot\omega_m \quad = \frac{\tau_{shaft}}{K\cdot\phi}\cdot R_A + \left(\frac{fric\cdot R_A}{K\cdot\phi} + K\cdot\phi\right)\cdot\omega_m$$

$$\tau_{shaft} = \left[V_T - \left(\frac{fric\cdot R_A}{K\cdot\phi} + K\cdot\phi\right)\cdot\omega_m\right]\cdot\frac{K\cdot\phi}{R_A}$$

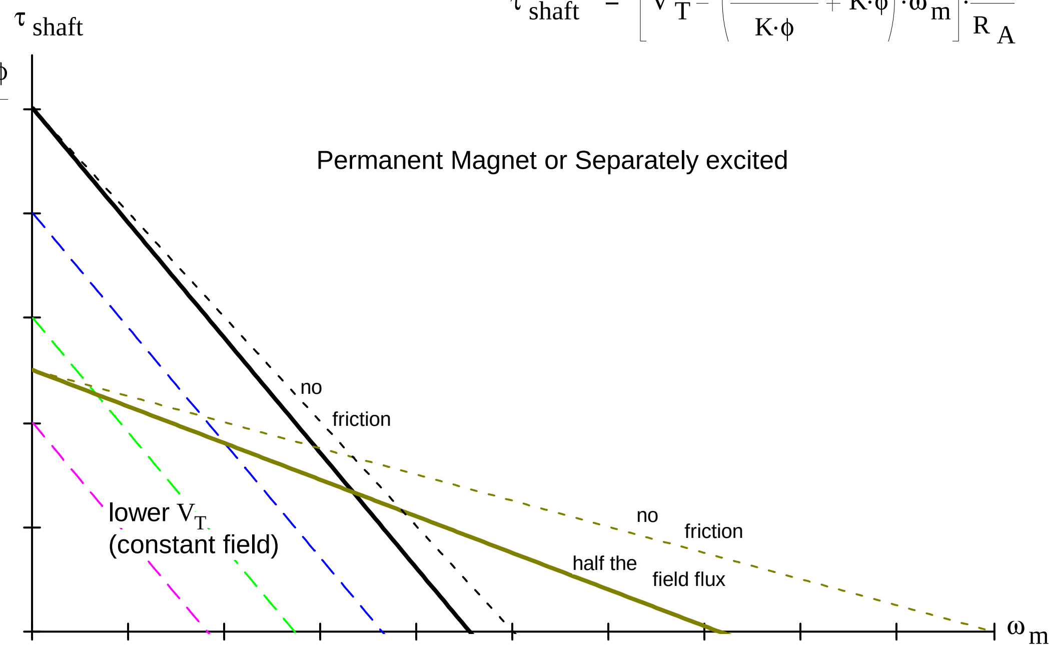
At 0 speed (locked rotor): $\frac{V_T\cdot K\cdot\phi}{R_A}$

At max speed:
induced torque all lost to friction
(no output or shaft torque)

$$V_T = \left(\frac{fric\cdot R_A}{K\cdot\phi} + K\cdot\phi\right)\cdot\omega_{max}$$

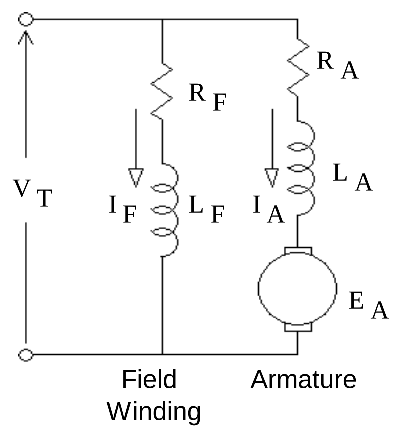
$$\omega_{max} = \frac{V_T}{\frac{fric\cdot R_A}{K\cdot\phi} + K\cdot\phi}$$

= Max speed or no-load speed



Shunt Excited

Field winding is connected in parallel with the armature to the same terminal voltage source, V_T.



If flux is proportional to field current $\phi = c\cdot I_F = c\cdot\frac{V_T}{R_F}$ c = the "core constant"

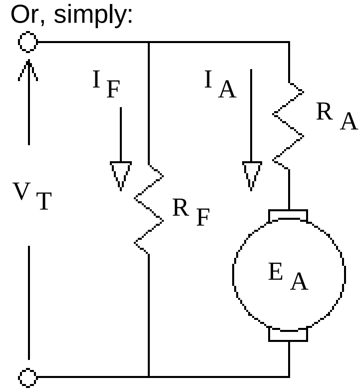
At 0 speed (locked rotor): $V_T = \frac{\tau_{ind}}{K\cdot\phi}\cdot R_A$

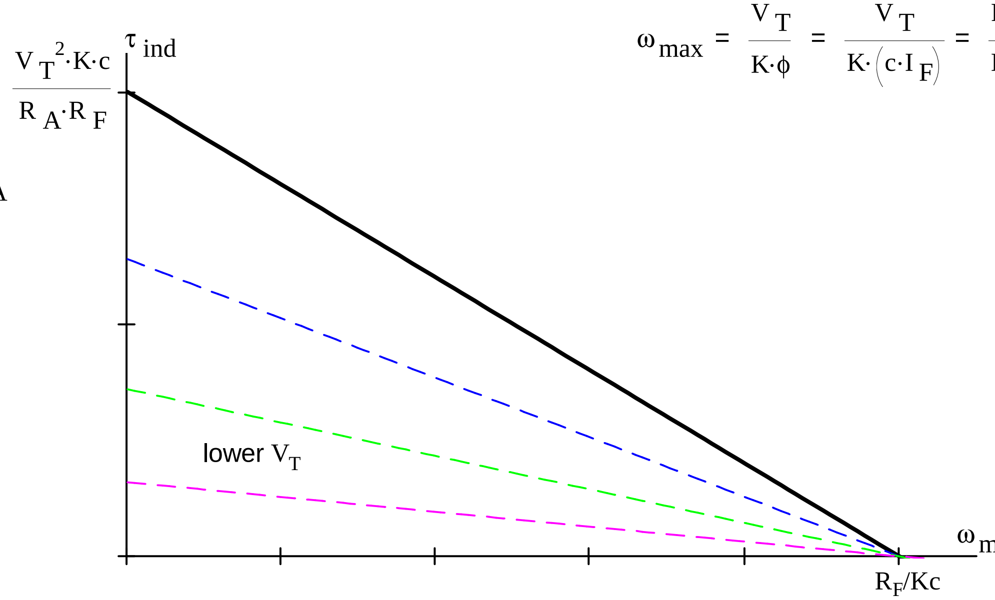
$$\text{so: } \tau_{ind} = \left(V_T\right)\cdot\frac{K\cdot\phi}{R_A} = \left(V_T\right)\cdot\frac{K\cdot\left(c\cdot\frac{V_T}{R_F}\right)}{R_A} = \frac{V_T^{\,2}\cdot K\cdot c}{R_A\cdot R_F}$$

At max speed (no induced torque): $V_T = K\cdot\phi\cdot\omega_{max}$

$$\omega_{max} = \frac{V_T}{K\cdot\phi} = \frac{V_T}{K\cdot\left(c\cdot I_F\right)} = \frac{R_F}{K\cdot c}$$

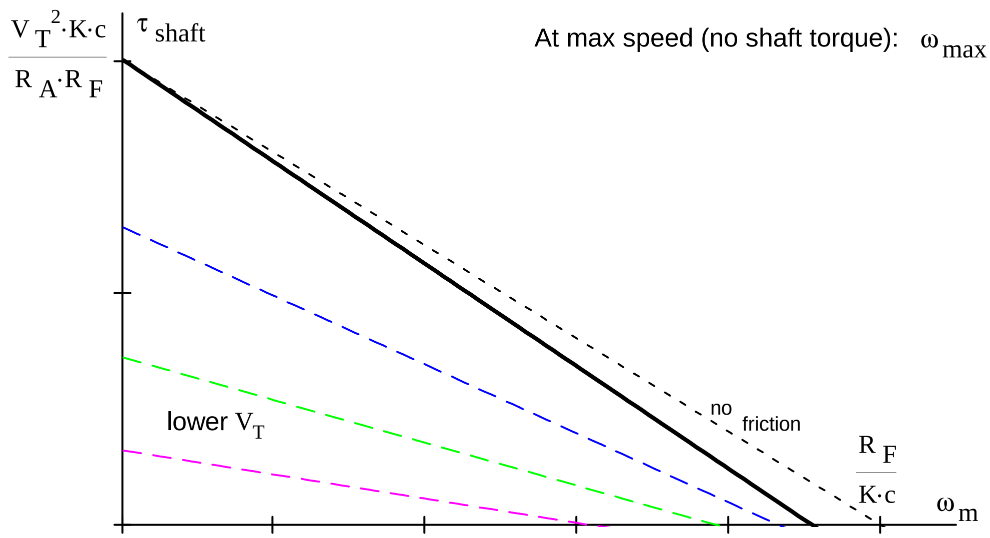
Or, simply:





Output Torque instead of Induced Torque, If lost torque is proportional to speed:

$$\tau_{shaft} = \tau_{ind} - fric\cdot\omega_m \qquad V_T = \frac{\tau_{shaft} + fric\cdot\omega_m}{K\cdot\phi}\cdot R_A + K\cdot\phi\cdot\omega_m = \frac{\tau_{shaft}}{K\cdot\phi}\cdot R_A + \left(\frac{fric\cdot R_A}{K\cdot\phi} + K\cdot\phi\right)\cdot\omega_m$$



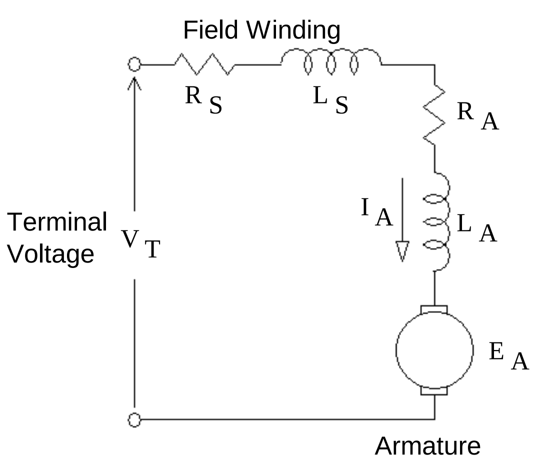
At max speed (no shaft torque):

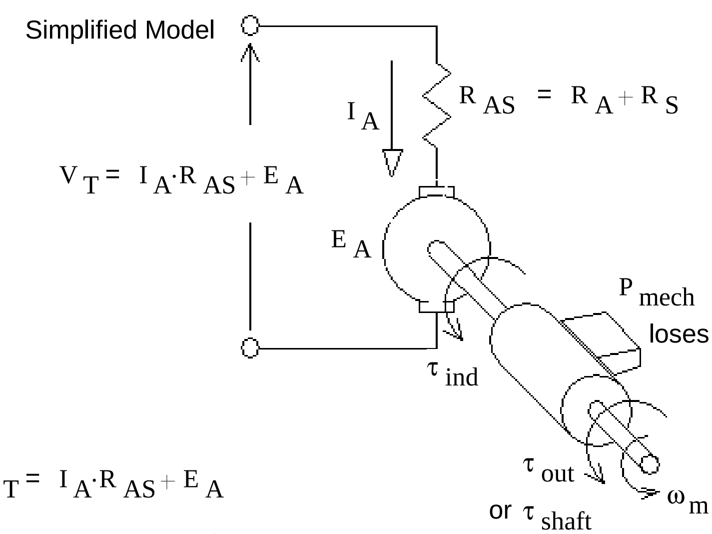
$$\omega_{max} = \frac{V_T}{\dfrac{fric\cdot R_A}{K\cdot\left(c\cdot\dfrac{V_T}{R_F}\right)} + K\cdot\left(c\cdot\dfrac{V_T}{R_F}\right)} = \frac{V_T}{\dfrac{fric\cdot R_A\cdot R_F}{K\cdot c\cdot V_T} + K\cdot c\cdot\dfrac{V_T}{R_F}} = \frac{1}{V_T^{2}\cdot\dfrac{fric\cdot R_A\cdot R_F}{K\cdot c} + \dfrac{K\cdot c}{R_F}}$$

$$\omega_{max} = \frac{1}{V_T^{2}\cdot\dfrac{fric\cdot R_A\cdot R_F}{K\cdot c} + \dfrac{K\cdot c}{R_F}}$$

Series Excited (Often called an AC/DC or Universal Motor)

Field winding is connected in series with the armature and is designed with much thicker windings, so it can handle much larger current and has much less resistance. The resistance of the field winding is now called R_S. Since R_S is in series with R_A, they are often combined into R_{AS}.





$$V_T = I_A\cdot R_{AS} + E_A = I_A\cdot R_{AS} + K\cdot\phi\cdot\omega_m = I_A\cdot R_{AS} + K\cdot(c\cdot I_A)\cdot\omega_m = I_A\cdot R_{AS} + K\cdot c\cdot I_A\cdot\omega_m$$

If flux is proportional to field current

$$\phi = c\cdot I_A$$

$$I_A = \frac{\tau_{ind}}{K\cdot\phi} = \frac{\tau_{ind}}{K\cdot(c\cdot I_A)}$$

$$I_A^{2} = \frac{\tau_{ind}}{K\cdot c} \qquad \text{so: } I_A = \sqrt{\frac{\tau_{ind}}{K\cdot c}}$$

$$V_T = \sqrt{\frac{\tau_{ind}}{K\cdot c}}\cdot(R_{AS} + K\cdot c\cdot\omega_m) \qquad \sqrt{\frac{\tau_{ind}}{K\cdot c}} = \frac{V_T}{(R_{AS} + K\cdot c\cdot\omega_m)}$$

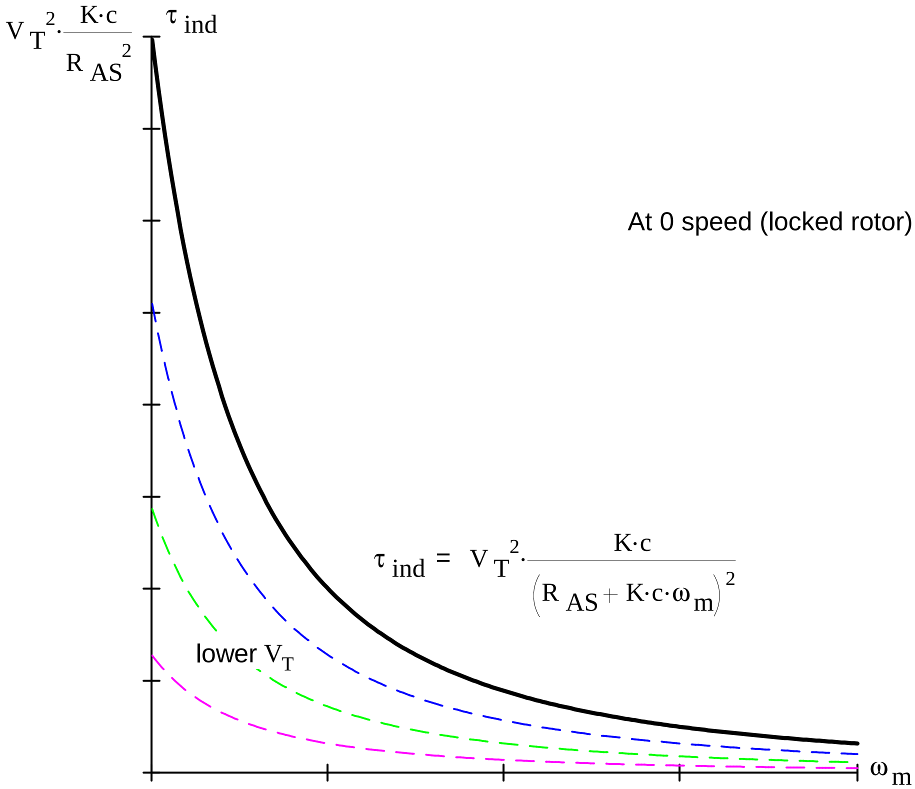
$$\tau_{ind} = K\cdot c\cdot\left[\frac{V_T}{(R_{AS} + K\cdot c\cdot\omega_m)}\right]^{2}$$

Series Excited Torque - Speed curve

$$\tau_{ind} = K\cdot c\cdot\left[\frac{V_T}{(R_{AS}+K\cdot c\cdot\omega_m)}\right]^2 = V_T^2\cdot\frac{K\cdot c}{(R_{AS}+K\cdot c\cdot\omega_m)^2}$$

At 0 speed (locked rotor): max $\tau_{ind} = V_T^2\cdot\dfrac{K\cdot c}{R_{AS}^2}$

R_{AS}. is usually small, so the maximum induced torque can be huge.



