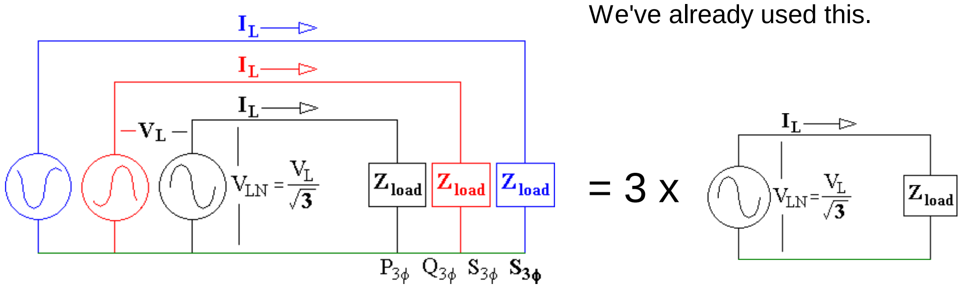
# ECE 3600 Power System Diagrams a

Full 3-phase diagrams can be very cumbersome. In a balanced system you only need to consider one phase

## Per-phase Analysis

Balanced 3-phase systems can be represented by just one phase.
We've already used this.




$$P_{1\phi} = \frac{P_{3\phi}}{3} \qquad Q_{1\phi} = \frac{Q_{3\phi}}{3}$$

$$\mathbf{S_{1\phi}} = \frac{\mathbf{S_{3\phi}}}{3}$$

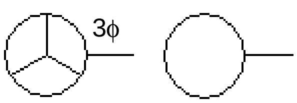
$$S_{1\phi} = |\mathbf{S_{1\phi}}| = \frac{S_{3\phi}}{3}$$

Anything not Y-connected can be converted to a Y- equivalent. $\mathbf{Z_Y} = \frac{\mathbf{Z_\Delta}}{3}$

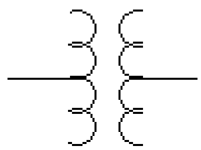
## One-Line Diagrams

In a balanced system neutral current is zero, so in one-line diagrams, even the neutral connections are omitted.

Some Important symbols




generator or motor



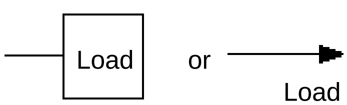
transformer

bus or bus

All items connected to one bus have same voltage. Like a circuit node, but actually represents 4 connections.

transmission line

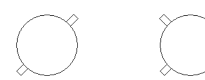
typ: oil or $SF_6$




Load

powered general

Circuit breakers



manual powered

Switches

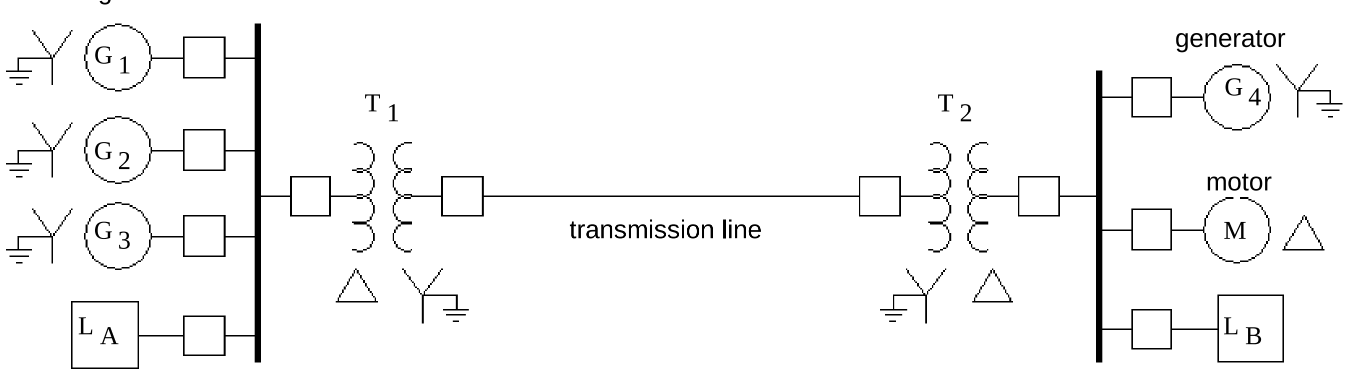
Connection symbols:

delta wye wye with ground

Can also include resistors, inductors, capacitors and impedances
Unfortunately these symbols are not as well standardized as you would think

