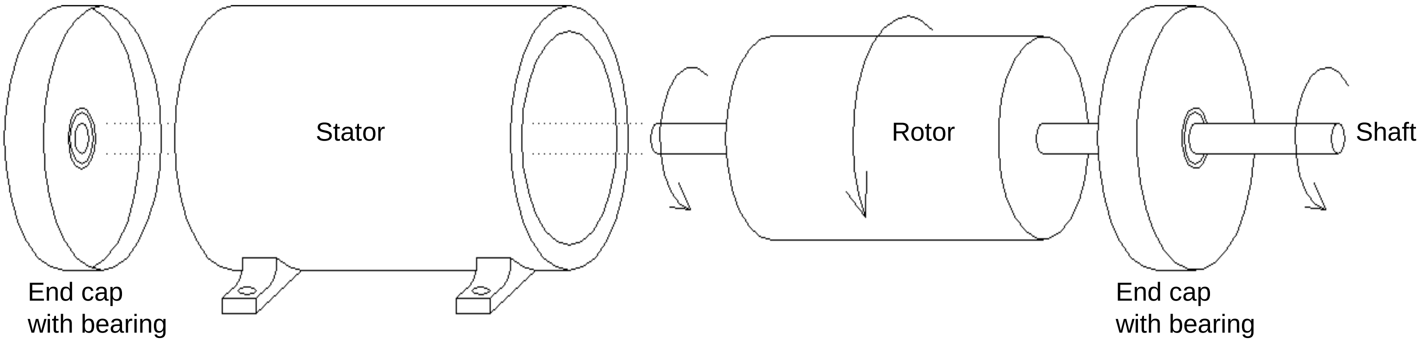
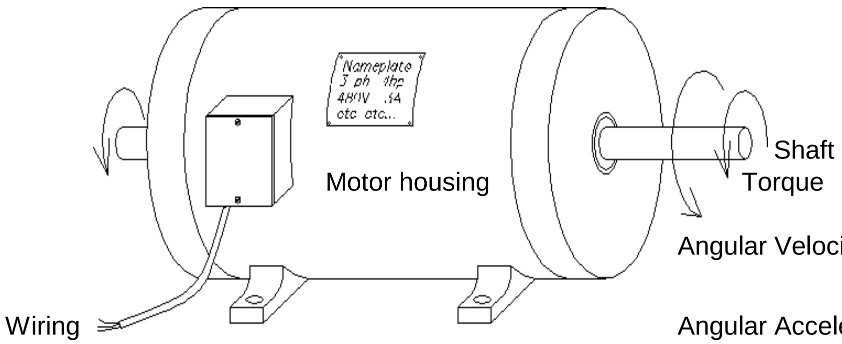
ECE 3600 Generator & Motor Basics

Simplified Drawing of a Motor or Generator (Exploded view)





Assembled Motor or Generator

Torque T or τ , $N \cdot m$

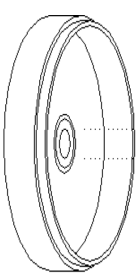
Angular Velocity ω , $\frac{rad}{sec}$ or n , $\frac{rev}{min}$ or rpm

Angular Acceleration α , $\frac{rad}{sec^2}$

Mechanical Power P_{shaft} or P_{out} $= \tau \cdot \omega$ W or hp (horse power)

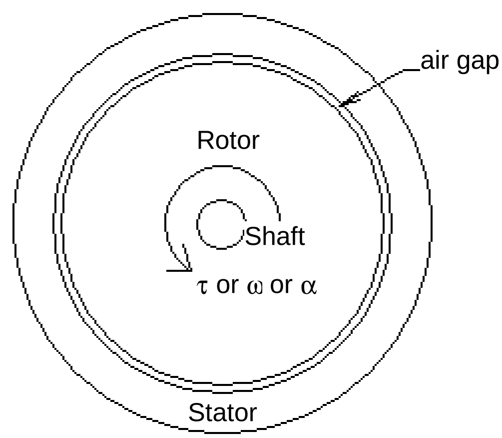
Generator: Torque as shown would come from some other rotating device, the *prime-mover* (not shown). The mechanical, shaft power would be into the generator.

Motor: Torque as shown would be caused by the motor overcoming a mechanical *load* resisting the motor's rotation (not shown). The mechanical, shaft power would be out of the motor. If the shaft were not coupled to some mechanical load the motor would spin freely (the no-load condition).

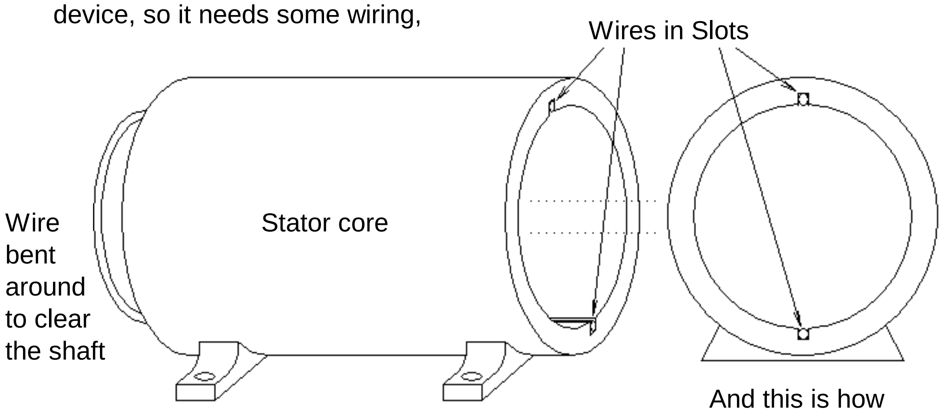


All of the parts are more complex than what I've shown. For instance, the end caps usually slip inside the main housing to insure good alignment.

Many drawings of motors, especially those designed to show how the motor works, show the motor from one end and only show the most important parts.