$$\tau_{ind} = V_T^2\cdot\frac{K\cdot c}{(R_{AS}+K\cdot c\cdot\omega_m)^2}$$

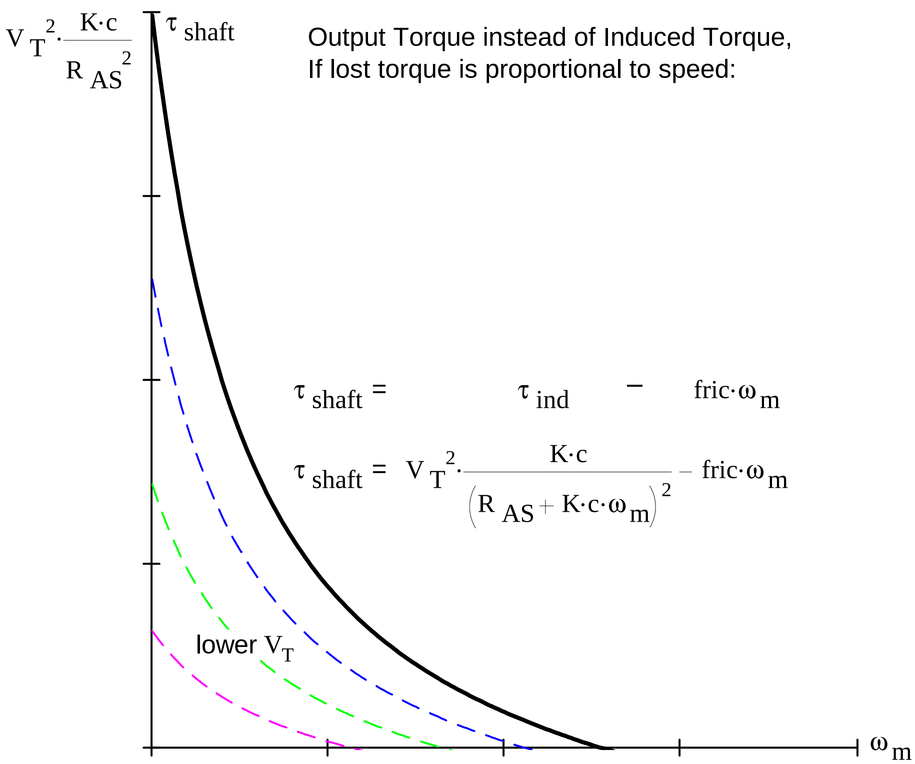
$$\omega_m = \frac{V_T}{K\cdot c\cdot I_A} - \frac{I_A\cdot R_{AS}}{K\cdot c\cdot I_A}$$

$$\omega_m = \frac{V_T}{\sqrt{K\cdot c}\cdot\sqrt{\tau_{ind}}} - \frac{R_{AS}}{K\cdot c}$$

Max speed (no induced torque) is undefined

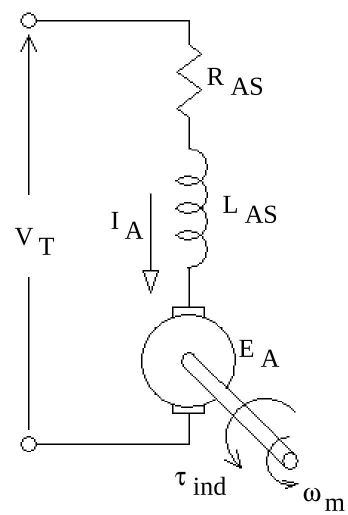
Output Torque instead of Induced Torque,
If lost torque is proportional to speed:

$$\tau_{shaft} = \tau_{ind} - fric\cdot\omega_m$$



$$\tau_{shaft} = \tau_{ind} - fric\cdot\omega_m$$

$$\tau_{shaft} = V_T^2\cdot\frac{K\cdot c}{(R_{AS}+K\cdot c\cdot\omega_m)^2} - fric\cdot\omega_m$$



AC/DC or Universal Motor (Series Excited DC motor)

Because the same current flows through both the field and the armature, the series-excited motor will turn the same direction even if the current flows the opposite direction-- or even if goes back and forth (AC). These motors are common in AC devices because they can provide a lot of power and torque in small, lightweight motor. They are very common in handheld power tools like drills, saws, grinders, weed eaters, hedge trimmers, etc.. They are also found in vacuum cleaners, blenders and food processors. If you look at the motor of an AC device and see brushes and a commutator, then it is a universal motor.

It is easy to vary the speed of these motors by changing the average voltage, usually with a thyristor-based control similar to a light dimmer.

Universal motors tend to be very noisy.

When used with AC supply, the inductance of the windings becomes important.

Compounded Motor

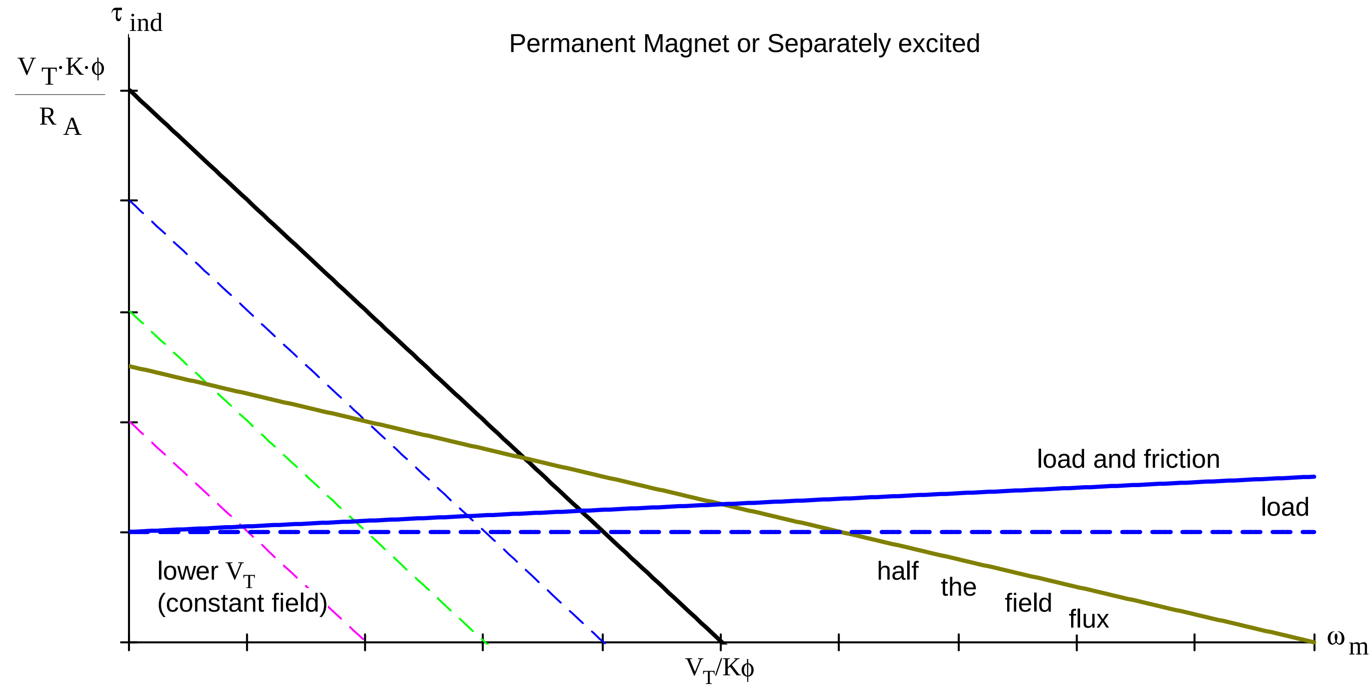
A motor with both shunt and series field windings. Covered in your book, starting on p. 420.