Example

3 generators




load A Station A Station B load B

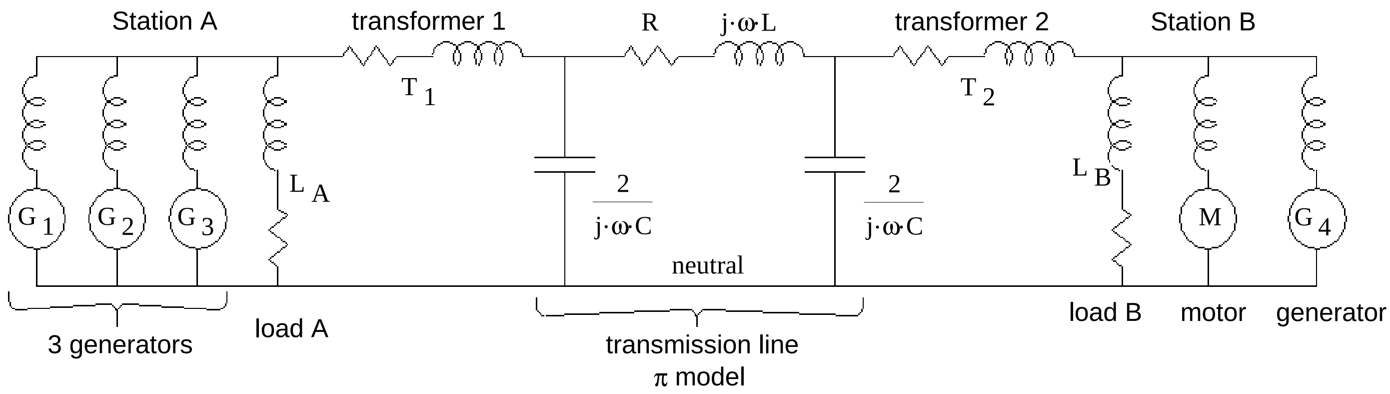
Many more real examples:
www.ece.utah.edu/~ece3600/SingleLineML102530301.pdf start p.13

## Impedance Diagrams

Same system

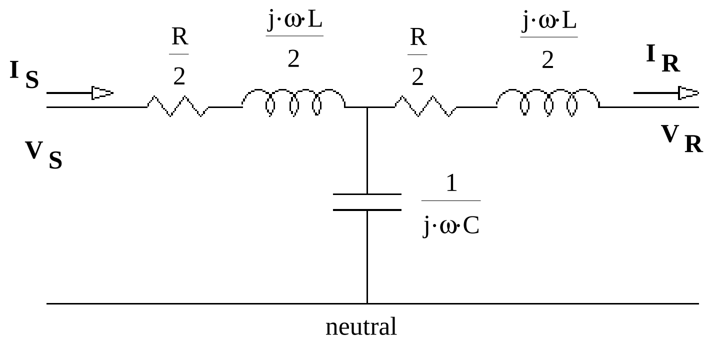
Component values are per-unit (pu).

If you didn't use pu values then you would have to transform impedances across the transformers.




A T model of the transmission line may be easier to work with

T model:

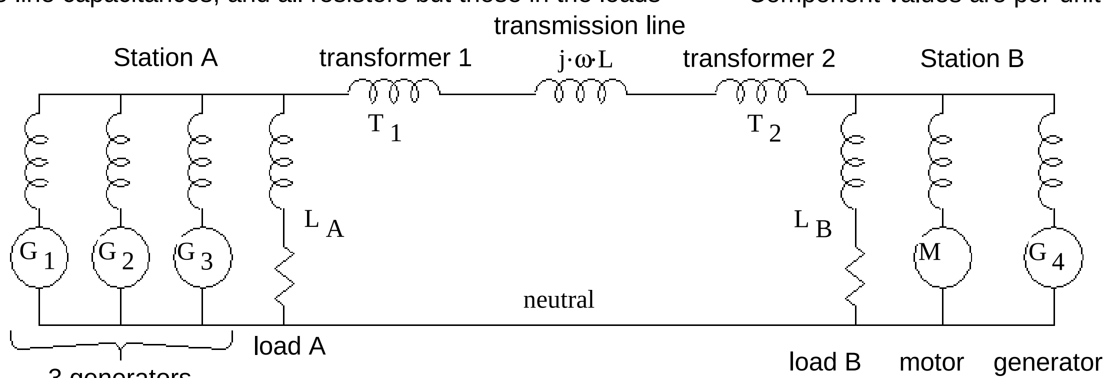



## Reactance Diagrams

Same system

Ignore the line capacitances, and all resistors but those in the loads

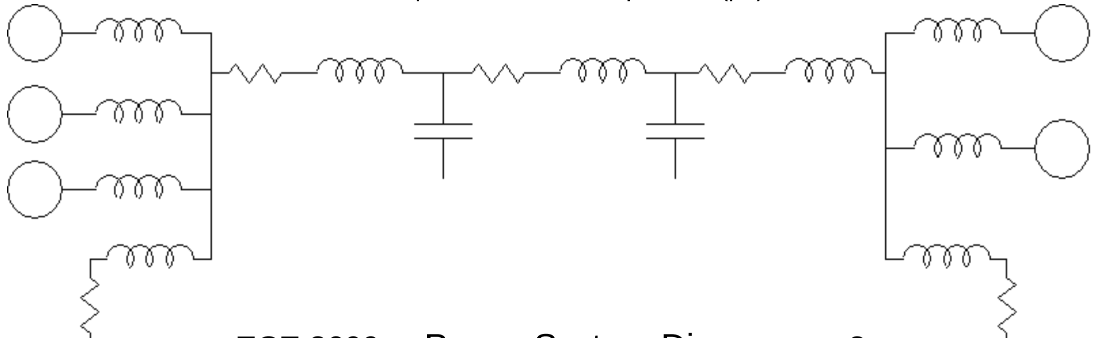
Component values are per-unit (pu).