This is, of course, an electrical device, so it needs some wiring,

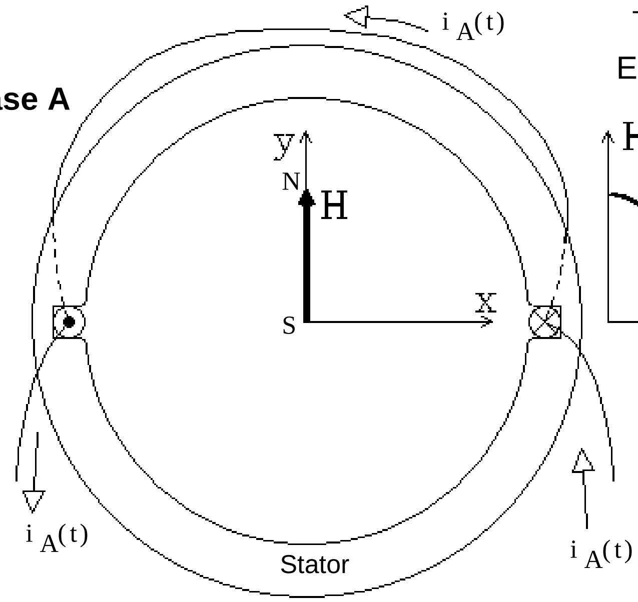


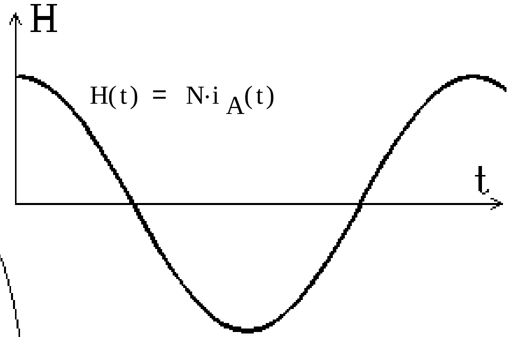
And this is how details will be shown

This is also an electromagnetic device

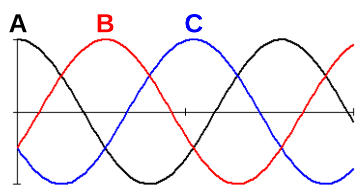
Establishing a Rotating Magnetic Field

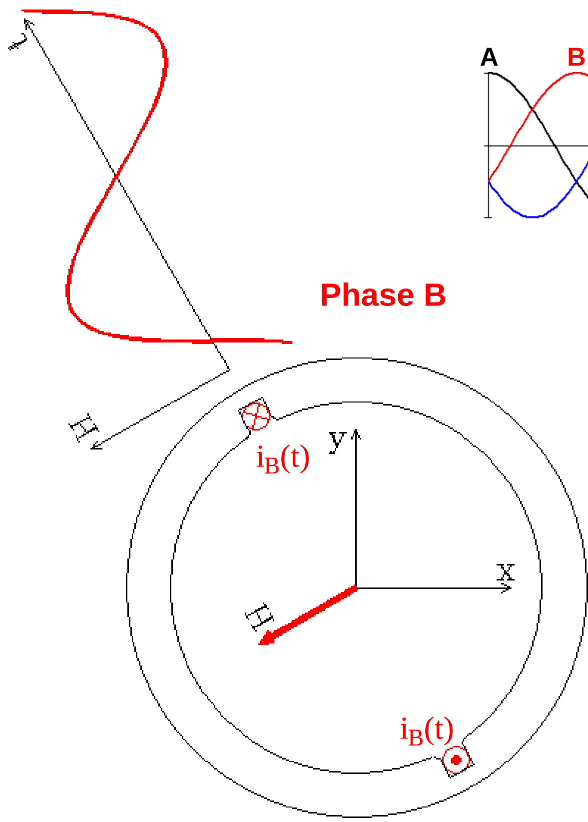
Phase A

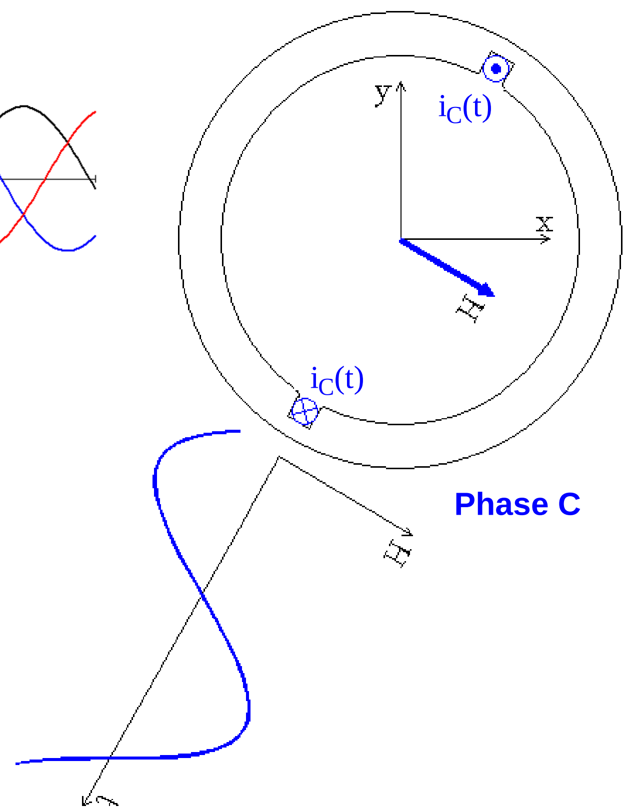


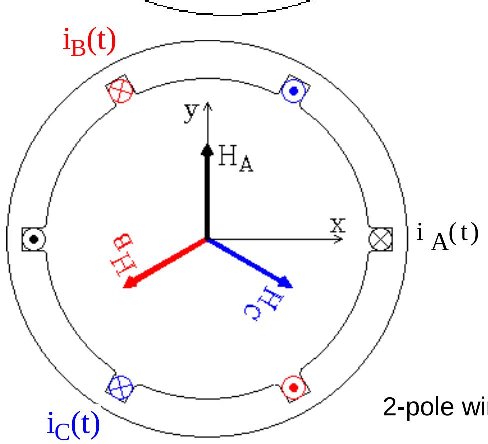


Field goes up and down, but no rotation



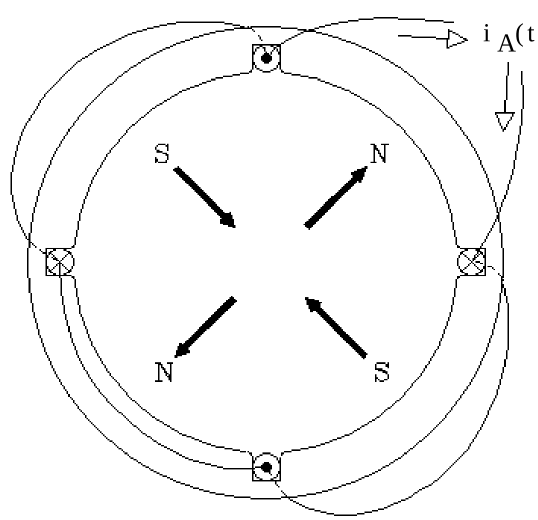






2-pole windings

All 3 phases create a counterclockwise rotating magnetic field.



Phase A of a 4-pole winding

Speed of the Rotating Magnetic Field

Depends on the number of poles $\omega = \dfrac{4\cdot\pi\cdot f}{N_{poles}} = \dfrac{2\cdot\left(377\cdot\frac{rad}{sec}\right)}{N_{poles}}$ for 60Hz systems

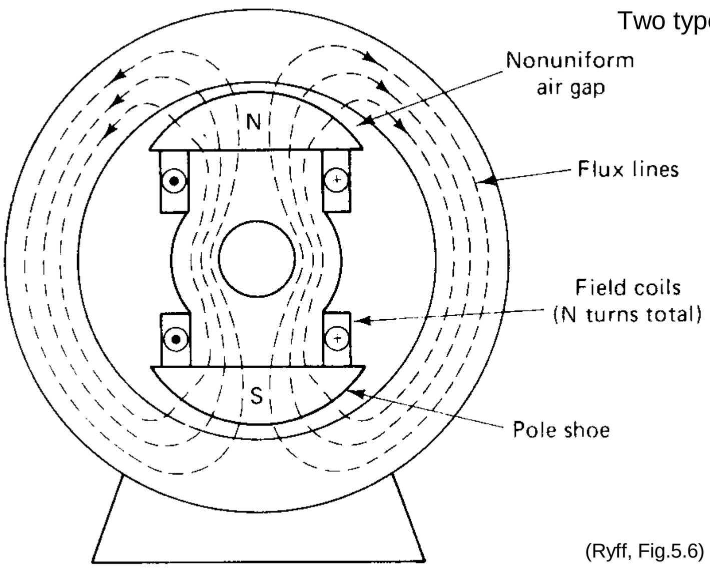
OR, n_m or $n_{sync} = \dfrac{f\cdot\frac{2\cdot poles}{cyc}\cdot\frac{60\cdot sec}{min}}{N_{poles}} = \dfrac{7200\cdot rpm}{N_{poles}}$ for 60Hz systems

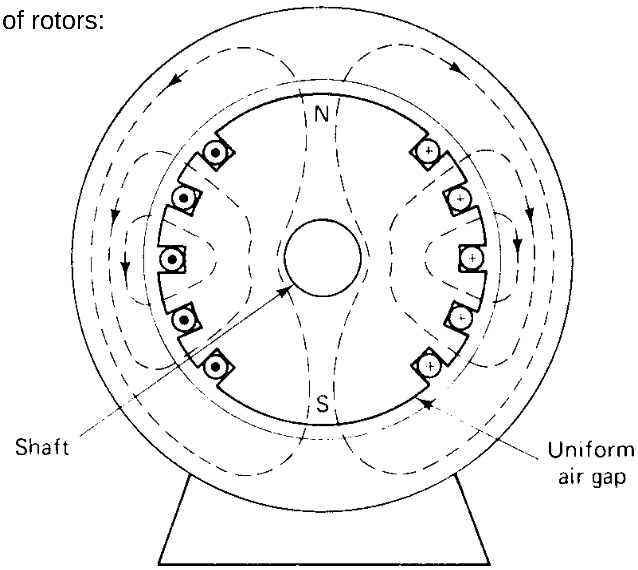
The same stator windings are used for 3-phase Synchronous Machines and for Induction Motors.
In Synchronous Machines the stator is often called the "**armature**".

Synchronous Generators & Motors

Rotor The rotors in Synchronous Machines are magnets which want to follow the rotating magnetic fields, usually DC electromagnets. The DC current usually flows through brushes and slip rings to reach the moving rotor. Sometimes the field current is generated and rectified right on the rotor. This DC field current is called the field current (I_f).

