## Electromagnetics basics
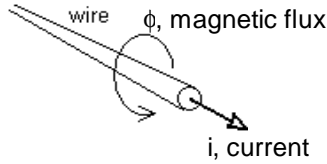
1. Electric currents produce magnetic fields.

wire    $\phi$, magnetic flux
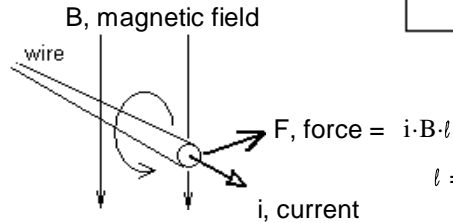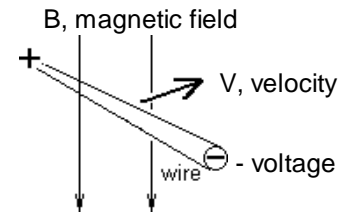
i, current

Right-hand-rule
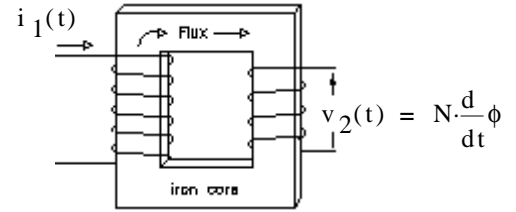
2. A fluctuating magnetic field passing through a coil of wire will induce a voltage in that coil.  Basis of transformer secondary, and primary too (back EMF).

$i_1(t)$     Flux ⟶

$v_2(t) = N \cdot \dfrac{d}{dt} \phi$

iron core

3. A wire with a current in the presence of another magnetic field feels a force.

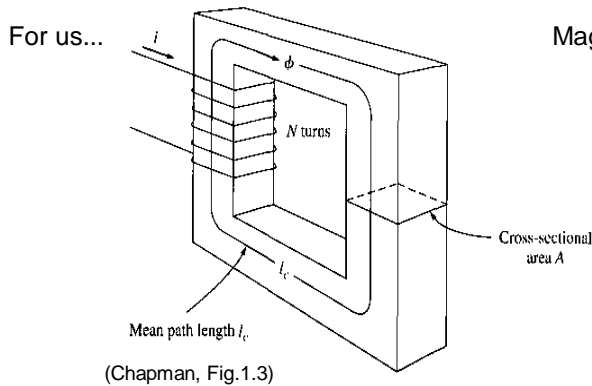(Basis of electric motors, also explains why generators resist the mechanical input)

B, magnetic field

wire

F, force = $i \cdot B \cdot \ell$

$\ell$ = length of the wire

i, current

4. A voltage will be induced on wires moving in the presence of a magnetic field. This is very similar to 2.     (Basis of electric generators)

B, magnetic field

V, velocity

wire ⊖ - voltage

## 1. Magnetic field from a current

Ampere's law:      $\oint \mathbf{H} \cdot \mathbf{dl} = I_{net}$

For us...

$i$

$\phi$

$N$ turns

Cross-sectional area $A$

$l_c$

Mean path length $l_c$

(Chapman, Fig.1.3)

Magnetic field intensity:    $H = \dfrac{N \cdot i}{\ell_c}$    $\left( \dfrac{A \cdot turns}{meter} \right)$

Ampere-turns: $= N \cdot i$
(like voltage)

Flux density:    $B = \mu \cdot H = \dfrac{\mu \cdot N \cdot i}{\ell_c}$   (tesla, T)

Flux:       $\phi = B \cdot A$   (weber)  (Wb)
(like current)

Permeability of free space:   $\mu_o := 4 \cdot \pi \cdot 10^{-7} \cdot \dfrac{henry}{m}$