Loads are often Omitted

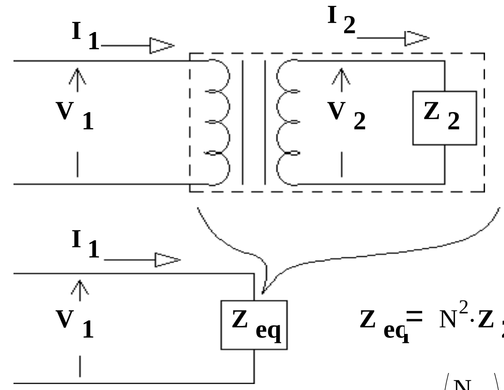
## One-Line Impedance Diagrams

Component values are per-unit (pu).



# ECE 3600    Per-Unit Notes

Think for a moment about systems which have transformers and impedances. We've learned that transformers don't just transform voltages and currents-- they also transform impedances (by the turns ratio squared). When there are impedances on both sides of a transformer, it's usually easier to transform impedances in such a way that the ideal part of the transformer can be eliminated from the calculations. Otherwise, you'll have to solve multiple equations simultaneously.

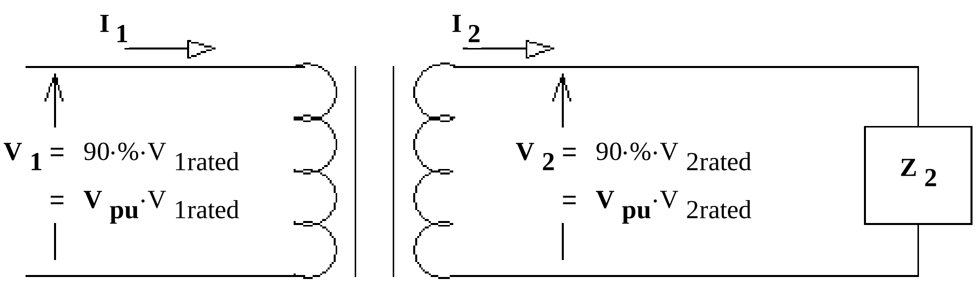



$$\mathbf{Z}_{eq} = N^2 \cdot \mathbf{Z}_2$$

$$= \left(\frac{N_1}{N_2}\right)^2 \cdot \mathbf{Z}_2$$

But, there is another way...

We could express our voltages as percentages of the transformer ratings instead of their actual values. As an example, what if the primary voltage was 90% of the rated primary voltage, then the secondary would also be 90% of the rated secondary voltage.




Let's find the rated currents so that the currents can also be expressed as percentages of rated values.

$$I_{1rated} = \frac{S_{rated}}{V_{1rated}} \qquad I_{2rated} = \frac{S_{rated}}{V_{2rated}}$$

And a "rated" impedance $\quad Z_{2rated} = \dfrac{V_{2rated}}{I_{2rated}} = \dfrac{V_{2rated}^{\;2}}{S_{rated}}$

If this "rated" impedance hooked to the secondary, then the rated voltage would make the rated current flow.

So let's say the actual $\mathbf{Z}_2$ is 50% greater than the "rated" impedance $\quad \mathbf{Z}_2 = 150\cdot\%\cdot Z_{2rated} = \mathbf{Z}_{pu} \cdot Z_{2rated}$

We could now calculate the current as a percentage of the rated current $\quad \dfrac{\mathbf{V}_{pu}}{\mathbf{Z}_{pu}} = \dfrac{90\cdot\%}{150\cdot\%} = 60\cdot\% = \mathbf{I}_{pu}$

$\mathbf{I}_2$ is 60% of the rated secondary current $\quad \mathbf{I}_2 = \mathbf{I}_{pu} \cdot I_{2rated} = 60\cdot\%\cdot I_{2rated}$

And the primary current is also 60% of the rated primary current $\quad \mathbf{I}_1 = 60\cdot\%\cdot I_{1rated} = \mathbf{I}_{pu} \cdot I_{1rated}$

And $\quad \dfrac{\mathbf{V}_{pu}}{\mathbf{Z}_{pu}} = \dfrac{90\cdot\%}{150\cdot\%} = 60\cdot\% = \mathbf{I}_{pu}$

And the upshot is, if we work with these percentages rather than the actual values, the percentages are the same on both sides of the transformer and the transformer "disappears". Notice that the percentages are the $\mathbf{V}_{pu}$, $\mathbf{I}_{pu}$ and $\mathbf{Z}_{pu}$ values. These "per unit" values may be complex.

I wouldn't recommend using this method for a circuit with a single transformer. But.., in circuits with many transformers, like the power grid, this method becomes very .. well.. powerful. There are a few differences:

1. Use "base" values rather than a single transformer's ratings, although $V_{base}$ values almost always match the transformer ratings.
2. The per-unit method is almost exclusively used in 3-phase systems, so a few of the base relationships include $3$ and $\sqrt{3}$ .