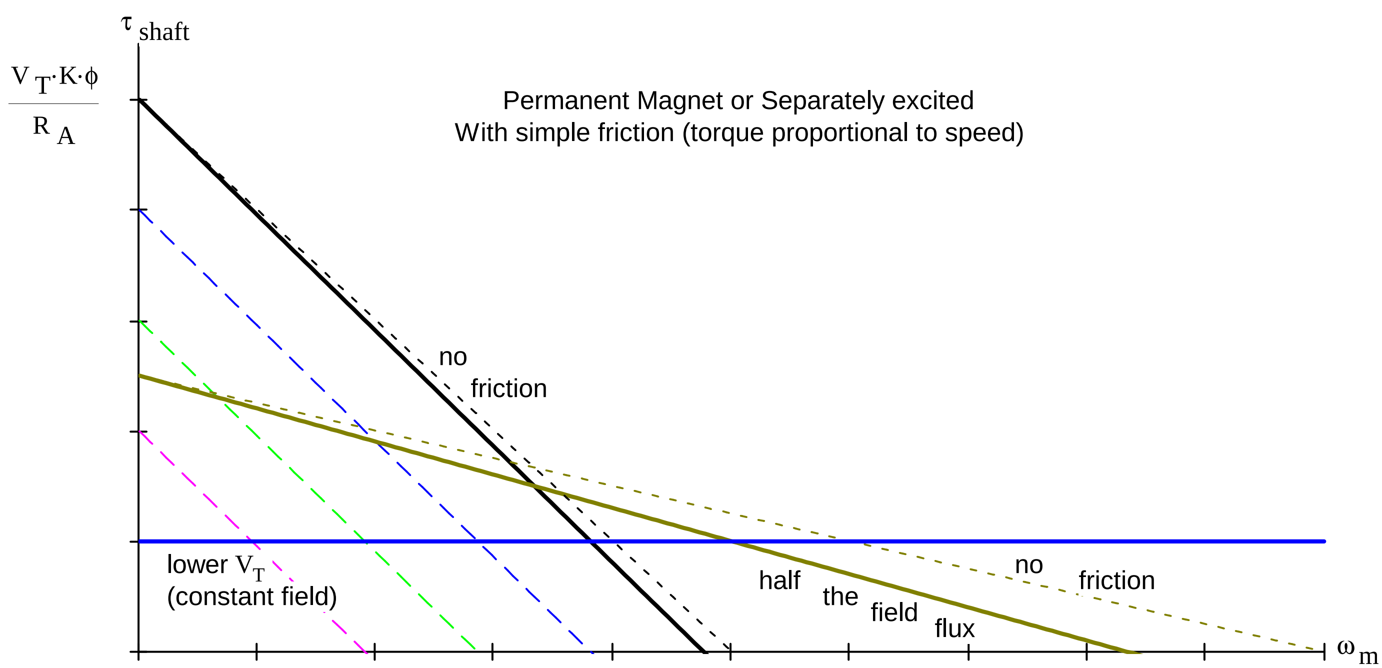
Adding Load Characteristics

Returning to the Separately Excited (or permanent Magnet) Motor, let's add Load Torque-Speed curves $P = \tau \cdot \omega_m$

Load is Power is Proportional to speed

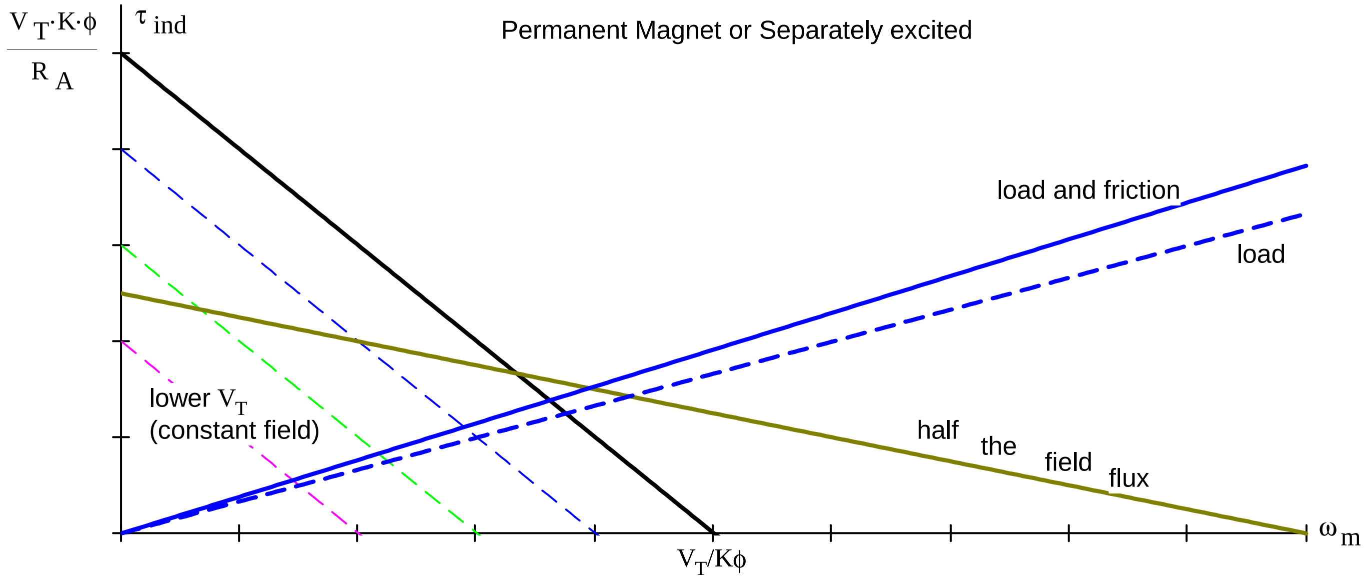
If a power is proportional to speed, then the torque is constant with speed.



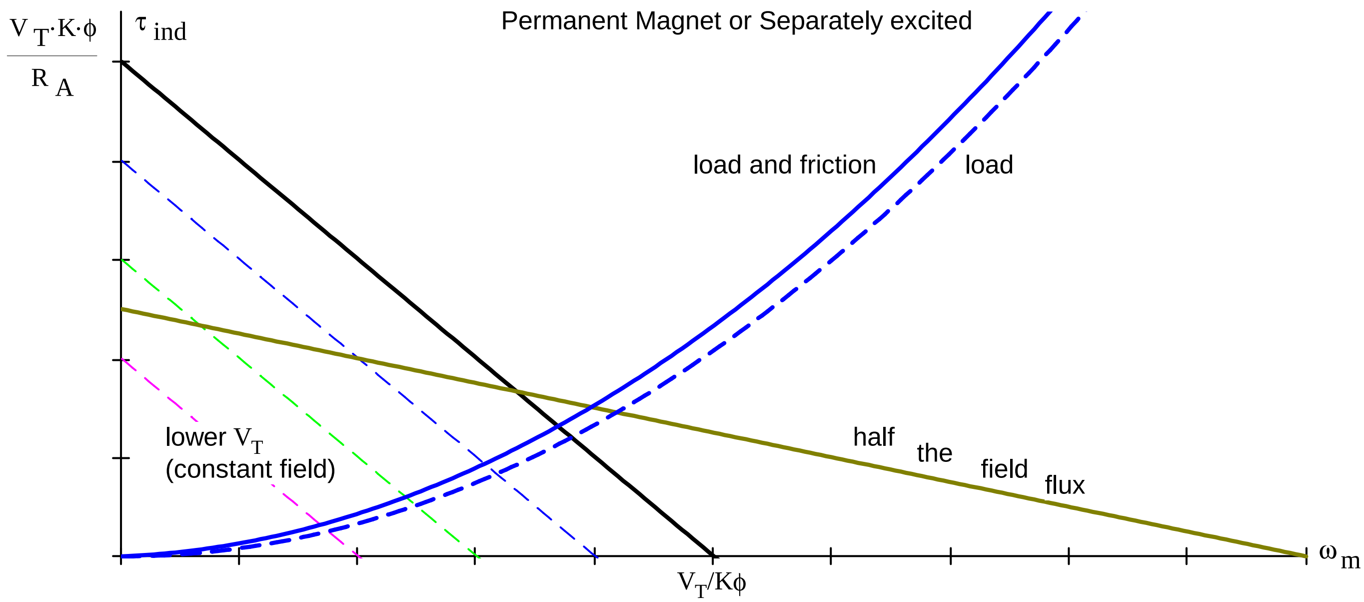


Load Torque proportional to speed

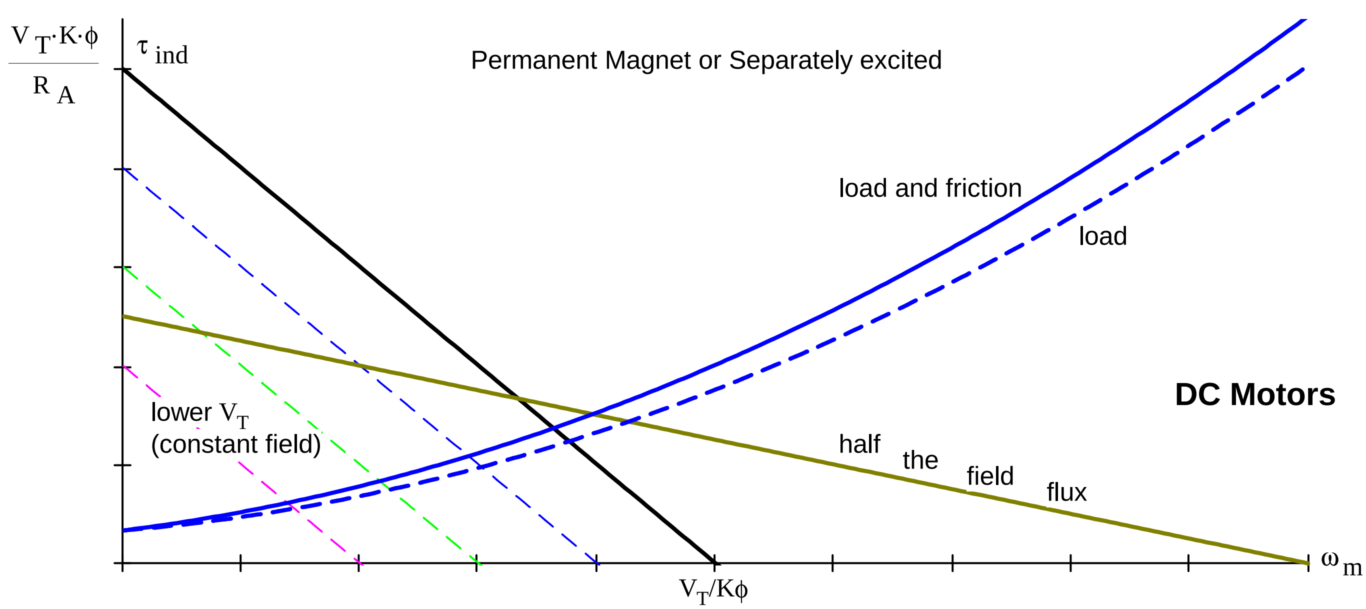
Load Power is proportional to the square of the speed