Relative permeability:   $\mu_r$

Permeability:     $\mu = \mu_r \cdot \mu_o$

Reluctance of core:   $\mathcal{R}_c = \dfrac{\ell_c}{\mu \cdot A_c}$    $\left( \dfrac{A \cdot turns}{Wb} \right)$
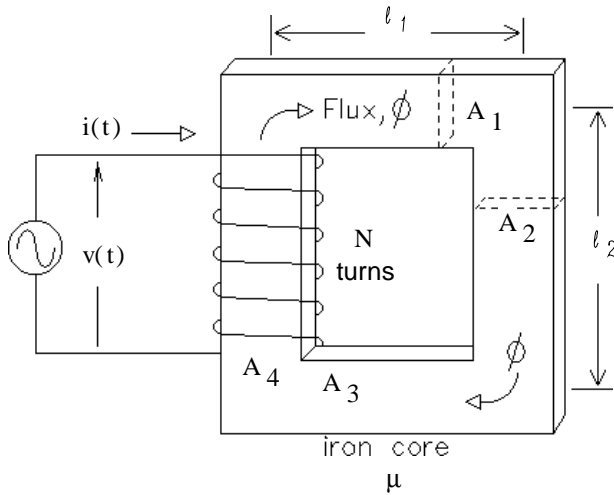(like resistance)

Flux:     $\phi = \dfrac{N \cdot i}{\mathcal{R}_c}$     (weber, Wb)

**Inductance:**    $L = \dfrac{N^2}{\mathcal{R}_c} = N^2 \cdot \left( \dfrac{\mu_r \cdot \mu_o \cdot A_c}{\ell_c} \right)$     (henry, H)
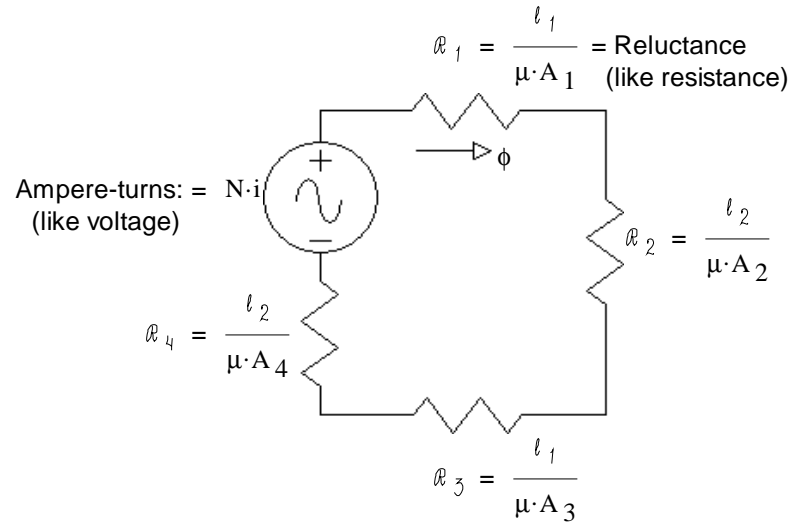
| material | relative permeability $\mu_r$ |
|---|---|
| Mu-metal | 20000 |
| Permalloy | 8000 |
| Electrical steel | 4000 |
| ferrite (nickel zinc) | 16 - 640 |
| ferrite (manganese zinc) | >640 |
| Steel | 700 |
| Nickel | 100 |

Magnetic "Circuits"

$\ell_1$

i(t) $\longrightarrow$

Flux, $\phi$    $A_1$

v(t)

N turns

$A_2$

$\ell_2$

$A_4$    $A_3$

$\phi$

iron core
$\mu$

Similar to this electric circuit

$\mathcal{R}_1 = \dfrac{\ell_1}{\mu \cdot A_1}$ = Reluctance (like resistance)

Ampere-turns: $= N \cdot i$
(like voltage)