## Base Values $S_{base}$, $V_{base}$, $I_{base}$, and $Z_{base}$

At least two base values must be specified in order to find all the other base values. The power base ($S_{base}$) is the most universal base value since it isn't changed by transformers-- it's the same across the entire system. Usually a power company will use a nice round number, like 10MVA or 100MVA. The second most common base is the voltage ($V_{base}$) which will, of course, change at each transformer. $I_{base}$, and $Z_{base}$ are calculated from the first two and also change at each transformer. All these base values only need to be calculated once for a given $S_{base}$ and system configuration. Once the base values are established, all load and power-flow calculations can be made much more easily and quickly.

Per unit analysis is usually done on 3-phase systems and then on a per-phase basis, BUT, $S_{base}$ and $V_{base}$ are 3-phase and line-to-line respectively, so calculations of $I_{base}$, and $Z_{base}$ will need to take that into account. $I_{base}$ is line current and $Z_{base}$ is for a Y-connected load. Other bases are sometimes defined, but it should be clear that they are not the normal bases of a 3-phase system

If per-unit values are given by the manufacturer of a generator, transformer or other device, then the manufacturer will use the device ratings as base values. For those devices, the per-unit impedances will have to be converted from the device's $S_{rated}$ (or $S_{base}$) to the $S_{base}$ of the system.

### Starting bases

$S_{base}$ = The 3-phase power base of the system, usually a nice round number, like 10MVA or 100MVA

$P_{base} = Q_{base} = S_{base}$ don't need to be separately defined

$V_{base}$ = The nominal $V_L$ ($V_{LL}$) in each region of the power system, where regions are regions are separated by transformers

$V_{base} = V_L = V_{LL}$ nominal in each region

### Finding the other bases

$$I_{base} = \frac{\left(\frac{S_{base}}{3}\right)}{\left(\frac{V_{base}}{\sqrt{3}}\right)} = \frac{S_{base}}{\sqrt{3}\cdot V_{base}}$$

= The base for line current = current in one phase of a Y-connected load or device.

\ most / common way to calculate

$$Z_{base} = \frac{\left(\frac{V_{base}}{\sqrt{3}}\right)}{I_{base}} = \frac{V_{base}}{\sqrt{3}\cdot I_{base}} = \frac{\left(\frac{V_{base}}{\sqrt{3}}\right)^2}{\left(\frac{S_{base}}{3}\right)} = \frac{{V_{base}}^2}{S_{base}}$$

\ most / common way to calculate

$R_{base} = X_{base} = Z_{base}$ don't need to be separately defined

It is possible (but not recommended) to define the following: $S_{1\phi\_base} = \frac{S_{base}}{3}$ and $V_{LN\_base} = \frac{V_{base}}{\sqrt{3}}$

In which case: $I_{base} = \frac{S_{1\phi\_base}}{V_{LN\_base}}$ and $Z_{base} = \frac{V_{LN\_base}}{I_{base}} = \frac{{V_{LN\_base}}^2}{S_{1\phi\_base}}$

### Base changes

Per-unit impedances given by a manufacturer of a generator, transformer or other device must be converted from the device ratings ($S_{rated}$ and $V_{rated}$) to the system base values ($S_{base}$ and $V_{base}$).

$$Z_{pu} = Z_{pu\_device}\cdot\frac{S_{base}\cdot\left(V_{rated}\right)^2}{S_{rated}\cdot\left(V_{base}\right)^2}$$

OR, more commonly

$$Z_{pu} = Z_{pu\_device}\cdot\frac{S_{base}}{S_{rated}}$$

When device $V_{rated} = V_{base}$

<u>Expressing values as "per-unit", pu</u> (Multiply by 100% to express as %, otherwise, use "pu" as the units)

$$S_{pu} = \frac{S_{3\phi}}{S_{base}} \qquad P_{pu} = \frac{P_{3\phi}}{S_{base}} \qquad Q_{pu} = \frac{Q_{3\phi}}{S_{base}}$$

$$V_{pu} = \frac{V_L}{V_{base}} \qquad I_{pu} = \frac{I_L}{I_{base}}$$

resistance $R_{pu} = \dfrac{R}{Z_{base}}$ conductance $G = \dfrac{1}{R}$ $G_{pu} = \dfrac{1}{R_{pu}}$

reactance $X_{pu} = \dfrac{X}{Z_{base}}$ susceptance $B = \dfrac{1}{X}$ $B_{pu} = \dfrac{1}{X_{pu}}$

impedance $Z_{pu} = \dfrac{Z}{Z_{base}}$ admittance $Y = \dfrac{1}{Z}$ $Y_{pu} = \dfrac{1}{Z_{pu}}$

common in power-flow calculations

The $V_{pu}$, $I_{pu}$, $Z_{pu}$ and $S_{pu}$ values are not affected by transformers.

The voltage, current and impedance bases WILL change at each transformer. The power base will NOT.