Load Torque is proportional to the square of the speed

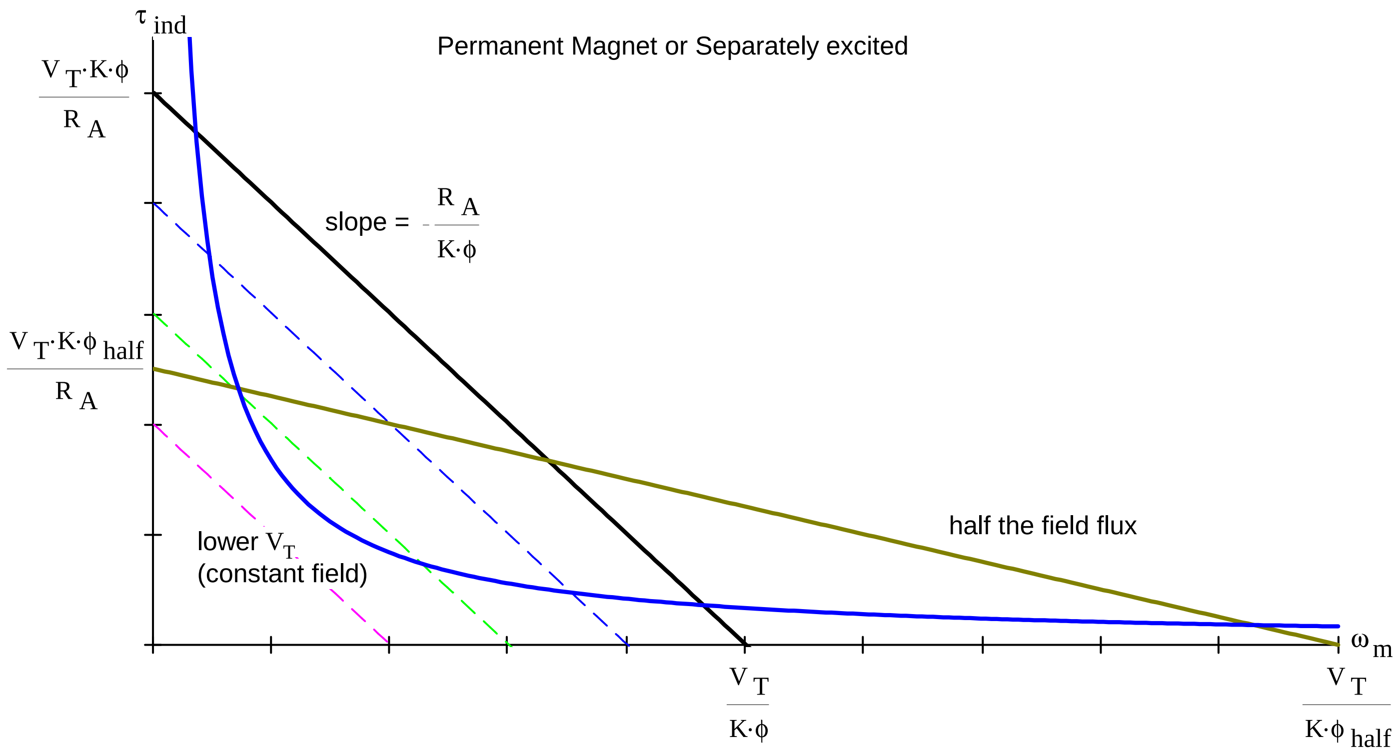


Combination Load



Constant power

If an output or loss power is constant for all speeds, then the torque is inversely proportional to the speed.



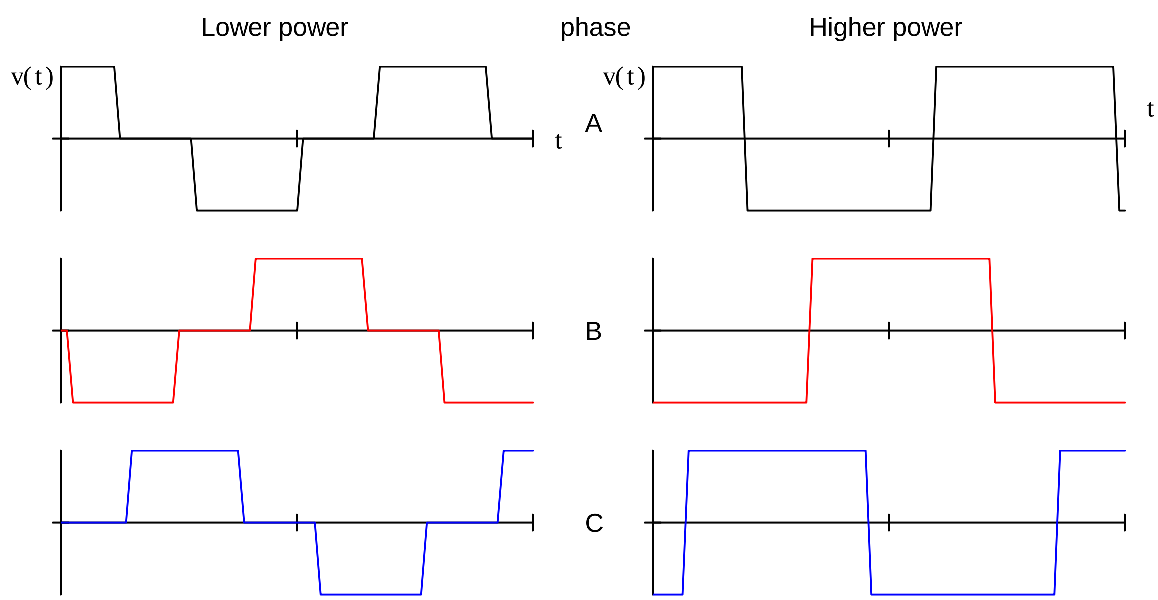
Brushless DC Motors

Many Brushless DC motors simply replace the commutator with a rotor position sensor (magnetic (Hall effect), optic, inductive, etc.) and power-electronic circuitry which switches the winding current. The field is usually on the rotor (permanent magnet or a winding fed through slip rings) and the armature windings are stationary. These motors are analyzed just like DC motors with brushes. Many DC fans are made this way.

Many Brushless DC motors are actually 3-phase Synchronous or Induction motors driven by power-electronic circuitry which produces variable-frequency, Pulse-Width-Modulated (PWM), 3-phase power to operate the motor. Actually, they don't have to be 3-phase, as long as they have at least 2 phases. 2, 3, 4, and 6 phases are common.

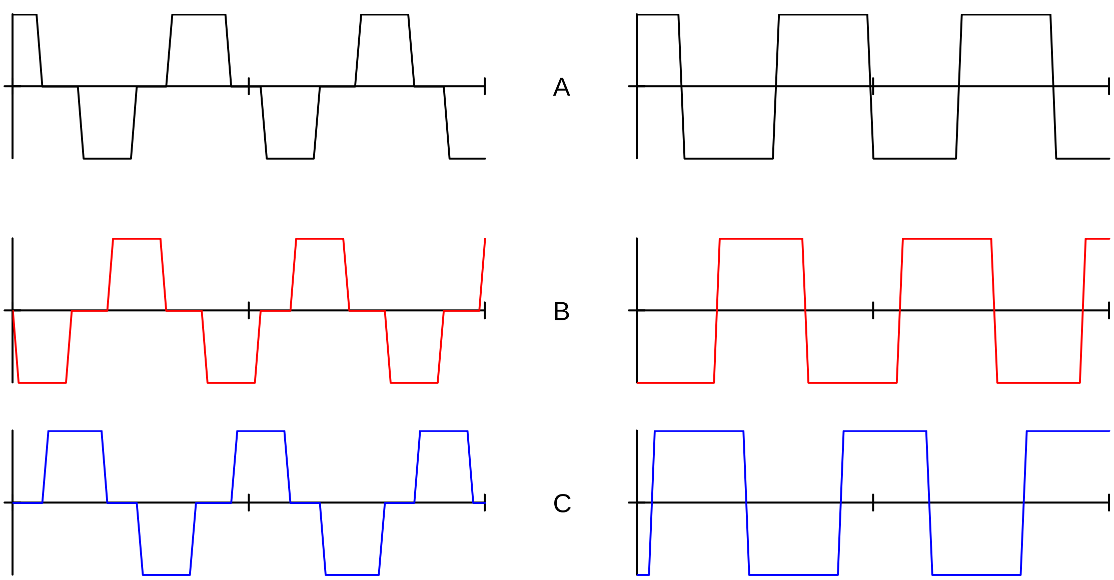
Brushless motors have some important advantages. They are mechanically simpler and more reliable. They can be operated in environments where the sparking between brushes and commutator would be undesirable or unsafe. They are relatively quiet.