$\phi$

$\mathcal{R}_2 = \dfrac{\ell_2}{\mu \cdot A_2}$

$\mathcal{R}_4 = \dfrac{\ell_2}{\mu \cdot A_4}$

$\mathcal{R}_3 = \dfrac{\ell_1}{\mu \cdot A_3}$

magnetic flux $= \phi = \dfrac{N \cdot i}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}$ (weber)
(like current)     (Wb)

i(t) $\longrightarrow$

$\phi$

N turns

Air gap

$\ell_3$

$\ell_A$

$\ell_4$

$\phi$

$\mathcal{R}_1 + \mathcal{R}_3 + \mathcal{R}_4$

$\phi$

$N \cdot i$

$\dfrac{\ell_3}{\mu \cdot A_2}$

$\mathcal{R}_A = \dfrac{\ell_A}{\mu_o \cdot A_A}$

$\dfrac{\ell_4}{\mu \cdot A_2}$
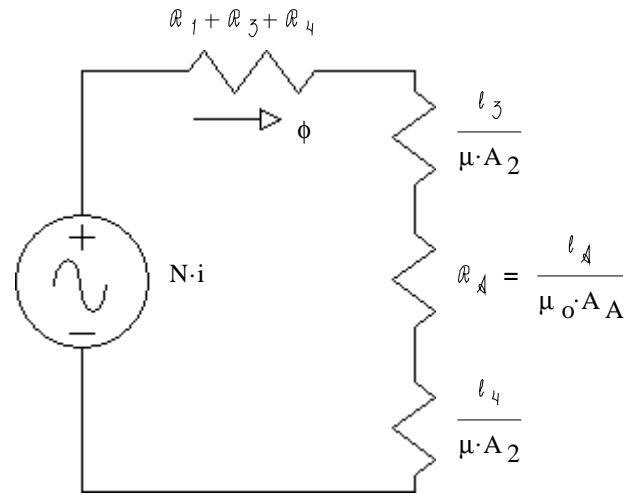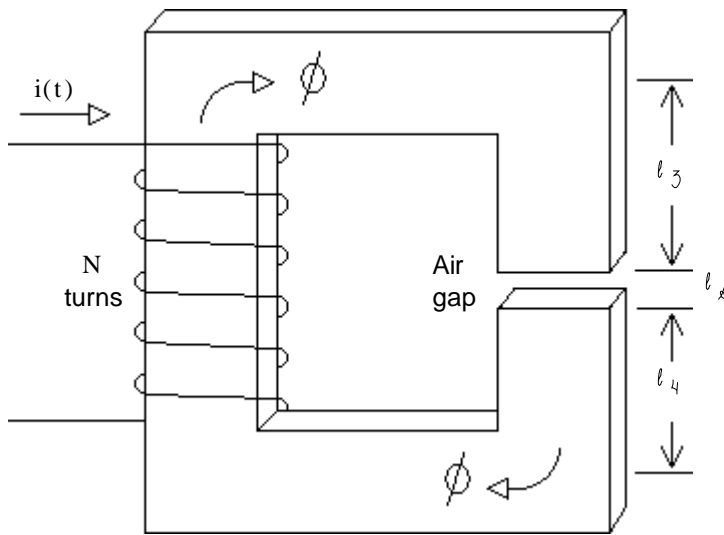
$\phi = \dfrac{N \cdot i}{\mathcal{R}_1 + \mathcal{R}_3 + \mathcal{R}_4 + \dfrac{\ell_3 + \ell_4}{\mu \cdot A_2} + \mathcal{R}_A}$  (weber) (Wb)

Flux density:    $B = \dfrac{\phi}{A} = \mu \cdot H$    (tesla, T)

Magnetic field intensity:    $H = \dfrac{B}{\mu} = \dfrac{\phi}{A \cdot \mu}$    $\left( \dfrac{A \cdot turns}{meter} \right)$