Please note that: $S_{pu} = \dfrac{S_{3\phi}}{S_{base}} = \dfrac{S_{1\phi}}{S_{1\phi\_base}}$ and $V_{pu} = \dfrac{V_L}{V_{base}} = \dfrac{V_{LN}}{V_{LN\_base}}$

\ / \ /

so these calculations (and the respective bases) are rarely needed or wanted

## Example

a) Find the bases for a simple system with two transformers.

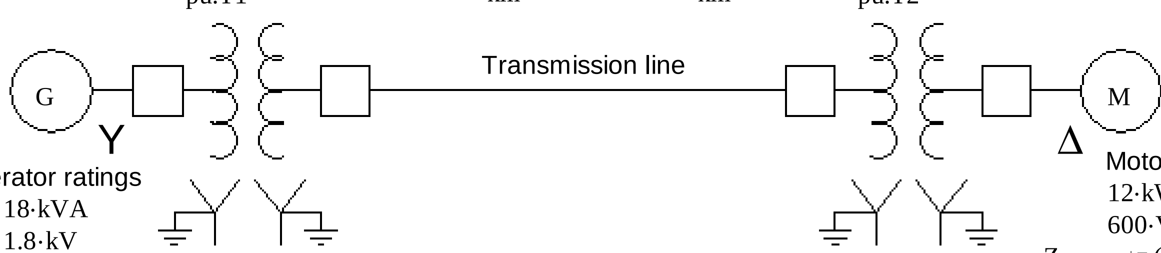
Since this is a very small, self-contained system, let's use an unusually small $S_{base} := 18 \cdot kVA$

Transmission line length = $len := 50 \cdot km$

1.8kV / 4kV
$18 \cdot kVA$
$Z_{pu.T1} := 0.1j \cdot pu$

$r := 0.3 \cdot \dfrac{\Omega}{km}$ $\omega l := 0.8 \cdot \dfrac{\Omega}{km}$

4kV / 500V
$15 \cdot kVA$
$Z_{pu.T2} := 9j \cdot \%$




Generator ratings
$18 \cdot kVA$
$1.8 \cdot kV$
$Z_{pu.G} := 1.5j \cdot \%$
actual voltage
$1732 \cdot V$

Motor ratings
$12 \cdot kW$
$600 \cdot V$
$Z_{pu.M} := (0.958 + 0.2j) \cdot pu$
per phase
at full load

\ Region 1 / \ Region 2 / \ Region 3 /

$V_{base1} := 1.8 \cdot kV$ $V_{base2} := 4 \cdot kV$ $V_{base3} := 500 \cdot V$

Set the $V_{base}$ values in each region by looking at the transformer ratings.

b) Find the other bases.

**Region 1**

$$I_{base1} := \frac{S_{base}}{\sqrt{3}\cdot V_{base1}}$$

$$I_{base1} = 5.774\cdot A$$

$$Z_{base1} := \frac{\left(\frac{V_{base1}}{\sqrt{3}}\right)}{I_{base1}} = \frac{V_{base1}^{\ 2}}{S_{base}} = 180\cdot\Omega$$

**Region 2**

$$I_{base2} := \frac{S_{base}}{\sqrt{3}\cdot V_{base2}}$$

$$I_{base2} = 2.598\cdot A$$

$$Z_{base2} := \frac{V_{base2}^{\ 2}}{S_{base}}$$

$$Z_{base2} = 888.889\cdot\Omega$$

**Region 3**

$$I_{base3} := \frac{S_{base}}{\sqrt{3}\cdot V_{base3}}$$

$$I_{base3} = 20.785\cdot A$$

$$Z_{base3} := \frac{V_{base3}^{\ 2}}{S_{base}}$$

$$Z_{base3} = 13.889\cdot\Omega$$

c) Find all the impedances in per-unit form. Make base changes as necessary.

**No changes necessary**

$$Z_{pu.G} = 0.015j\cdot pu$$

$$Z_{pu.T1} = 0.1j\cdot pu$$

**Transmission line**

$$Z_{pu.TL} := \frac{\left(0.3\cdot\frac{\Omega}{km} + 0.8\cdot j\cdot\frac{\Omega}{km}\right)\cdot len}{Z_{base2}}$$

$$Z_{pu.TL} = 1.687 + 4.5j \ \cdot\%$$

**Transformer 2**

$$S_{rated\_T2} := 15\cdot kVA$$

$$Z_{pu.T2} := 0.09j\cdot\frac{S_{base}}{S_{rated\_T2}}$$

$$Z_{pu.T2} = 0.108j\cdot pu$$

**Motor**

$$S_{rated\_M} := 12\cdot kVA$$

$$Z_{pu.M} = 0.958 + 0.2j \ \cdot pu$$

$$Z_{pu.m} := Z_{pu.M}\cdot\frac{S_{base}\cdot(600\cdot V)^2}{S_{rated\_M}\cdot\left(V_{base3}\right)^2}$$