Two types of rotors:





(Ryff, Fig.5.6)

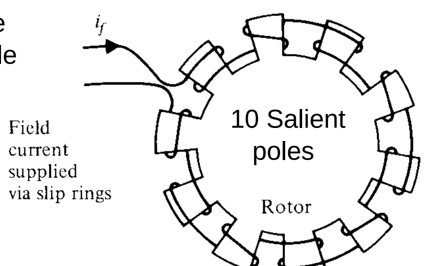
2 Salient poles

Common for a motor or generator with many poles

2 Non-salient poles, Cylindrical rotor

Common for a motor or generator with few poles

10-pole example

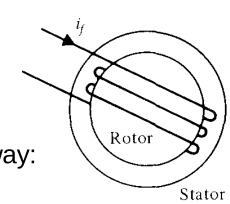


(Brown, Fig.4.3)

10-poles - 720rpm
(fast for hydro)

Typically low-speed motor or water turbine driven generator. Typically short in length and large in diameter. (typ diameter is 1.5xlength)

Sometimes shown this way:



(Brown, Fig.4.2)

Typically high-speed motor or steam turbine driven generator, long and small diameter. (typ length is 3xdiameter)

2-poles - 3600rpm 4-poles - 1800rpm

Motor

If the stator currents flow in from a 3-phase power source and the rotor is a magnet, the rotor will follow the rotating magnetic field at the synchronous speed (in sync with the rotating field). That would be a synchronous motor. However, when the magnetic rotor is spinning within the stator windings it will induce voltages on those windings, just like a generator. The induced voltages (called the back EMF, $\mathbf{E_A}$) will oppose the input voltages that caused the original currents to flow.

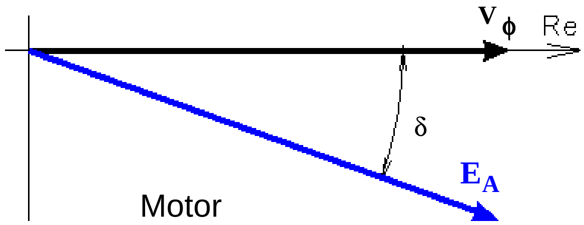
When the motor shaft is connected to a mechanical load (spins something which resists spinning), the rotor tries to slow down, but it only succeeds in lagging behind the rotating magnetic field a little (unless the motor is overloaded). When the rotor lags behind the field, the induced voltages ($\mathbf{E_A}$)s will also lag the input voltages.

Generator

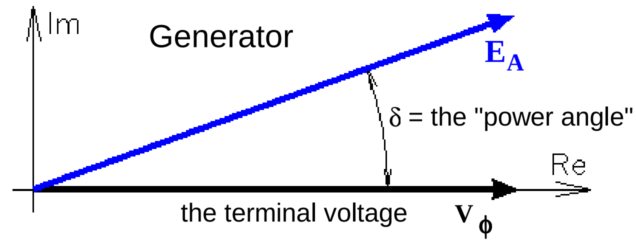
If, instead of a mechanical load, the shaft of this same device is connected to a source of mechanical power which tries to make it spin faster than the synchronous speed, it will act as a generator. If the generator is connected to the power grid (as they usually are) the only way the mechanical power source (the prime mover) can increase the speed would be to push the frequency of the entire grid higher than 60 Hz -- not likely. So all it succeeds in doing is to make the rotor lead the rotating magnetic field a little and along with it the induced voltages ($\mathbf{E_A}$)s will also lead the grid voltages..

Phasor diagrams of one phase.

We usually consider the the terminal or phase voltage ($\mathbf{V_\phi}$) be set and held constant by the entire power grid.

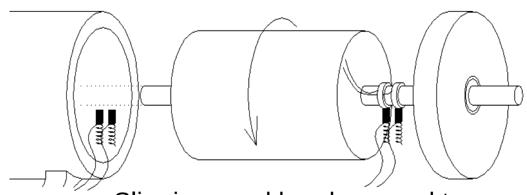


When operated as a motor, the induced armature voltage ($\mathbf{E_A}$) lags the terminal voltage, $\mathbf{V_\phi}$.



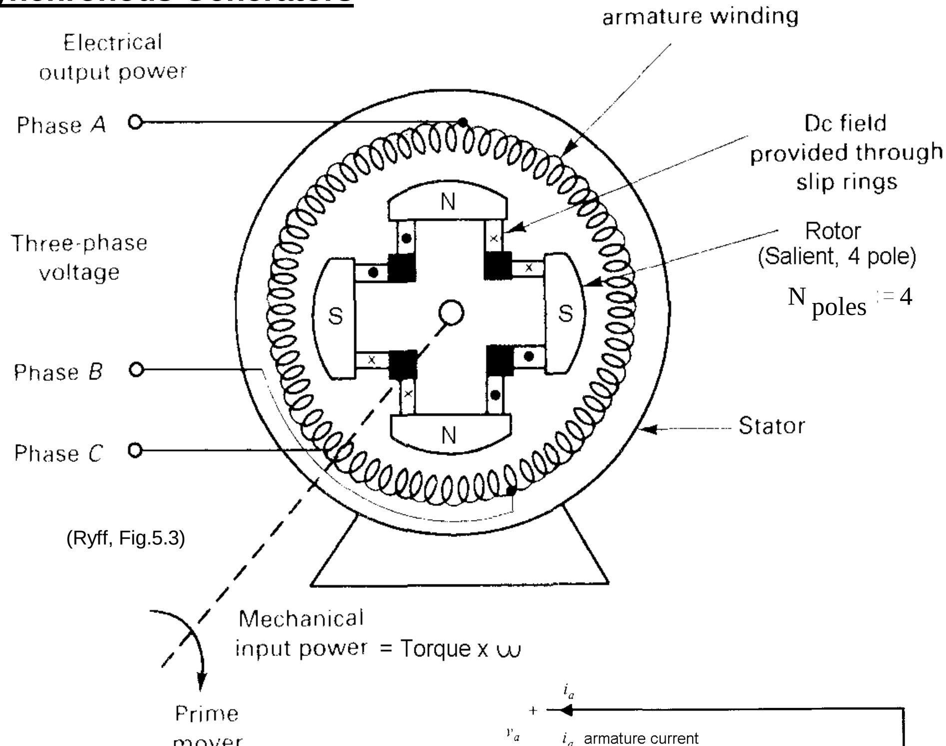
When operated as a generator, the induced armature voltage ($\mathbf{E_A}$) leads the terminal voltage, $\mathbf{V_\phi}$.

The magnitude of the induced armature voltages ($\mathbf{E_A}$ for our phase) depends on the field current, I_f . I_f causes the field flux (called **excitation**). The DC current may come from an external supply or it may be generated on the rotor. Either way there are usually brushes and slip rings, if not for DC current, then for control of that current.



Slip rings and brushes used to connect a DC supply to the rotor. (brushes shown twice)

Synchronous Generators



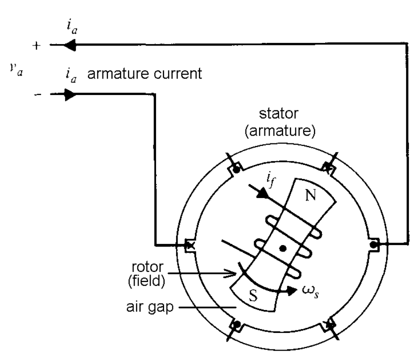
(Ryff, Fig.5.3)

(mechanical input power)

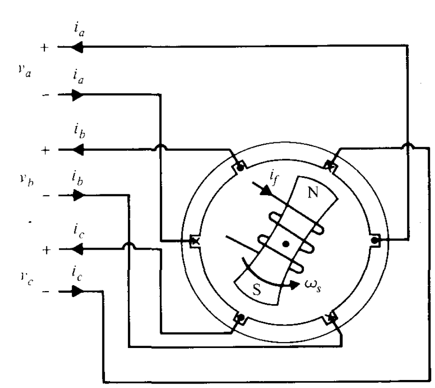
$N_{poles} := 4$

$$\omega = \frac{377 \cdot \frac{rad}{sec}}{2} = 188.5 \cdot \frac{rad}{sec}$$

$$n = \frac{7200 \cdot rpm}{4} = 1800 \cdot rpm$$



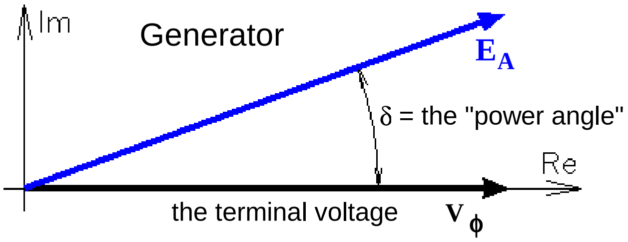
Stator (armature) winding



2-pole, 3-phase synchronous generator

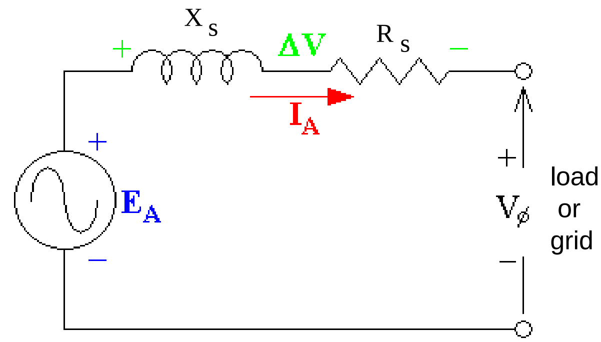
(Brown, Fig.4.1)

Electrical analysis on a per-phase basis



When operated as a generator, the induced armature voltage (E_A) leads the terminal voltage, V_ϕ .