$v(t) = N \cdot \dfrac{d}{dt} \phi = N \cdot \dfrac{d}{dt} B \cdot A$

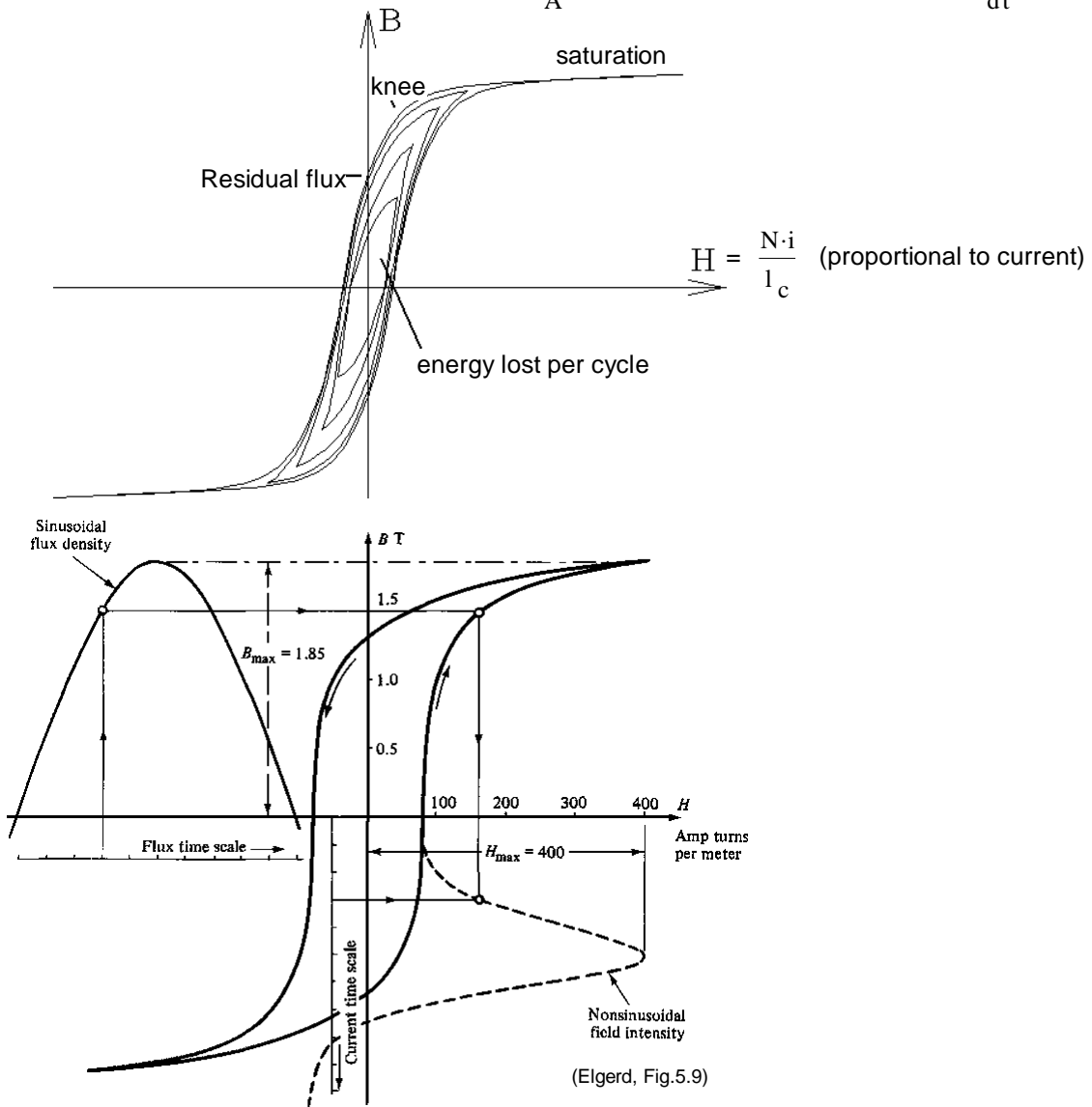$= -N \cdot \dfrac{d}{dt} \phi = -N \cdot \dfrac{d}{dt} B \cdot A$  often shown with a negative sign

- indicates that this voltage tries to
produce a current to oppose the change.