Variable-Frequency, Pulse-Width-Modulated (PWM), 3-phase Power

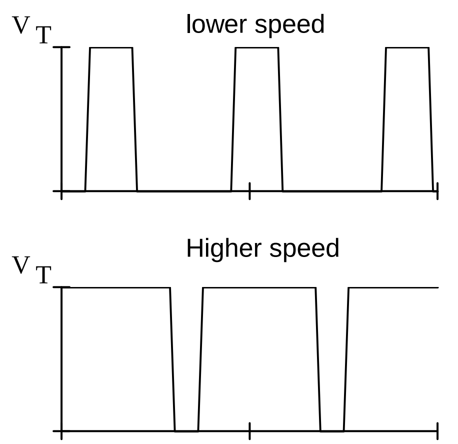


More sample waveforms are shown on the next page.

Higher speed

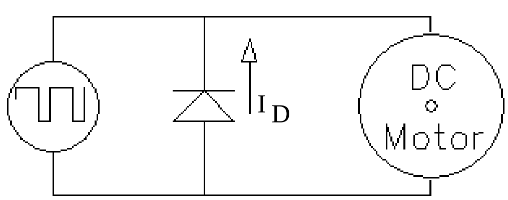


Pulse Width Modulation (PWM) for speed control

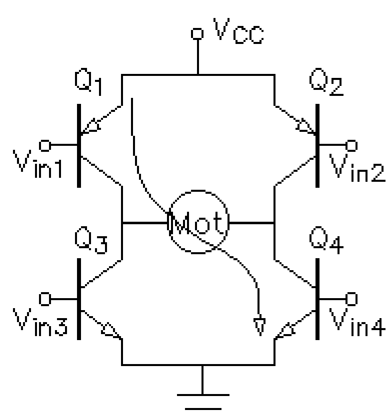


As you can see by the torque - speed curves above, regulating the terminal voltage, V_T, is a very effective way to control the speed of a DC motor. Unfortunately, it is often an inefficient process. Pulse Width Modulation, shown here, is a very efficient. It is also more linear, especially at low speeds. The torque - speed curves do not show these non-linearities-- due largely to the difference between static and dynamic friction. (Motors are often a bit sticky at startup.)

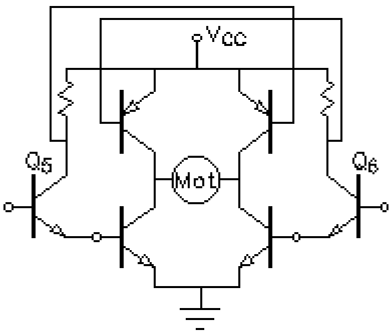
If you do use PWM to control a motor, it is important to remember that the inductance within the windings will not allow the current to go to zero instantaneously. A diode (called a flyback, flywheel, or freewheel diode when used like this) provides a path for the current still flowing through the motor when a pulse is switched off.



H-bridge: Of course, if you want to make the motor turn in both directions you'll need a more complex circuit. Look at the circuit at right, it's has the shape of an H, hence the name. If transistors Q_1 and Q_4 are on, then the current flows as shown, left-to-right through the motor. If transistors Q_2 and Q_3 are on, then the current flows the other way through the motor and the motor will turn in the opposite direction. (The motor here is a permanent-magnet DC motor.) In my circuit, the top two transistors are PNPs, which makes the circuit more efficient. The H-bridge could also be made with all NPNs or with power MOSFET transistors.



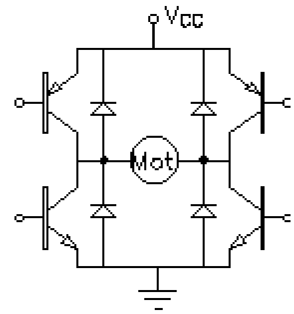
An H-bridge requires four inputs, all operated in concert. To turn on Q_1 and Q_4, as shown, V_{in1} would have to be low and V_{in4} would have to be high. At the same time, the other two transistors would have to be off, so V_{in2} would have to be high and V_{in3} would have to be low.



If the control circuit makes a mistake and turns on Q_1 and Q_3 (or Q_2 and Q_4) at the same time you'll have a toaster instead of a motor driver, at least for a short while.

The circuit at left requires only two inputs. Transistors Q_5 and Q_6 work as *inverters*, when their inputs are high, their outputs are low and vice-versa. The resistors are known as *pull-up* resistors.

The H-bridge should also include flyback diodes.



Regenerative Braking

Electric motors are not limited to converting electrical energy to mechanical. They can also convert energy from mechanical to electrical. If that is done for the purpose of mechanical braking, say in an electric car, then it's called Regenerative Braking. It is a way recover kinetic energy when slowing a moving mass, or potential energy of a mass moving from a higher to a lower elevation. Examples: a car coming to a stop at a traffic light or driving down Parley's canyon. This recovered energy can be used to recharge batteries or simply be wasted in resistors. Electric lawnmowers and some electric drills use this technique to stop the moving parts very quickly for safety.

Armature Reaction

This phenomenon is well explained in your book, starting on p.372. One effect is a shift of the neutral plane of the field flux. This is a small twisting of the overall North - South orientation and can increase the sparking at the brushes. If the load and rotation direction are known and constant, a small twist of the brush location (in the same direction) will help mitigate the sparking. Interpoles (shown on p. 379) are a better solution. Please note that Fig. 8-11 on p. 374 is for a generator and that ω will be in the opposite direction for a motor. The other effect is an overall flux weakening due to core saturation. A few series windings to shore up the flux at high loads can help with this.

Brush Loss

Sometimes the voltage drop across the brush-commutator connection is also considered. This voltage drop is usually estimated at about 2V for both brushes, regardless of the armature current. (p. 384 in book.)

Characterizing an Unknown DC motor

For a motor that can be operated as separately excited and as a generator;

Motor Constant: Operate the motor as a generator with no load ($I_A = 0$), then $V_T = E_A$. Calculate $K\phi$ from speed and E_A measurements. You may wish to calculate this at various field currents.

R_A: Hook an electrical load to your still-spining generator. Adjust input power to return to no-load speed. Measure V_T and I_A and calculate R_A. You may wish to repeat at several loads and take an average.

Alternatively, measure V_T, I_A, and ω at 2 different mechanical loads, solve 2 equations for 2 unknowns, $K\phi$ and R_A .

$$V_{T1} = I_{A1} \cdot R_A + E_{A1} = I_{A1} \cdot R_A + K \cdot \phi \cdot \omega_1$$
$$V_{T2} = I_{A2} \cdot R_A + K \cdot \phi \cdot \omega_2$$

You may wish to calculate this at various field currents.

If you can't measure the rotational speed, but can measure the time required to move something a fixed distance, that time would be inversely proportional to speed:

$$V_{T1} = I_{A1} \cdot R_A + \frac{K \cdot \phi \cdot K_T}{t_1} \qquad V_{T2} = I_{A2} \cdot R_A + \frac{K \cdot \phi \cdot K_T}{t_2}$$

K_T is just another constant which is found together with $K\phi$ as $K\phi K_T$.

ECE 3600 DC Motor Examples

A.Stolp
11/29/11
rev 12/1/14

Ex.1 A 3-hp dc motor has the following nameplate information: 150 V, 1400 rpm, 18 A, $R_A = 0.8\ \Omega$, and $R_F = 300\ \Omega$. The field is shunt connected and the 18 A <u>includes</u> the field current. Assume rotational losses are constant.

a) Find the efficiency of the motor at nameplate operation. (Include the field in your calculations) $1\cdot hp = 745.7\cdot W$

$V_T := 150\cdot V$ $n_{FL} := 1400\cdot rpm$ $I_{FL} := 18\cdot A$ $R_A := 0.8\cdot\Omega$ $R_F := 300\cdot\Omega$

$P_{out} := 3\cdot hp\cdot\frac{745.7\cdot W}{hp}$ $P_{out} = 2.237\cdot kW$

$P_{in} := V_T\cdot I_{FL}$ $P_{in} = 2.7\cdot kW$ $\eta = \frac{P_{out}}{P_{in}} = 82.86\cdot\%$

b) Find the rotational losses at nameplate operation.

field current: $I_F := \frac{V_T}{R_F}$ $I_F = 0.5\cdot A$

armature full-load current: $I_{AFL} := I_{FL} - I_F$ $I_{AFL} = 17.5\cdot A$

$E_{AFL} := V_T - I_{AFL}\cdot R_A$ $E_{AFL} = 136\cdot V$

$P_{conv} := E_{AFL}\cdot I_{AFL}$ $P_{conv} = 2.38\cdot kW$

$P_{rot} := P_{conv} - P_{out}$ $P_{rot} = 142.9\cdot W$ $P_{rot} = 0.192\cdot hp$

either answer

c) Find the required current for a developed power of 1.5 hp with $V_T = 150$ V.