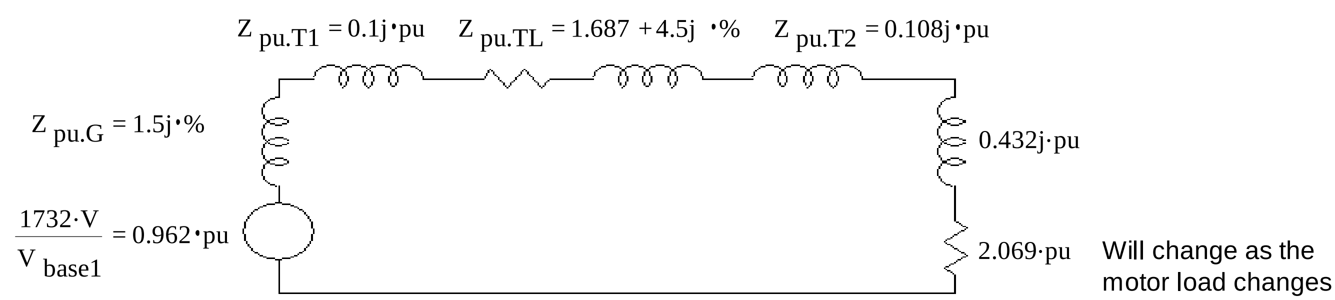
$$Z_{pu.m} = 2.069 + 0.432j \ \cdot pu$$

at full load

ALL calculations made to this point **ONLY need to be made ONCE!!**

With the exception of the equivalent resistance of the motor. That will depend on the mechanical load placed on the motor.

d) Make a per-phase drawing that could be used to make current calculations.




e) Calculate the currents and motor power when the motor in under full load.

The per-unit current: $$I_{pu} := \frac{0.962\cdot pu}{\left[\sqrt{(0.017 + 2.069)^2 + (0.015 + 0.1 + 0.045 + 0.108 + 0.432)^2}\right]\cdot pu} \qquad I_{pu} = 0.437\cdot pu$$

**Region 1**

$$I_{L1} = I_{pu}\cdot I_{base1} = 2.524\cdot A$$

**Region 2**

$$I_{L2} = I_{pu}\cdot I_{base2} = 1.136\cdot A$$

**Region 3**

$$I_{L3} = I_{pu}\cdot I_{base3} = 9.087\cdot A$$

Motor Power $$P_{Mpu} := I_{pu}^{\ 2}\cdot 2.069 \qquad P_{Mpu} = 0.395\cdot pu$$

$$P_M := P_{Mpu}\cdot S_{base} \qquad P_M = 7.119\cdot kW$$

A little less than the full load of 12kW because the voltage is a little low.

Notice that once the bases and per-unit impedances are established, the actual calculations are very easy!

Note: In these notes, I have retained "pu" in the subscripts of my variables as well as using "pu" or % as a unit of the number. If this seems redundant to you, it is. Most texts drop the "pu" in the variable subscripts.

Name ____________________________ ECE 3600 homework # 10 Due: W 2/15 d

1. A 3-phase system operates at $200\ \text{kVA}$ and $10\ \text{kV}$. Using these quantities as base values, find:

   a) The base current and base impedance for the system.

   Use these bases below:

   b) Express the following as a per-unit values

   $V_L := 8 \cdot \text{kV}$

   $I_L := 12 \cdot \text{A}$

   $\mathbf{I_L} := (5 + 2j) \cdot \text{A}$

   $P := 40 \cdot \text{kW}$

   $Q_{1\phi} := 20 \cdot \text{kVAR}$

   $\mathbf{Z} := 1.2 \cdot \text{k}\Omega \cdot e^{-j \cdot 10 \cdot \text{deg}}$

   c) The line voltage represented by $V_{pu} := 0.98 \cdot \text{pu}$

   d) The line-to-neutral voltage represented by $V_{pu} := 1.04 \cdot \text{pu}$

   e) The real power represented by $P_{pu} := 0.3 \cdot \text{pu}$

   f) The single-phase power represented by $P_{pu} := 0.3 \cdot \text{pu}$

   g) The single-phase reactive power represented by $\mathbf{S_{pu}} := 0.4 \cdot e^{j \cdot 14 \cdot \text{deg}} \cdot \text{pu}$

   h) The line current represented by $\mathbf{I_{pu}} := (0.5 + 0.2j) \cdot \text{pu}$

   i) The impedance represented by $\mathbf{Z_{pu}} := 2.8 \cdot \text{pu} \cdot e^{j \cdot 24 \cdot \text{deg}}$

2. If $26.1\ \Omega$ is the impedance base and $124\ \text{A}$ is the current base for a 3-phase system, find the power base and voltage base.

3. A 3-phase transmission line supplies a reactive load at a lagging power factor. The load draws $0.5\ \text{pu}$ current at $1.1\ \text{pu}$ voltage while using $0.8\ \text{pu}$ real power. If the base voltage is $20\ \text{kV}$ and the base current is $16\ \text{A}$, calculate the power factor and the values of the resistance and reactance of the load. Give both the pu and $\Omega$.