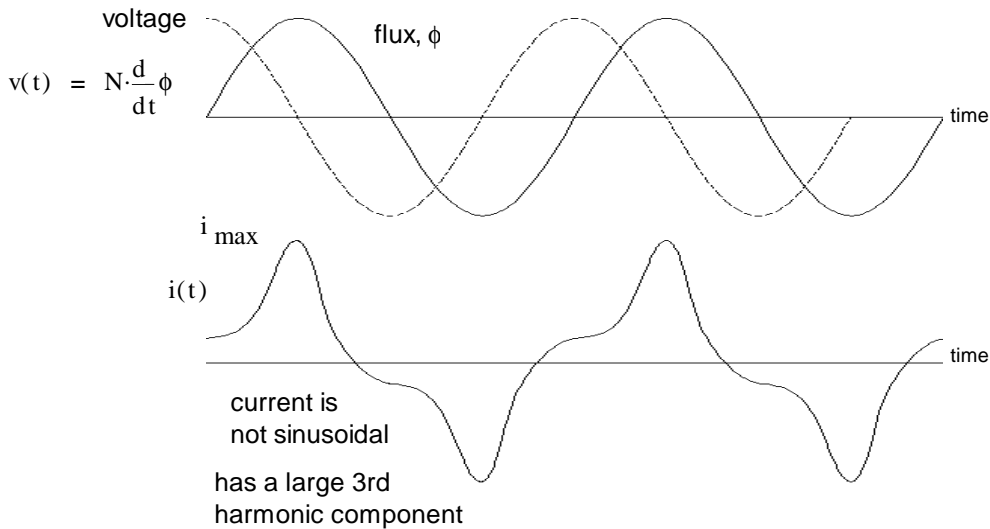## Non-ideal Ferrromagnetic materials (B-H curve)    Magnetics are not really linear

The B-H or Hystereses curve

$$B = \mu \cdot H = \frac{\phi}{A} \text{ (proportional to voltage)} \quad v(t) = N \cdot \frac{d}{dt}\phi = -N \cdot \frac{d}{dt}B \cdot A$$

B

saturation

knee

Residual flux

$$H = \frac{N \cdot i}{l_c} \text{ (proportional to current)}$$

energy lost per cycle

Sinusoidal
flux density

*B* T

1.5

$B_{max} = 1.85$

1.0

0.5

100   200   300   400   *H*

Flux time scale

Amp turns
per meter

$H_{max} = 400$

Current time scale

Nonsinusoidal
field intensity

(Elgerd, Fig.5.9)

voltage         flux, $\phi$

$$v(t) = N \cdot \frac{d}{dt}\phi$$

time

$i_{max}$

i(t)

time

current is
not sinusoidal

has a large 3rd
harmonic component

Sources:   Electric Machinery and Power System Fundamentals, Stephen J. Chapman
           Basic Electric Power Enineering, Ollie I. Elgerd

## Transformer basics and ratings

A Transformer is two coils of wire that are magnetically coupled.

Transformers are only useful for AC, which is one of the big reasons electrical power is generated and distributed as AC.

Transformer turns and turns ratios are rarely given, $V_p/V_s$ is much more common where $V_p/V_s$ is the rated primary over rated secondary voltages.  You may take this to be the same as $N_1/N_2$ although in reality $N_2$ is usually a little bit bigger to make up for losses.  Also common: $V_p : V_s$.

Both RMS

Transformers are rated in VA    Transformer Rating $(VA)$ = (rated V) x (rated I) , on either side.
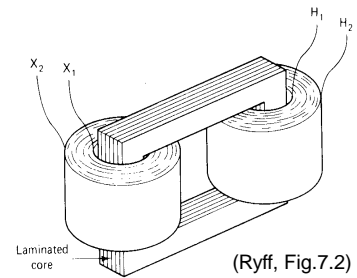
Don't allow voltages over the rated $V$ , regardless of the actual current.
Don't allow currents over the rated $I$ , regardless of the actual voltage.