The magnitude E_A depends on the DC field current, I_f .



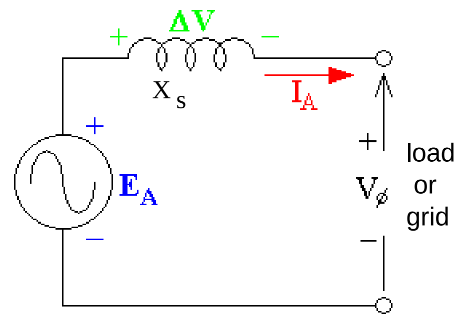
The electrical model of an armature winding

X_s is the armature inductance
(armature windings and leakage)
(magnetization)

R_s is the armature winding resistance

This is almost always simplified to this:
(Especially in our class)

$$E_A = I_A \cdot j \cdot X_s + V_\phi$$



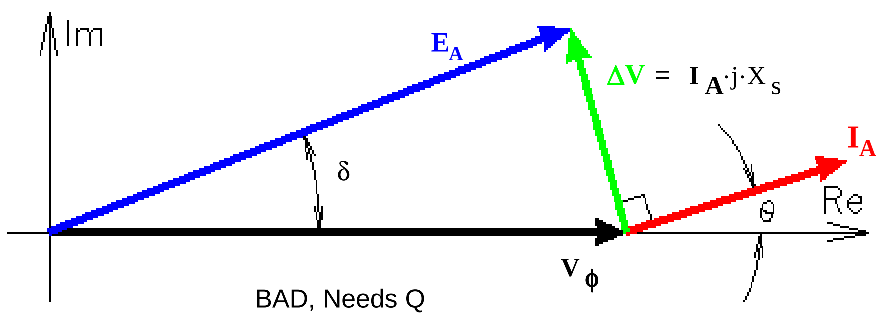
Low, or Under-excited ($\theta > 0$)

Low DC field current

Low E_A

Makes -Q

"Uses" Q like an inductive load



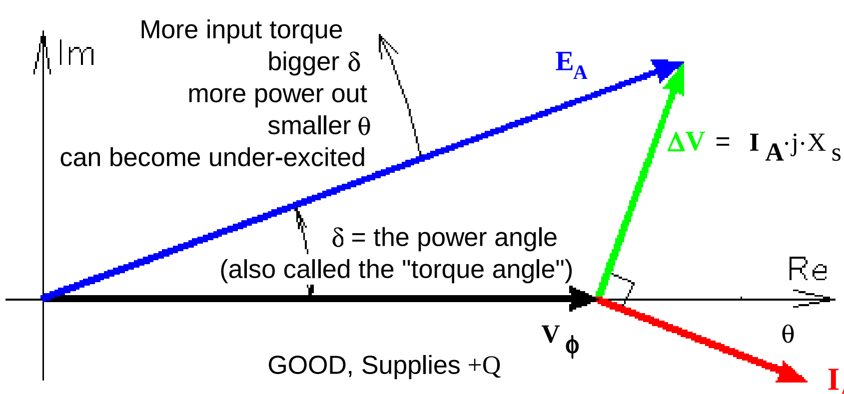
The **under-excited** condition, the current leads the terminal voltage, V_ϕ . The generator supplies -Q (-VARs), that is, it absorbs +Q (+VARs), just like an inductive load. Usually not desirable.

High, or Over-excited ($\theta < 0$)

Higher DC field current

High E_A

Makes Q

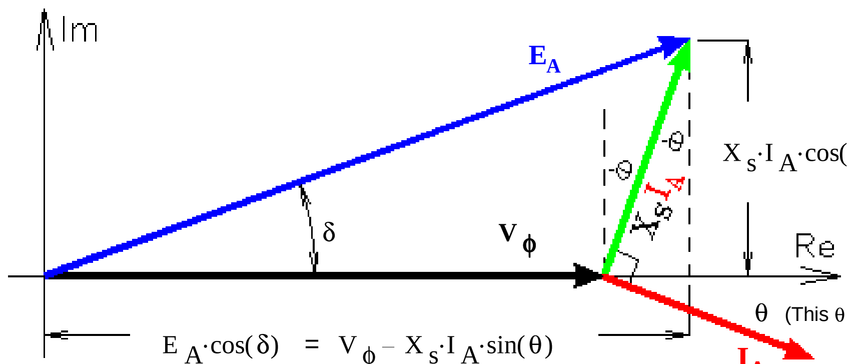


$$I_A = \frac{\Delta V}{j \cdot X_s}$$

current lags, matching the need of most loads

The **over-excited** condition, the current lags the terminal voltage, V_ϕ . The generator supplies +Q (+VARs), that is, it absorbs -Q (-VARs), just like an capacitive load. Usually desirable.

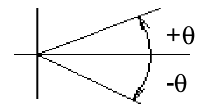
Important relations



Note: Voltages and currents are magnitudes, not complex numbers

The **signs** of the angles are **important!**

$$X_s \cdot I_A \cdot \cos(\theta) \ = \ E_A \cdot \sin(\delta)$$



under excited

over

$$E_A \cdot \cos(\delta) \ = \ V_\phi - X_s \cdot I_A \cdot \sin(\theta)$$

(θ is negative)

$$X_s \cdot I_A \cdot \cos(\theta) \ = \ E_A \cdot \sin(\delta)$$

$$-I_A \cdot \sin(\theta) \ = \ \frac{E_A \cdot \cos(\delta) - V_\phi}{X_s}$$

$$I_A \cdot \cos(\theta) \ = \ \frac{E_A \cdot \sin(\delta)}{X_s}$$

$$Q_{1\phi} \ = \ -V_\phi \cdot I_A \cdot \sin(\theta)$$

$$P_{1\phi} \ = \ V_\phi \cdot I_A \cdot \cos(\theta)$$

Important Equations →

$$Q_{1\phi} \ = \ \frac{V_\phi \cdot E_A \cdot \cos(\delta) - V_\phi^{\,2}}{X_s}$$

$$P_{1\phi} \ = \ \frac{V_\phi \cdot E_A \cdot \sin(\delta)}{X_s}$$

← Important Equations

→

$$\mathbf{E_A} = \mathbf{V_\phi} + \mathbf{I_A} \cdot j \cdot X_s$$

$$\delta \ = \ \text{asin}\left(\frac{P_{1\phi} \cdot X_s}{V_\phi \cdot E_A}\right)$$

$$E_A = \sqrt{\left(V_\phi - X_s \cdot I_A \cdot \sin(\theta)\right)^2 + \left(X_s \cdot I_A \cdot \cos(\theta)\right)^2}$$

Be careful with the sign of θ.

$$\text{asin} \ = \ \sin^{-1}$$

Pullout power

If δ reaches 90^o, the generator will lose synchronization.
Pullout power is the maximum power a generator can produce for a given excitation, at $\delta := 90 \cdot \text{deg}$

$$P_{po} \ = \ \frac{E_A \cdot V_\phi}{X_s} \cdot \sin(90 \cdot \text{deg}) \ = \ \frac{E_A \cdot V_\phi}{X_s}$$

To Bring a Synchronous Generator "On Line"

1. Bring speed to the correct rpm so that the generator frequency matches the line frequency.
2. Adjust the field current, I_f so that the generator voltage matches the line voltage.
3. Readjust speed if necessary, check that the phases are in the correct sequence if necessary.
4. Wait until the phases align (0 volts difference between generator terminal and the line phase). Connect to the line at just the right moment.
5. Increase input torque to produce real electrical power and and field current to produce reactive power.

Most (~99%) of the world's electrical energy is produced by 3-phase synchronous generators.