$P_{conv} := 1.5\cdot hp$ $P_{conv} = 1.119\cdot kW = E_A\cdot I_A$

$V_T = E_A + I_A\cdot R_A = \frac{P_{conv}}{I_A} + I_A\cdot R_A$ Rearrange $0 = R_A\cdot I_A^2 - V_T\cdot I_A + P_{conv}$

Solving for $$I_A = \begin{bmatrix} \frac{1}{(2\cdot R_A)}\cdot\left(V_T + \sqrt{V_T^2 - 4\cdot R_A\cdot P_{conv}}\right) \\ \frac{1}{(2\cdot R_A)}\cdot\left(V_T - \sqrt{V_T^2 - 4\cdot R_A\cdot P_{conv}}\right)\end{bmatrix} = \begin{pmatrix}179.72\\7.78\end{pmatrix}\cdot A$$

$I_A := 7.78\cdot A$

$I_S := I_A + I_F$

$I_S = 8.28\cdot A$

d) Find the output power if the developed power is 1.5 hp with $V_T = 150$ V.

$P_{out} := P_{conv} - P_{rot}$ $P_{out} = 975.7\cdot W$ $P_{out} = 1.308\cdot hp$

e) Find the shaft speed if the developed power is 1.5 hp with $V_T = 150$ V.

$E_A := \frac{P_{conv}}{I_A}$ $E_A = 143.773\cdot V$ $n := \frac{E_A}{E_{AFL}}\cdot n_{FL}$ $n = 1480\cdot rpm$ either answer

$\omega := n\cdot\frac{2\cdot\pi\cdot rad}{60\cdot\frac{sec}{min}\cdot rev}$ $\omega = 155\cdot\frac{rad}{sec}$

f) A deranged Mouse chews through part of the field winding so that the field current drops and the field flux drops to 40% of its former value. Find the shaft speed if the developed power is still 1.5 hp with $V_T = 150$ V.

$E_A = K\cdot\phi\cdot\omega$ so... $\frac{n}{n_{FL}} = \frac{\omega}{\omega_{orig}} = \frac{\left(\frac{E_A}{\phi_{new}}\right)}{\left(\frac{E_A}{\phi_{orig}}\right)} = \frac{E_A}{E_A}\cdot\frac{\phi_{orig}}{\phi_{new}}$ $n_{new} := \frac{E_A}{E_A}\cdot\frac{100\cdot\%}{40\cdot\%}\cdot n$ $n_{new} = 3700\cdot rpm$ either answer

g) Find the load torque if the developed power is still 1.5 hp with $V_T = 150$ V.

$\tau := \frac{P_{out}}{n_{new}\cdot\left(\frac{2\cdot\pi\cdot rad}{60\cdot\frac{sec}{min}\cdot rev}\right)}$ $\tau = 2.518\cdot N\cdot m$

Ex.2 A dc motor has the following nameplate information: 5-hp, 1200 rpm, Armature: 200 V, 22 A, 1 Ω; Field: 200 V, 1 A

a) Find the rotational losses at nameplate operation.

$V_T := 200 \cdot V$ $\quad n_{FL} := 1200 \cdot rpm$ $\quad I_{FL} := 22 \cdot A$ $\quad R_A := 1 \cdot \Omega$

$P_{outFL} := 5 \cdot hp \cdot \frac{745.7 \cdot W}{hp}$ $\quad P_{outFL} = 3.728 \cdot kW$

$E_{AFL} := V_T - I_{FL} \cdot R_A$ $\quad E_{AFL} = 178 \cdot V$

$P_{convFL} := E_{AFL} \cdot I_{FL}$ $\quad P_{convFL} = 3.916 \cdot kW$

$P_{rotFL} := P_{convFL} - P_{outFL}$ $\quad P_{rotFL} = 187.5 \cdot W$ $\quad P_{rotFL} = 0.251 \cdot hp$

b) The load torque is constant (not dependent on speed) and the rotational losses are proportional to speed, find the shaft speed at half the rated armature voltage and full rated field voltage.

$P = \tau \cdot \omega_m$ so, if a power is proportional to speed, then the torque is constant.

OR, conversely, if the torque is constant, the power is proportional to speed.

In this case, ALL the power converted is proportional to speed and ALL the the induced torque is constant. $\quad P_{conv} = \frac{n_{new}}{n_{FL}} \cdot P_{convFL}$

One way: $\tau_{ind} = K \cdot \phi \cdot I_A$ so, if τ_{ind} and the field current are constant, then I_A is constant. $\quad I_A := I_{FL}$

$V_T := \frac{200 \cdot V}{2}$ $\quad E_A := V_T - I_A \cdot R_A$ $\quad E_A = 78 \cdot V$ $\quad n_{new} = \frac{E_A}{E_{AFL}} \cdot n_{FL} = 525.8 \cdot rpm$

Another solution: recognize that: $E_A = \frac{n_{new}}{n_{FL}} \cdot E_{AFL}$ because the field is constant

$$V_T = E_A + I_A \cdot R_A = E_A + \frac{P_{conv}}{E_A} \cdot R_A = E_A + \frac{\frac{n_{new}}{n_{FL}} \cdot P_{convFL}}{\frac{n_{new}}{n_{FL}} \cdot E_{AFL}} \cdot R_A = E_A + \frac{P_{convFL}}{E_{AFL}} \cdot R_A$$

$E_A := \frac{200 \cdot V}{2} - \frac{P_{convFL}}{E_{AFL}} \cdot R_A$ $\quad E_A = 78 \cdot V$ $\quad n_{new} = \frac{E_A}{E_{AFL}} \cdot n_{FL} = 525.8 \cdot rpm$

c) The torque is constant (like part b)) , find the shaft speed at half the rated voltage for both the armature and field. You may assume that the flux is proportional to the field current. $\quad \phi_{100} = \frac{\phi_{200}}{2}$

One way: $\tau_{ind} = K \cdot \phi \cdot I_A$ so, if τ_{ind} is constant and the field current (and flux) is halved, then I_A is constant at twice the value it used to be. $\quad I_A := 2 \cdot I_{FL}$ $\quad I_A = 44 \cdot A$

$V_T := \frac{200 \cdot V}{2}$ $\quad E_A := V_T - I_A \cdot R_A$

$E_A = 56 \cdot V = K \cdot \phi_{100} \cdot \omega_m$ $\quad n_{new} = \frac{\left(\frac{E_A}{\phi_{100}}\right)}{\left(\frac{E_{AFL}}{\phi_{200}}\right)} \cdot n_{FL} = \frac{2 \cdot E_A}{E_{AFL}} \cdot n_{FL} = 755.1 \cdot rpm$

Another solution: recognize that: $E_A = K \cdot \phi \cdot \omega_m$ is halved because the flux is halved $\quad E_A = \frac{1}{2} \cdot \frac{E_{AFL}}{n_{FL}} \cdot n_{new}$

$$V_T = E_A + I_A \cdot R_A = E_A + \frac{P_{conv}}{E_A} \cdot R_A = E_A + \frac{\frac{n_{new}}{n_{FL}} \cdot P_{convFL}}{\frac{1}{2} \cdot \frac{n_{new}}{n_{FL}} \cdot E_{AFL}} \cdot R_A = E_A + 2 \cdot \frac{P_{convFL}}{E_{AFL}} \cdot R_A$$

$E_A := \frac{200 \cdot V}{2} - 2 \cdot \frac{P_{convFL}}{E_{AFL}} \cdot R_A$ $\quad E_A = 56 \cdot V$ $\quad n_{new} = \frac{2 \cdot E_A}{E_{AFL}} \cdot n_{FL} = 755.1 \cdot rpm$

d) If the load torque and rotational loss torque are both proportional to speed (most like real life), find the shaft speed when the motor armature is hooked to half the rated voltage. The field is at rated voltage.

$$\tau_{ind} = \frac{n_{new}}{n_{FL}} \cdot \tau_{indFL} \qquad P = \tau \cdot \omega_m \qquad \text{which leads to:} \qquad P_{conv} = \left(\frac{n_{new}}{n_{FL}}\right)^2 \cdot P_{convFL}$$