## Ideal Transformers

Iron-core transformer
primary                secondary
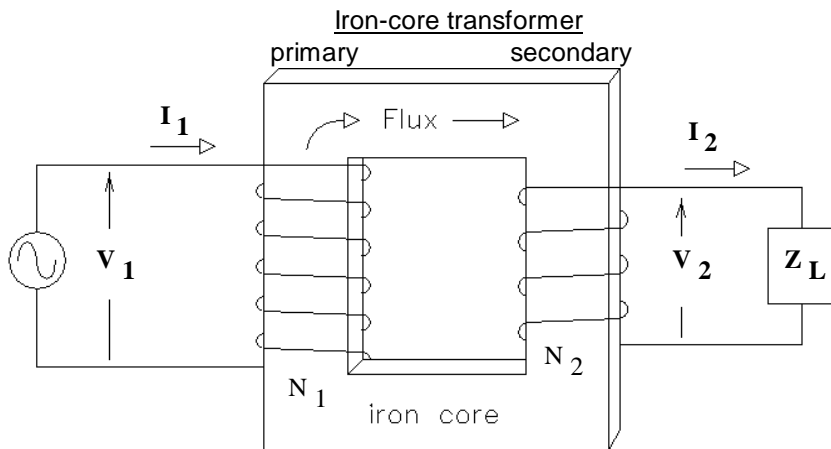


Ideal:        $P_1 = P_2$

power in = power out

(Ryff, Fig.7.2)

Transformation of voltage and current

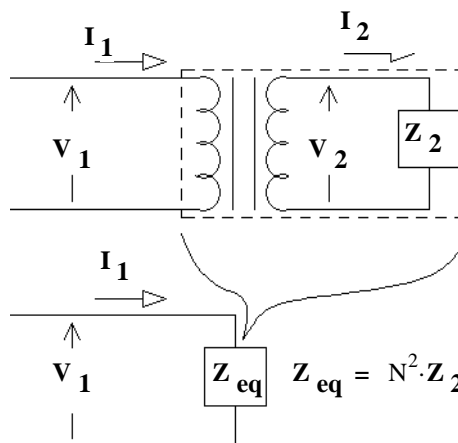$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

Turns ratio

Turns ratio as defined in Chapman text:  $a = \dfrac{N_1}{N_2}$ , same as  $N = \dfrac{N_1}{N_2}$        Note: some other texts define the turns ratio as:  $\dfrac{N_2}{N_1}$

Be careful how you and others use this term
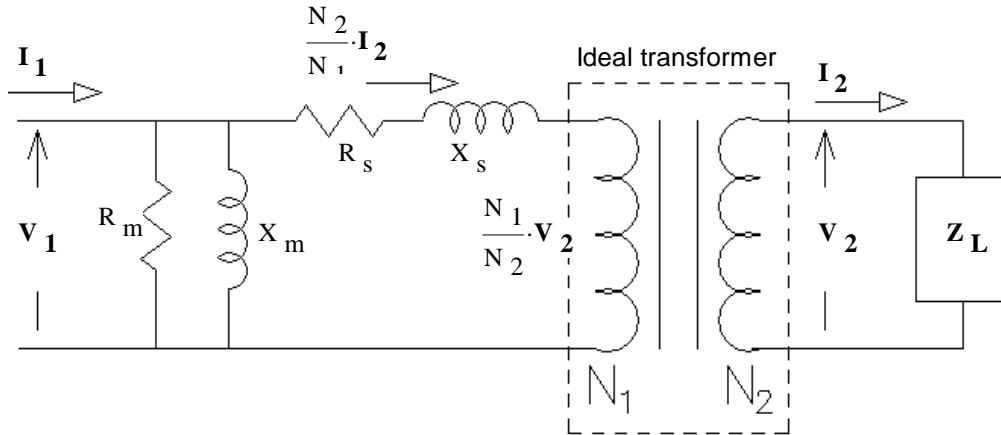
Transformation of impedance



You can replace the entire transformer and load with ($Z_{eq}$).  This "impedance transformation" can be very handy.