Mechanical speed, torque, and power

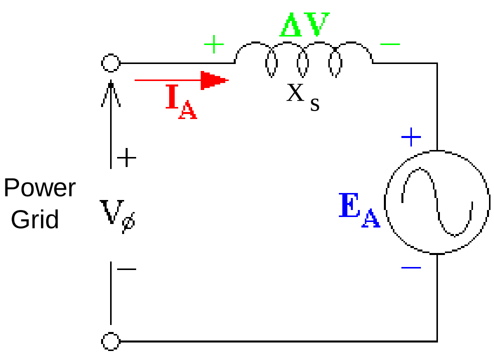
Shaft speed in rad/sec $\quad \omega_{mech} \ = \ \frac{4 \cdot \pi \cdot f}{N_{poles}} \ = \ \frac{2 \cdot \left(377 \cdot \frac{rad}{sec}\right)}{N_{poles}}$ for 60Hz systems

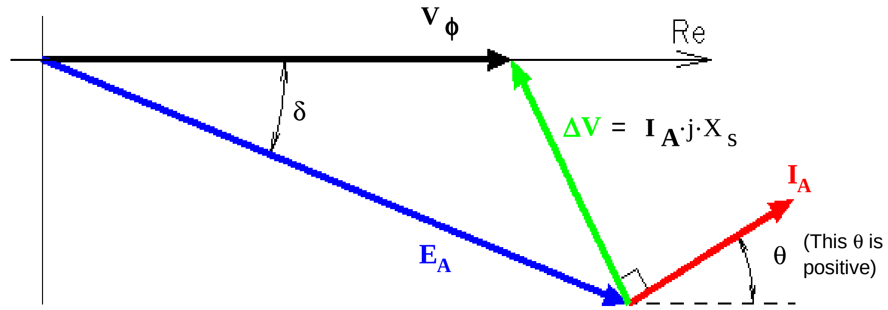
Shaft speed in rev/min $\quad n \ = \ \frac{f \cdot \frac{2 \cdot poles}{cyc} \cdot \frac{60 \cdot sec}{min}}{N_{poles}} \ = \ \frac{7200 \cdot rpm}{poles}$ for 60Hz systems

$\tau_{mech} \cdot \omega_{mech} \ = \ P_{3\phi}$ (electrical) neglecting losses

τ_{mech} = mechanical torque

Synchronous Motors

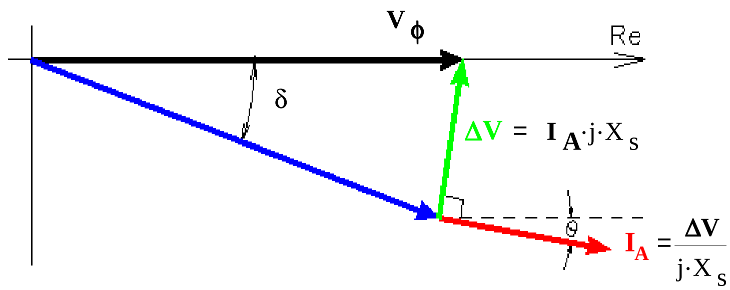




Over-excited condition, GOOD, supplies +Q (+VARs).

(θ is measured in opposite direction to a regular load)

$$Q_{1\phi} = V_\phi \cdot I_A \cdot \sin(-\theta)$$



Under-excited condition, BAD, absorbs +Q (+VARs).

Important relations (all)

$$E_A \cdot \sin(|\delta|) = X_s \cdot I_A \cdot \cos(\theta)$$

$$P_{1\phi} = \frac{E_A \cdot V_\phi}{X_s} \cdot \sin(|\delta|)$$

$$Q_{1\phi} = \frac{V_\phi^2 - E_A \cdot V_\phi \cdot \cos(\delta)}{X_s}$$

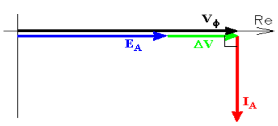
(Bigger E_A makes Q negative (good))

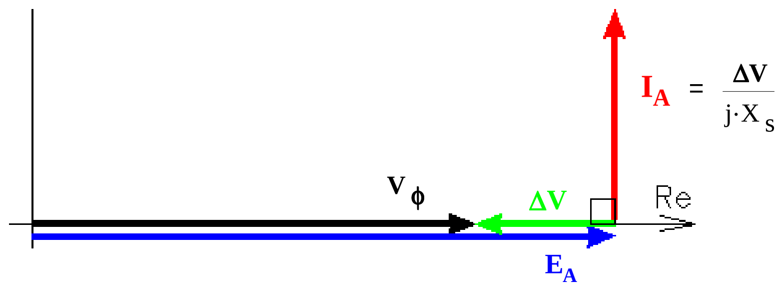
$$= V_\phi \cdot I_A \cdot \sin(-\theta)$$

Synchronous Condenser (Capacitor)

A special case of the over-excited motor with no mechanical load (and neglecting friction)

An under-excited motor with no mechanical load (and neglecting friction) will look like an inductor. Called synchronous reactance.

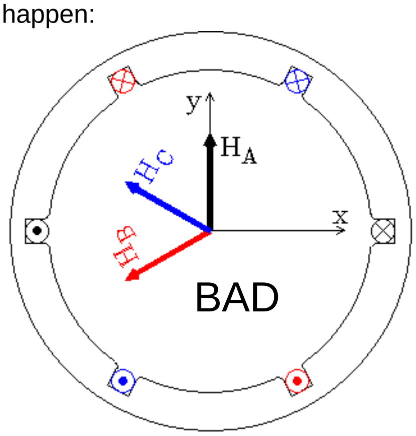


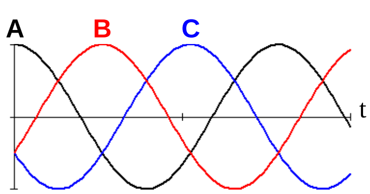


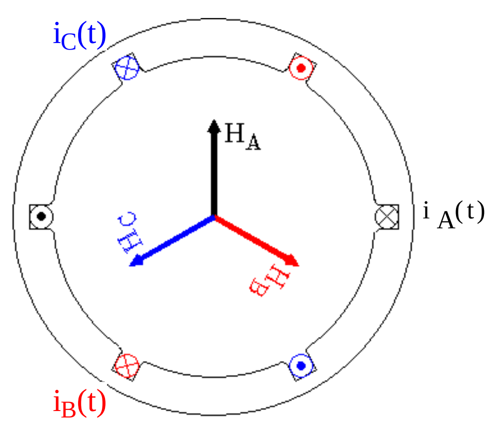
Motor Connections and Changing the Direction of Rotation

DO NOT alter the manufacturer's wiring within the motor, other than to change from Y to Δ or reverse. And then follow directions carefully. Otherwise something like this could happen:

It is OK to change the connections between the power panel and the motor as long as you don't mess with the neutral and/or ground connections. Swapping any two phases from the power panel will reverse the direction of rotation. Works for both Y and Δ connections







The 3 phases create a clockwise rotating magnetic field.

Core losses

In steady-state synchronous operation, the rotor of a synchronous machine does not experience a changing magnetic flux so there are no core losses in the rotor and it can be made of solid ferromagnetic material. The stator, on the other hand, *does* experience a changing magnetic flux (at 60 Hz) so there are both hysteresis and eddy-current core losses in the stator. The stator is constructed of laminated, silconized material to minimize the eddy currents.

Stator windings in practice

The nonlinearity of the stator core also causes the stator current to be nonsinusioidal, including a significant third-harmonic (just like in a transformer). The reduce the harmonics, the phase windings are designed to overlap each other a little and don't always span exactly 180^o of flux.

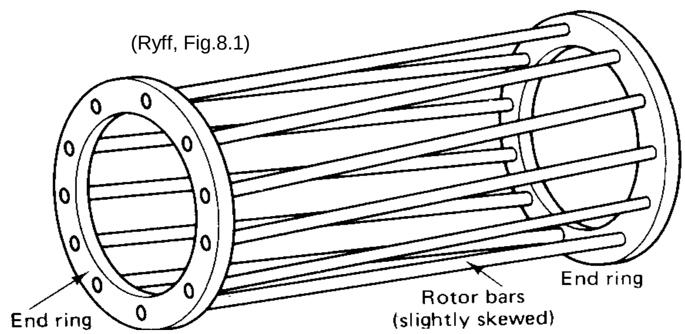
Effect on the network (grid)

Our analysis regularly assumes that the generator feeds an "infinite" network bus. Then we can assume the network voltage, or the terminal voltage, is constant in magnitude, frequency and phase (The slack-bus idea). In reality, large generators *do* affect the network (the larger the generator, the larger the effect). Increasing the prime-mover torque will raise the network voltage (especially in the local area) and slightly increase the entire network frequency. Matching the generation of reactive power to the local needs will help to optimize the network power flow.