**Answers** 1. a) $11.6 \cdot \text{A}$ $500 \cdot \Omega$

b) $0.8 \cdot \text{pu}$ $1.039 \cdot \text{pu}$ $(0.433 + 0.173 \cdot j) \cdot \text{pu}$ $0.2 \cdot \text{pu}$ $0.3 \cdot \text{pu}$ $(2.364 - 0.417 \cdot j) \cdot \text{pu}$ c) $9.8 \cdot \text{kV}$ d) $6 \cdot \text{kV}$

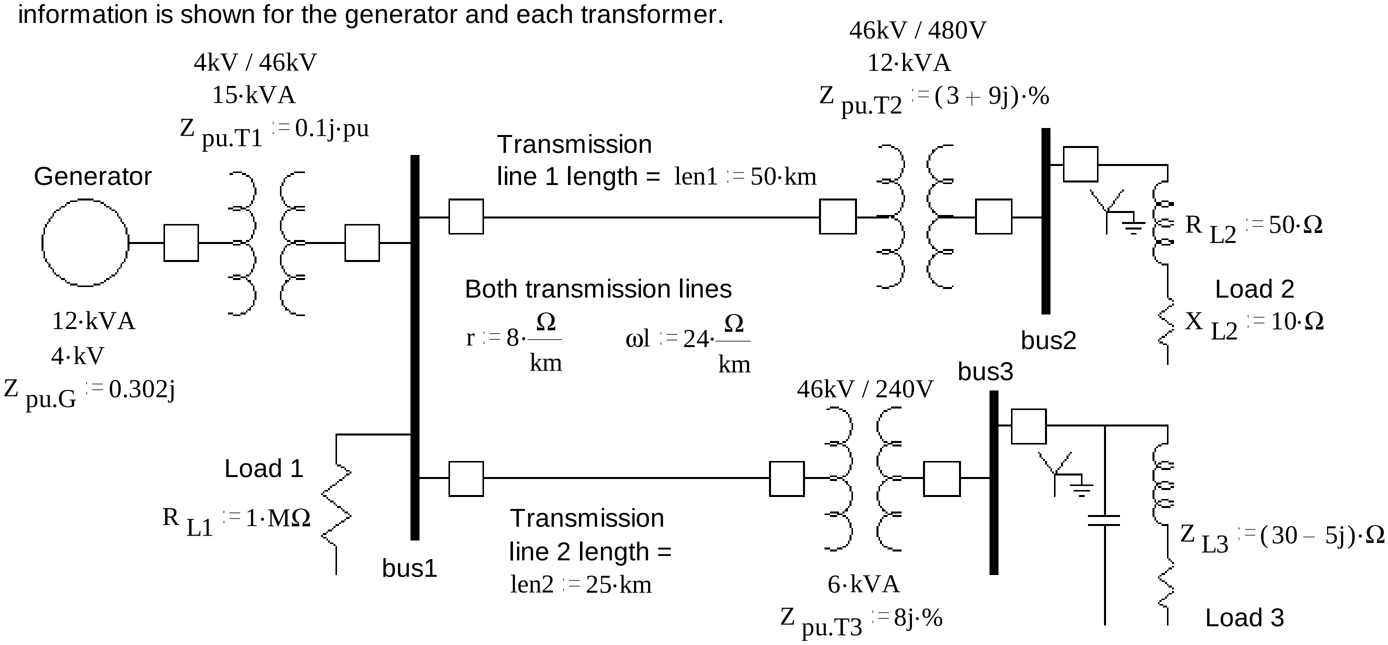
e) $60 \cdot \text{kW}$ f) $20 \cdot \text{kW}$ g) $6.45 \cdot \text{kVAR}$ h) $(5.77 + 2.31j) \cdot \text{A}$ i) $(1.28 + 0.57j) \cdot \text{k}\Omega$ 2. $1.2 \cdot \text{MVA}$ $5.61 \cdot \text{kV}$

3. $84 \cdot \%$ If R & X are in parallel $1.513 \cdot \text{pu}$ $1.092 \cdot \text{k}\Omega$ $2.34 \cdot \text{pu}$ $1.688 \cdot \text{k}\Omega$

If R & X are in series $3.2 \cdot \text{pu}$ $2.309 \cdot \text{k}\Omega$ $2.069 \cdot \text{pu}$ $1.493 \cdot \text{k}\Omega$

Name ______________________________ ECE 3600 homework # 11 Due: Sat. , 2/18/23 d

1. A one-line diagram of a 3ϕ system is shown below. Manufacturer's information is shown for the generator and each transformer.




a) Choose an $S_{base}$ to minimize the per-unit base conversions. Then choose regions and a $V_{base}$ for each region.

b) Find $I_{base}$ and $Z_{base}$ in each of the regions.

c) Make the necessary per-unit $S_{base}$ conversions.

d) Find the impedances of the two transmission lines and convert to pu.

e) Draw the per-phase diagram on separate paper, showing all the per-unit numbers found or given so far.

ALL calculations made to this point **ONLY need to be made ONCE** for this system and $S_{base}$ **!!**

f) Find the pu values of the 3 loads and add that information to the per-phase diagram.

g) The line voltage at bus1 is measured and found to be $V_{bus1} := 46.00 \cdot kV$ Assume the phase angle is $0^{o}$.