One way: $\tau_{ind} = K \cdot \phi \cdot I_A$ so, if τ_{ind} is proportional to speed and the field current is constant,

then I_A is also proportional to speed. $I_A := \frac{n_{new}}{n_{FL}} \cdot I_{FL}$

$$V_T = E_A + I_A \cdot R_A = E_A + \frac{n_{new}}{n_{FL}} \cdot I_{FL} \cdot R_A = E_A + \frac{n_{new}}{n_{FL}} \cdot I_{FL} \cdot R_A = \frac{n_{new}}{n_{FL}} \cdot E_{AFL} + \frac{n_{new}}{n_{FL}} \cdot I_{FL} \cdot R_A$$

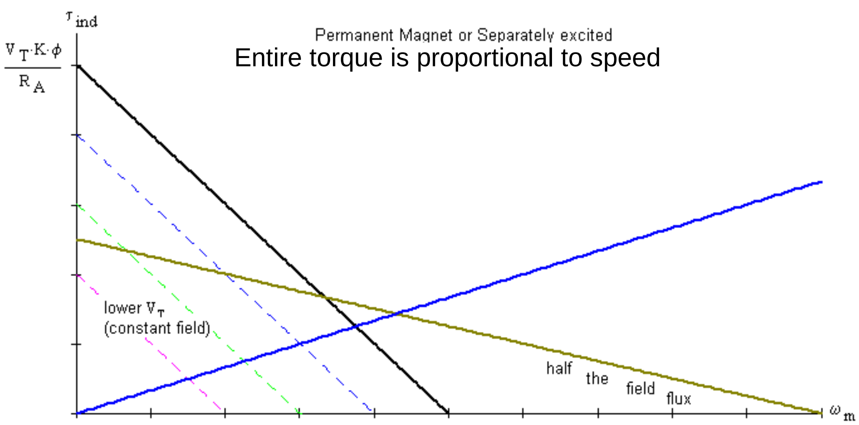
$$= \frac{n_{new}}{n_{FL}} \cdot \underbrace{\left(E_{AFL} + I_{FL} \cdot R_A\right)}_{V_{TFL}} = \frac{n_{new}}{n_{FL}} \cdot (200 \cdot V) \qquad n_{new} = \frac{100 \cdot V}{200 \cdot V} \cdot n_{FL} = 600 \cdot rpm$$

Another solution: $E_A = \frac{n_{new}}{n_{FL}} \cdot E_{AFL}$ because the field is constant

$$V_T = E_A + I_A \cdot R_A = E_A + \frac{P_{conv}}{E_A} \cdot R_A = E_A + \frac{\left(\frac{n_{new}}{n_{FL}}\right)^2 \cdot P_{convFL}}{\frac{n_{new}}{n_{FL}} \cdot E_{AFL}} \cdot R_A = \frac{n_{new}}{n_{FL}} \cdot E_{AFL} + \frac{\frac{n_{new}}{n_{FL}} \cdot P_{convFL}}{E_{AFL}} \cdot R_A$$

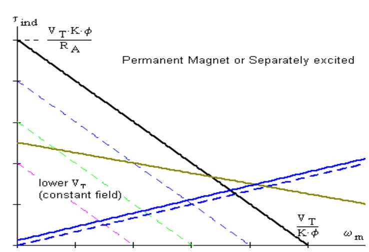
$$= \frac{n_{new}}{n_{FL}} \cdot \underbrace{\left(E_{AFL} + \frac{P_{convFL}}{E_{AFL}} \cdot R_A\right)}_{V_{TFL}}$$

same as above



Speed is proportional to V_T $\frac{V_T}{K \cdot \phi}$

e) If the load torque is proportional to speed and rotational loss torque is constant, find the shaft speed when the motor armature is hooked to half the rated voltage. The field is at rated voltage.



$$\tau_{load} = \frac{n_{new}}{n_{FL}} \cdot \tau_{loadFL} \qquad \tau_{loss} = \tau_{lossFL} = \frac{P_{rotFL}}{\omega_{mFL}}$$

$$\tau_{ind} = \frac{n_{new}}{n_{FL}} \cdot \tau_{loadFL} + \tau_{lossFL} = K \cdot \phi \cdot I_A$$

$$I_A = \frac{n_{new}}{n_{FL}} \cdot \frac{\tau_{loadFL}}{K \cdot \phi} + \frac{\tau_{lossFL}}{K \cdot \phi} = \frac{n_{new}}{n_{FL}} \cdot I_{AloadFL} + I_{AlossFL}$$

the current can be thought of as composed of two parts

$I_{AloadFL} := \frac{P_{outFL}}{E_{AFL}}$ $\quad I_{AloadFL} = 20.947 \cdot A \qquad I_{AlossFL} := \frac{P_{rotFL}}{E_{AFL}}$ $\quad I_{AlossFL} = 1.053 \cdot A$

Note: $I_{AloadFL} + I_{AlossFL} = 22 \cdot A = I_{AFL}$, exactly as it should be

$$V_T = E_A + I_A \cdot R_A \quad = \frac{n_{new}}{n_{FL}} \cdot E_{AFL} + \left(\frac{n_{new}}{n_{FL}} \cdot I_{AloadFL} + I_{AlossFL}\right) \cdot R_A$$

$$= \frac{n_{new}}{n_{FL}} \cdot E_{AFL} + \frac{n_{new}}{n_{FL}} \cdot I_{AloadFL} \cdot R_A + I_{AlossFL} \cdot R_A$$

$$V_T - I_{AlossFL} \cdot R_A \quad = \frac{n_{new}}{n_{FL}} \cdot \left(E_{AFL} + I_{AloadFL} \cdot R_A\right) \qquad n_{new} = \frac{V_T - I_{AlossFL} \cdot R_A}{E_{AFL} + I_{AloadFL} \cdot R_A} \cdot n_{FL} = 596.8 \cdot rpm$$

Ex.3 An unknown, permanent-magnet dc motor is tested at two different loads. In each case the armature voltage is: $24\ V$.

$V_T := 24 \cdot V$ Load 1: $I_{A1} := 10 \cdot A$ $n_1 := 163 \cdot rpm$ Load 2: $I_{A2} := 30 \cdot A$ $n_2 := 127 \cdot rpm$

a) Find the parameters of this motor.

Load 1: $I_{A1} := 10 \cdot A$ $n_1 := 163 \cdot rpm$

$$\omega_1 := n_1 \cdot \left(\frac{2 \cdot \pi \cdot rad}{60 \cdot \frac{sec}{min} \cdot rev}\right) \qquad \omega_1 = 17.069 \cdot \frac{rad}{sec}$$

$$V_{T1} = 24 \cdot V = I_{A1} \cdot R_A + K \cdot \phi \cdot \omega_1$$

Load 2: $I_{A2} := 30 \cdot A$ $n_2 := 127 \cdot rpm$

$$\omega_2 := n_2 \cdot \left(\frac{2 \cdot \pi \cdot rad}{60 \cdot \frac{sec}{min} \cdot rev}\right) \qquad \omega_2 = 13.299 \cdot \frac{rad}{sec}$$

$$V_{T2} = 24 \cdot V = I_{A2} \cdot R_A + K \cdot \phi \cdot \omega_2$$ solve for $R_A = \frac{24 \cdot V - K \cdot \phi \cdot \omega_2}{I_{A2}}$

Solve:

$$V_{T1} = 24 \cdot V = I_{A1} \cdot R_A + K \cdot \phi \cdot \omega_1$$ substitute in for R_A

$$V_{T1} = 24 \cdot V = I_{A1} \cdot \left(\frac{24 \cdot V - K \cdot \phi \cdot \omega_2}{I_{A2}}\right) + K \cdot \phi \cdot \omega_1$$

$$= \frac{I_{A1}}{I_{A2}} \cdot 24 \cdot V - \frac{I_{A1}}{I_{A2}} \cdot K \cdot \phi \cdot \omega_2 + K \cdot \phi \cdot \omega_1 \quad = \frac{I_{A1}}{I_{A2}} \cdot 24 \cdot V + K \cdot \phi \cdot \left(\omega_1 - \frac{I_{A1}}{I_{A2}} \cdot \omega_2\right)$$

$$K \cdot \phi = \frac{24 \cdot V - \frac{I_{A1}}{I_{A2}} \cdot 24 \cdot V}{\left(\omega_1 - \frac{I_{A1}}{I_{A2}} \cdot \omega_2\right)} = 1.266 \cdot V \cdot sec$$

$$R_A = \frac{24 \cdot V - K \cdot \phi \cdot \omega_2}{I_{A2}} = \frac{24 \cdot V - (1.266 \cdot V \cdot sec) \cdot \omega_2}{I_{A2}} = 0.239 \cdot \Omega$$

b) The rotational loss torque is proportional to speed.
Find the parameters of this motor.

Notice that the induced torque is NOT part of the calculation above. Therefore it doesn't matter how it is split between the loss and the load. The calculations are exactly the same.