Damper Bars

The rotor of an *induction* motor includes a number of thick conductors called "rotor bars". Current is induced in these bars because the rotor normally turns at speed which is slower than the synchronous speed (the speed of the rotating flux caused by the stator windings). The interaction between the induced current and the rotating flux provides the motor torque.

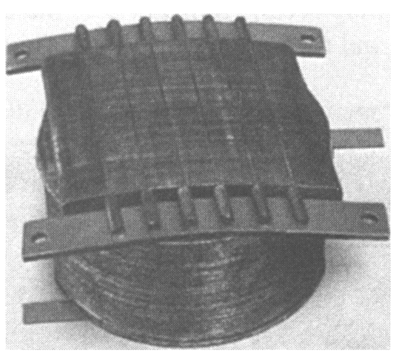
(Ryff, Fig.8.1)



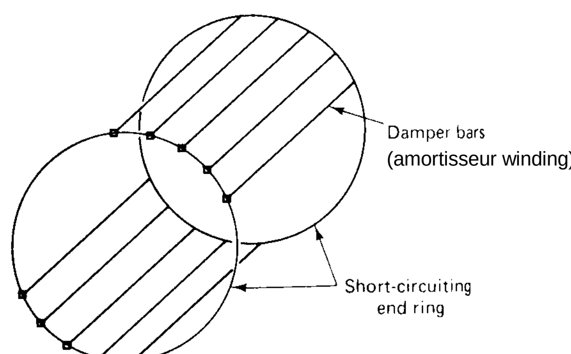
Synchronous machines usually have very similar bars in their rotors. In steady-state synchronous operation, they have no affect. The purpose of these bars is to resist, or dampen, transients. Currents will be induced in these windings when the stator current magnitudes change or when the input rotational shaft speed changes. By Lenz's law, those currents will be induced in a direction to oppose the change that caused them. In solid iron rotors, the eddy currents have the same effect. Without damping, the shaft speed can oscillate.

See textbook section 5.11, p.243 for more details.

See also textbook fig 5.41, p.245



(Ryff, Fig.10.3)



(Ryff, Fig.10.3)

Note: These notes and Chapter 5 of our textbook assumes that the DC supply of the field current is robust enough to withstand high voltage transients. It also assumes the source resistance and the field winding resistance are so small that the field winding itself can perform the transient damping function. It is more reasonable to assume that the synchronous machine is constructed with damper bars, but the results of the different assumptions is about the same.

Transient Conditions

The armature currents normally create a rotating flux in sync with the rotor motion and the flux through the rotor is constant. These steady-state currents see large synchronous reactances (X_s) due in large part to the low reluctance of the rotor. X_s can be 1 per-unit based on the machine's bases. When armature current magnitudes change, the armature flux has to change as well. A rotor with damping bars (or other low-resistance windings or eddy current paths), will strongly resist any changes in flux through the rotor, so much of the changing flux will go around the rotor, taking a much higher reluctance path. A higher reluctance path results in a lower inductance and a lower reactance.

X_s = steady-state synchronous reactance, nearly all flux goes through rotor

X''_s = sub transient synchronous reactance, no additional flux goes through rotor
first few cycles only

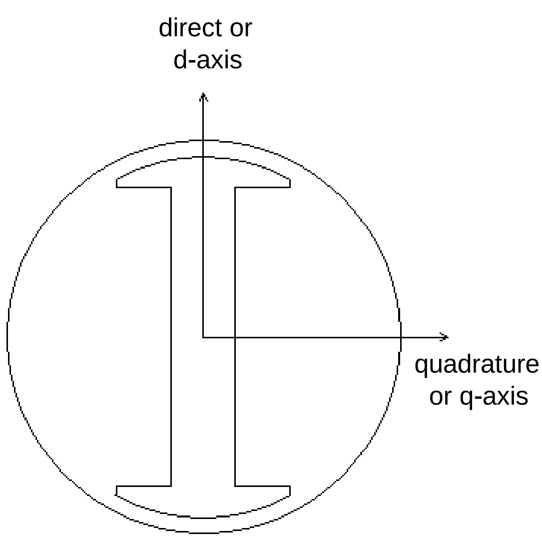
$$= \frac{X_s}{10} \text{ to } \frac{X_s}{4}$$

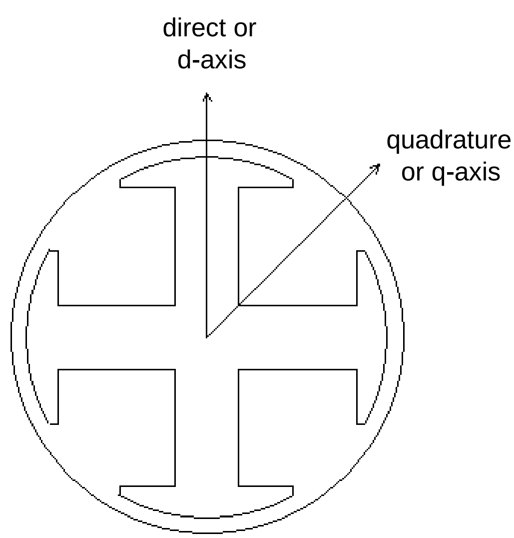
X'_s = transient synchronous reactance, some additional flux goes through rotor
after first few cycles until new steady-state.

$$\cong 2 \cdot X''_s$$

Variations of the magnetizing reactance in salient pole machines

A large part of the synchronous reactance (X_s) is the magnetizing reactance (X_m) and arises from the rotating magnetic flux produced by the armature current. Our analysis up to this point, assumes that the rotating flux depends proportionally on the current magnitude alone. That, in turn, assumes that the magnetic reluctance is the same for all angles of the rotor. This is a bad assumption for a salient-pole rotor. The reluctance along the "direct" or "d"-axis of the rotor is less than the reluctance along the "quadrature" or "q"-axis.





To fully analyze the salient-pole machine, the armature magnetizing reactance (X_m) needs to be broken up into X_{md} and X_{mq}, corresponding to the direct and quadrature axes. The armature voltages and currents are then also broken up into v_d and v_q, and i_d and i_q, respectively. This is beyond the scope of our class.

ECE 3600 Synchronous Generator & Motor Examples

A.Stolp
10/25/11
rev10/9/14,
10/3/22

Ex. 1

(F08 E2) A 60 Hz, 4-pole, 3-phase synchronous generator supplies 90 kW of power to a 4 kV bus. The synchronous reactance is 50 Ω/phase. The generator emf is 3 kV. Find the following.

a) The power angle, δ.

givens $f := 60 \cdot Hz$ $\quad N_{poles} := 4$ $\quad$ Assume Y-connected: Otherwise E_A would be very low. $\quad V_\phi := \dfrac{4 \cdot kV}{\sqrt{3}}$

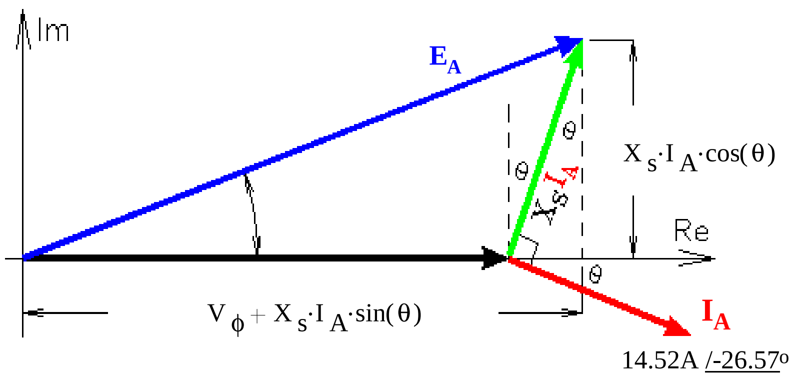
$X_s := 50 \cdot \Omega$ $\quad E_A := 3 \cdot kV$ $\quad P_{3\phi} := 90 \cdot kW$

$$P_{1\phi} := \frac{P_{3\phi}}{3} \qquad P_{1\phi} = 30 \cdot kW \quad = \frac{V_\phi \cdot E_A \cdot \sin(\delta)}{X_s} \qquad \delta := \operatorname{asin}\left(\frac{P_{1\phi} \cdot X_s}{V_\phi \cdot E_A}\right) \qquad \delta = 12.5 \cdot deg$$

b) The total reactive power generated.