Find all 3 load line-current magnitudes and the magnitude of the generator line-current. Please remember that you can't add magnitudes, so may need some complex values.

h) Find the power delivered to Load 2, both in pu and in kW.

i) Find the line voltage at Load 2 (magnitude).

j) Find the line voltage at the generator (magnitude).

k) The line voltage at the generator drops by 10% to: $3.688 \cdot kV$

Find the magnitude of Load-3 line current and repeat parts h) and i) for this new generator voltage.

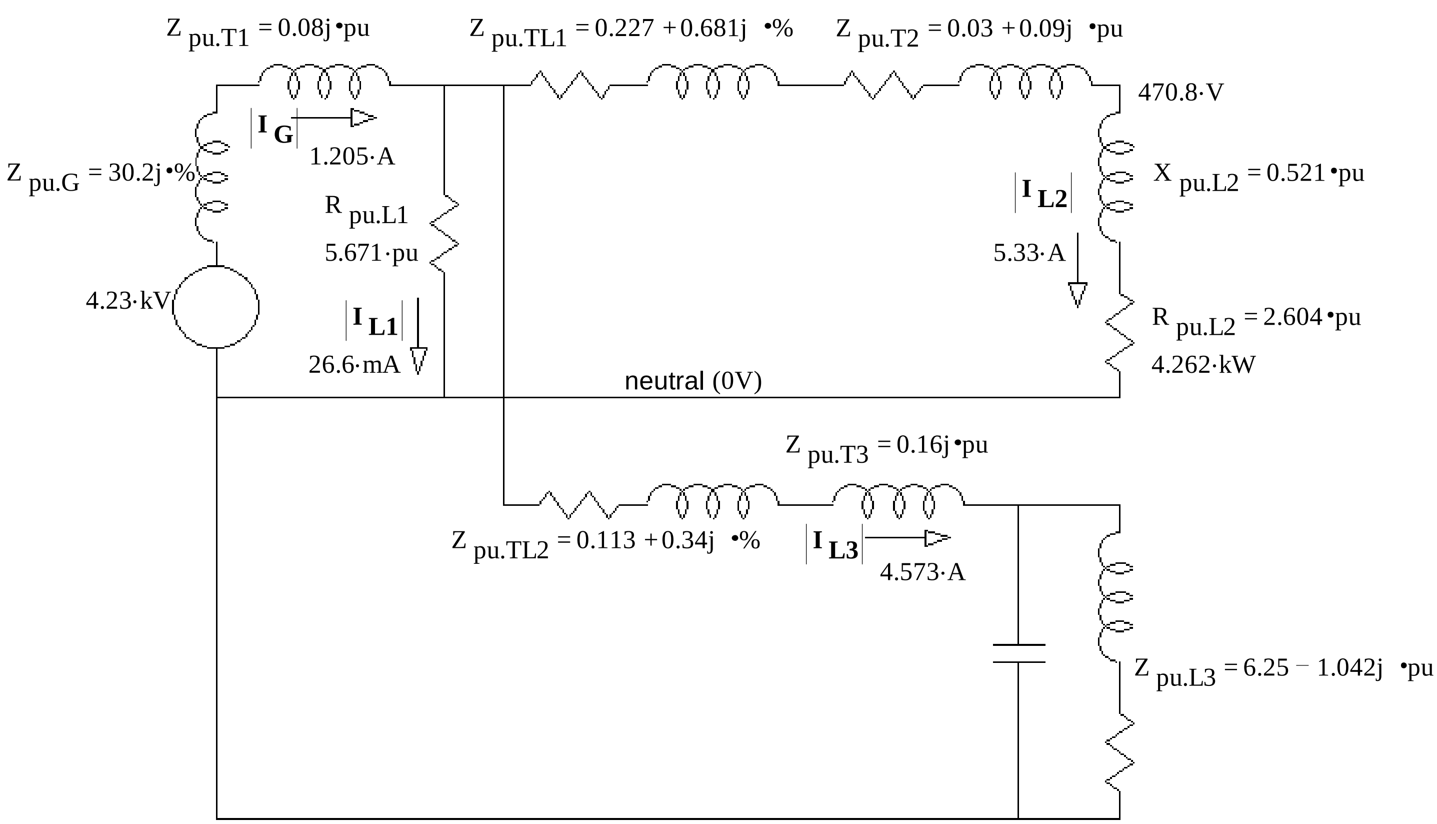
Note: It may be helpful to realize that if one voltage in the system drops by 10%, so do all the rest, and so do all the currents. Drop by 10% means multiply by 0.9. All powers drop too, but use $(0.9)^2$ as the factor.

**Answers**

1. a) $12 \cdot kVA$ $4 \cdot kV$ $46 \cdot kV$ etc

   b) $1.732 \cdot A$ $1.333 \cdot k\Omega$ $0.151 \cdot A$ $176.3 \cdot k\Omega$ etc

   c) through j) see drawing




   k) $4.1 \cdot A$ $3.452 \cdot kW$ $423.7 \cdot V$