$$Q_{1\phi} = \frac{V_\phi \cdot E_A \cdot \cos(\delta) - V_\phi^{\,2}}{X_s} \qquad Q_{3\phi} = 3 \cdot \frac{V_\phi \cdot E_A \cdot \cos(\delta) - V_\phi^{\,2}}{X_s} = 85.83 \cdot kVAR$$

c) Find a new magnitude of the generator emf so that $Q := 45 \cdot kVAR$



$$Q_{1\phi} := \frac{Q}{3} \qquad S_{1\phi} := \sqrt{P_{1\phi}^{\,2} + Q_{1\phi}^{\,2}}$$

$$I_A := \frac{S_{1\phi}}{V_\phi} \qquad I_A = 14.52 \cdot A$$

$$\theta := \operatorname{atan}\left(\frac{Q_{1\phi}}{P_{1\phi}}\right) \qquad \theta = 26.57 \cdot deg$$

Notice that θ is positive in the downward direction, contrary to the notes. Also notice how that affects the figure at left. Know what you're doing, don't just use formulas!

by Pythagorean theorem: $E_A := \sqrt{\left(V_\phi + X_s \cdot I_A \cdot \sin(\theta)\right)^2 + \left(X_s \cdot I_A \cdot \cos(\theta)\right)^2}$ $\qquad E_A = 2.713 \cdot kV$

OR $\mathbf{I_A} := I_A \cdot e^{-(j \cdot \theta)}$ $\quad \mathbf{E_A} := V_\phi + \mathbf{I_A} \cdot (j \cdot X_s)$ $\quad E_A = |\mathbf{E_A}| = 2713 \cdot V$ $\quad \delta = \arg(\mathbf{E_A}) = 13.851 \cdot deg$

d) The shaft speed.

$$n := \frac{7200 \cdot rpm}{N_{poles}} \qquad n = 1800 \cdot rpm \qquad \omega_m := n \cdot \frac{2 \cdot \pi \cdot rad}{rev} \cdot \left(\frac{min}{60 \cdot sec}\right) \qquad \omega_m = 188.496 \cdot \frac{rad}{sec}$$

$$\text{OR} \qquad \omega_m = \frac{\left(377 \cdot \dfrac{rad}{sec}\right)}{\left(\dfrac{N_{poles}}{2}\right)} = 188.5 \cdot \frac{rad}{sec}$$

e) The shaft torque.
Often called the "Prime-mover torque"

$$P_{3\phi} = \omega T \qquad T := \frac{P_{3\phi}}{\omega_m} \qquad T = 477.5 \cdot N \cdot m$$

f) The shaft torque is decreased to half the value found in part e). What is the new P and Q?

$$P'_{3\phi} := \frac{1}{2} \cdot P_{3\phi} \qquad P'_{3\phi} = 45 \cdot kW \qquad \delta := \operatorname{asin}\left(\frac{P'_{3\phi} \cdot X_s}{3 \cdot V_\phi \cdot E_A}\right) \qquad \delta = 6.87 \cdot deg$$

$$Q_{3\phi} = 3 \cdot \frac{V_\phi \cdot E_A \cdot \cos(\delta) - V_\phi^{\,2}}{X_s} = 53.23 \cdot kVAR$$

Ex. 2

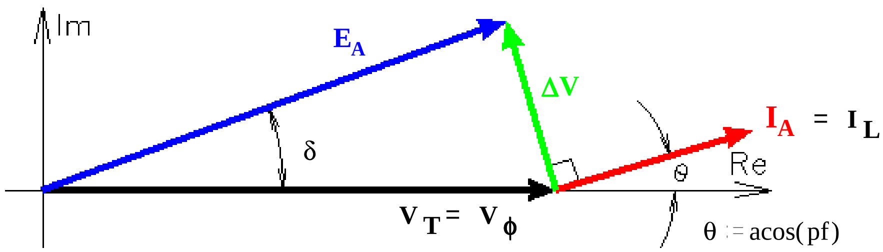
(F09 Fin) You make the following measurements on a 3-phase, Y-connected, synchronous generator.

$P_{3\phi} := 120 \cdot kW$ $\quad V_{LL} := 480 \cdot V$ $\quad I_L := 160 \cdot A$ $\quad X_s := 1.2 \cdot \Omega$

Unfortunately, you don't know the phase angle of current.

a) Draw a phasor diagram of one of the two possible interpretations of these numbers.
Find the induced armature voltage (E_A) and the power angle, δ. $\quad E_A = ?$ $\quad \delta = ?$

My first assumption: I_L leads V_T



$V_T := \frac{V_{LL}}{\sqrt{3}}$ $\quad V_T = 277.13 \cdot V = V_\phi$

$P_{1\phi} := \frac{P_{3\phi}}{3}$ $\quad P_{1\phi} = 40 \cdot kW$

$pf := \frac{P_{1\phi}}{I_L \cdot V_T}$ $\quad pf = 0.902$

$\theta := acos(pf)$

$\theta = 25.563 \cdot deg$

$\mathbf{I_L} := I_L \cdot e^{j \cdot \theta}$

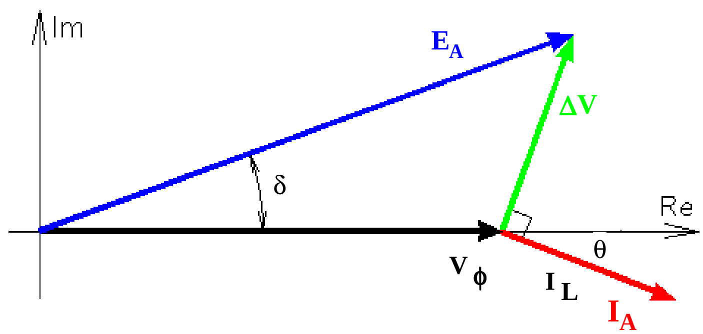
Produces negative Q

$\mathbf{E_A} := V_T + \mathbf{I_L} \cdot (j \cdot X_s)$ $\quad E_A = |\mathbf{E_A}| = 260.28 \cdot V$ $\quad \delta = arg(\mathbf{E_A}) = 41.718 \cdot deg$

b) Draw a phasor diagram of other possible interpretation of these numbers.
Find the induced armature voltage (E_A) and the power angle, δ. $\quad \delta = ?$

My second assumption:

I_L lags V_T



$\theta := -acos(pf)$

$\theta = -25.563 \cdot deg$

$\mathbf{I_L} := I_L \cdot e^{j \cdot \theta}$

Produces positive Q

$\mathbf{E_A} := V_T + \mathbf{I_L} \cdot (j \cdot X_s)$ $\quad E_A = |\mathbf{E_A}| = 399.48 \cdot V$ $\quad \delta = arg(\mathbf{E_A}) = 25.695 \cdot deg$

c) A traveling carnival uses a combination of this generator and the local power company to run its load, mainly induction motors. When the generator is connected to the carnival's power distribution network, it supplies half of the required power, but the current from the power company only decreases by about 30%. Which of the calculations above is most likely correct?

Assumption in a)

Give me the reasoning behind your answer (no calculations required).

The induction motors represent a lagging pf load, they use lots of VARs. If the local generator were supplying those VARs, then the current would go down by about half and quite possibly more. The small reduction in current implies that the generator also consumes VARs (creates negative VARs). That is condition a).

d) What do you change at the generator to reduce the current flow from the power company?
Tell me what you adjust and if you turn it up or down.

Turn up the field current.

Ex. 3

A 60 Hz, 2-pole, Y-connected, 3-phase synchronous generator supplies 15 MW of power to a 18 kV bus. The synchronous reactance is 6 Ω/phase. The magnitude of the generator emf equals the magnitude of the bus voltage.

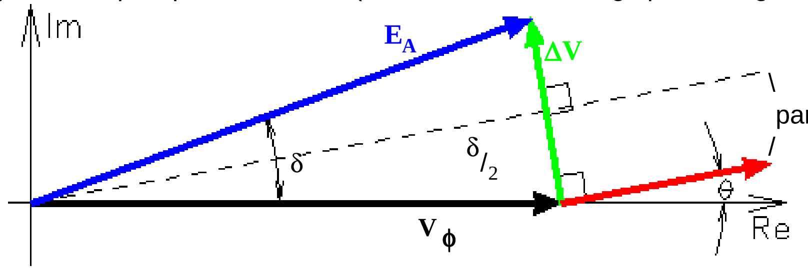
givens $V_\phi := \frac{18\cdot kV}{\sqrt{3}}$ $V_\phi = 10.392\cdot kV$ $X_s := 6\cdot\Omega$ $P_{1\phi} := \frac{15\cdot MW}{3}$ $P_{1\phi} = 5\cdot MW$

$E_A := V_\phi$

Find:

a) The power angle, δ. $P_{1\phi} = \frac{E_A\cdot V_\phi}{X_s}\cdot\sin(\delta)$ $\delta := \operatorname{asin}\left(\frac{P_{1\phi}\cdot X_s}{E_A\cdot V_\phi}\right)$ $\delta = 16.13\cdot deg$

b) The complex phase current, (Assume the bus voltage phase angle is 0º).



$\theta := \frac{\delta}{2}$ $\theta = 8.06\cdot deg$ lead

$P_{1\phi} = V_\phi\cdot I_A\cdot\cos(\theta)$ $I_A := \frac{P_{1\phi}}{V_\phi\cdot\cos(\theta)}$ $I_A = 485.9\cdot A$

c) The magnitude and direction of reactive power.

$Q_{3\phi} := 3\cdot V_\phi\cdot I_A\cdot\sin(-\theta)$ $Q_{3\phi} = -2.125\cdot MVAR$

Since the current leads the voltage, this generator absorbs reactive power (produces -VARs)

Ex. 4

A 60-Hz, three-phase, 6-pole, Δ-connected synchronous motor is connected to 480 V and produces 80 hp. The motor draws minimum current with an excitation voltage of $E_A = 520$ V per phase. Mechanical losses are 5hp .

givens $N_{poles} := 6$ $V_\phi := 480\cdot V$ $P_{3\phi} := 80\cdot hp + 5\cdot hp$ $E_A := 520\cdot V$

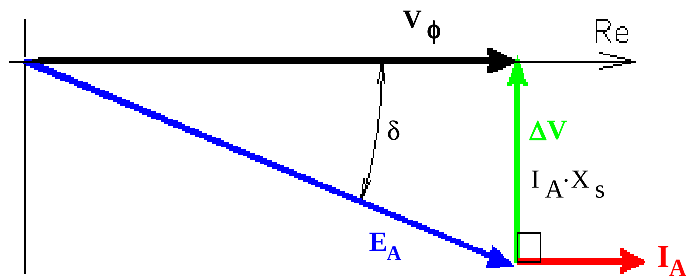
$P_{1\phi} := \frac{85\cdot hp}{3}\cdot\frac{745.7\cdot W}{hp}$ $P_{1\phi} = 21.1\cdot kW$

Determine the following:

a) The current.

Minimum current implies $pf := 1$ so...

$I_A := \frac{P_{1\phi}}{V_\phi}$ $I_A = 44.02\cdot A$



b) The line current. $I_L := \sqrt{3}\cdot I_A$ $I_L = 76.24\cdot A$

c) The synchronous reactance.

by Pythagorean theorem: $I_A\cdot X_s = \sqrt{E_A^{\,2} - V_\phi^{\,2}}$ $X_s := \frac{\sqrt{E_A^{\,2} - V_\phi^{\,2}}}{I_A}$ $X_s = 4.544\cdot\Omega$

d) The torque.

$\omega_{mech} := \frac{4\cdot\pi\cdot f}{N_{poles}}$ $T_{mech} = \frac{80\cdot hp\cdot\frac{745.7\cdot W}{hp}}{\omega_{mech}} = 475\cdot N\cdot m$

e) The rotor power angle. $\delta = \operatorname{acos}\left(\frac{V_\phi}{E_A}\right) = 22.62\cdot deg$

f) The maximum power this motor could provide at this excitation voltage.

$\delta := 90\cdot deg$ $P_{3\phi} = 3\cdot\frac{E_A\cdot V_\phi}{X_s}\cdot\sin(\delta)\cdot\frac{1\cdot hp}{745.7\cdot W} - 5\cdot hp = 216\cdot hp$

Note: The current rating of the motor may be exceeded at this load.

(F09 E2, p4) An industrial plant is powered from a 480-V, 3-phase bus and currently draws 60 kW at a power factor of 0.8 lagging. A new mill is to be added at the plant. This mill requires a shaft torque of 600 Nm at 1200 rpm. Your job is to specify a motor which will run the mill and correct the plant power factor at the same time. Be sure to specify the type of motor including the number of poles. Tell me how the motor should be connected to the bus (This is an arbitrary decision here, but it will affect many of your other answers). Specify its minimum hp, voltage, and current ratings. Tell me what the back emf should be. You may assume the synchronous reactance is 1 Ω/phase and that losses are negligible.

Plant, as is: $P_{3\phi} := 60 \cdot kW$ $\quad P_{1\phi} := \frac{P_{3\phi}}{3}$ $\quad P_{1\phi} = 20 \cdot kW$ $\quad pf := 0.8$

$S_{1\phi} := \frac{P_{1\phi}}{pf}$ $\quad S_{1\phi} = 25 \cdot kVA$ $\quad Q_{1\phi} := \sqrt{S_{1\phi}^2 - P_{1\phi}^2}$ $\quad Q_{1\phi} = 15 \cdot kVAR$

Motor basics

$N_{poles} := \frac{7200}{1200}$ $\quad N_{poles} = 6$ $\quad \omega_{mech} := \frac{4 \cdot \pi \cdot f}{N_{poles}}$ $\quad \omega_{mech} = 125.7 \cdot \frac{rad}{sec}$ $\quad \left(\frac{377}{3}\right)$

Use a 6-pole synchronous motor

$T_{mech} := 600 \cdot N \cdot m$ motor power = $P_{m3\phi}$ = $T_{mech} \cdot \omega_{mech} = 75.398 \cdot kW$ $\quad 1 \cdot hp = 745.7 \cdot W$

$T_{mech} \cdot \omega_{mech} \cdot \frac{1 \cdot hp}{745.7 \cdot W} = 101.111 \cdot hp$ = min hp rating

$P_{m1\phi} := \frac{T_{mech} \cdot \omega_{mech}}{3}$ $\quad P_{m1\phi} = 25.133 \cdot kW$

To fix the plant pf, $Q_{m1\phi} := -Q_{1\phi}$ $\quad Q_{m1\phi} = -15 \cdot kVAR$

Phase angle of the current: $\theta := atan\left(\frac{-Q_{m1\phi}}{P_{m1\phi}}\right)$ $\quad \theta = 30.83 \cdot deg$

If you select Y-connected

$V_\phi := \frac{480 \cdot V}{\sqrt{3}}$ $\quad V_\phi = 277.1 \cdot V$ = min voltage rating

Current per phase: $I := \frac{\sqrt{P_{m1\phi}^2 + Q_{m1\phi}^2}}{V_\phi}$ $\quad I = 105.61 \cdot A$ = min current rating

$X_s := 1 \cdot \Omega$ $\quad \mathbf{\Delta V} := I \cdot e^{j \cdot \theta} \cdot X_s \cdot j$ $\quad \mathbf{\Delta V} = (-54.127 + 90.69j) \cdot V$

motor back emf $\quad \mathbf{E_A} := V_\phi - \mathbf{\Delta V}$ $\quad \mathbf{E_A} = (331.255 - 90.69j) \cdot V$ $\quad |\mathbf{E_A}| = 343.44 \cdot V$ = required motor emf

$\delta = arg(\mathbf{E_A}) = -15.311 \cdot deg$ (unneeded)

If you select Δ-connected

$V_\phi := 480 \cdot V$ $\quad V_\phi = 480 \cdot V$ = min voltage rating

Current per phase: $I := \frac{\sqrt{P_{m1\phi}^2 + Q_{m1\phi}^2}}{V_\phi}$ $\quad I = 60.98 \cdot A$ = min current rating

$X_s := 1 \cdot \Omega$ $\quad \mathbf{\Delta V} := I \cdot e^{j \cdot \theta} \cdot X_s \cdot j$ $\quad \mathbf{\Delta V} = (-31.25 + 52.36j) \cdot V$

motor back emf $\quad \mathbf{E_A} := V_\phi - \mathbf{\Delta V}$ $\quad \mathbf{E_A} = (511.25 - 52.36j) \cdot V$ $\quad |\mathbf{E_A}| = 513.92 \cdot V$ = required motor emf

$\delta = arg(\mathbf{E_A}) = -5.848 \cdot deg$