Transformers can be used for "impedance matching"

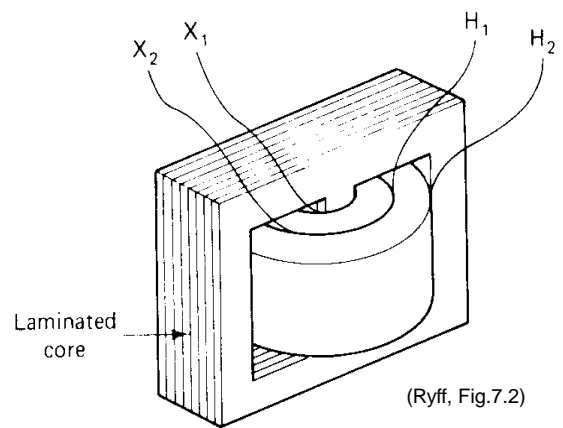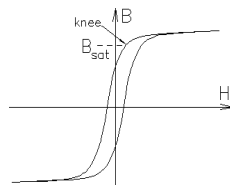$$Z_{eq} = N^2 \cdot Z_2 = \left(\frac{N_1}{N_2}\right)^2 \cdot Z_2$$

This also works the opposite way, to move an impedance from the primary to the secondary, multiply by: $\left(\dfrac{N_2}{N_1}\right)^2$

**ECE 3600    Transformer notes  p1**

$R_m$ - Core losses

Eddy-current losses - minimized by laminating the core
and adding silicon to raise the resistivity

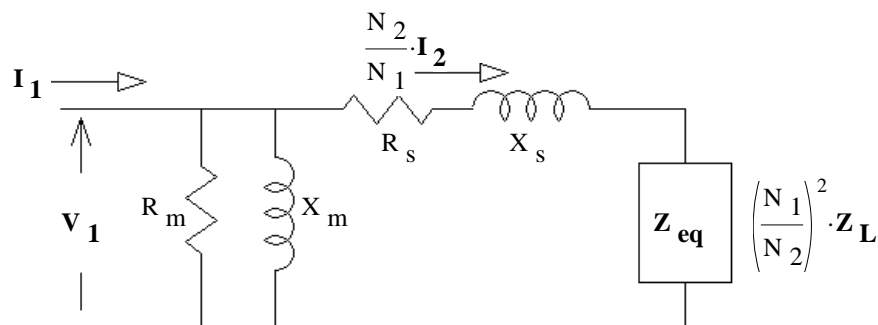Hysteresis losses - caused by the B-H hysteresis curve





(Ryff, Fig.7.2)

$X_m$ - basic inductance caused by the need to magnetize the core

$R_s$ - Winding resistance (copper) losses

$X_s$ - Reactance caused by flux leakage (leakage reactance)

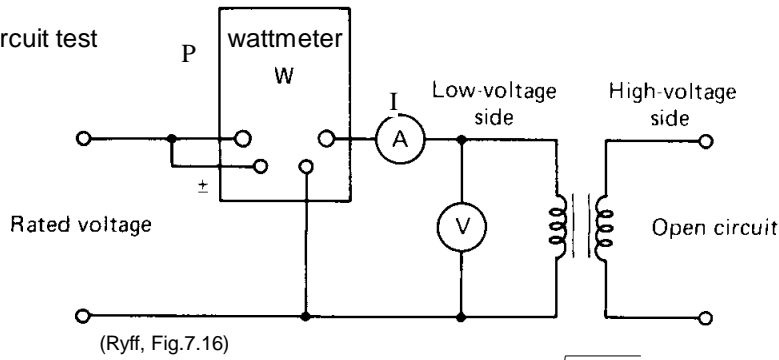Move the load impedance to simplification the  analysis



Typical calculations

$$\text{Voltage regulation} \quad \%VR \;=\; \frac{V_{no\_load} - V_{full\_load}}{V_{full\_load}} \cdot 100 \cdot \%$$

$$\text{Efficiency} \qquad \eta \;=\; \frac{P_{out}}{P_{in}} \cdot 100 \cdot \%$$
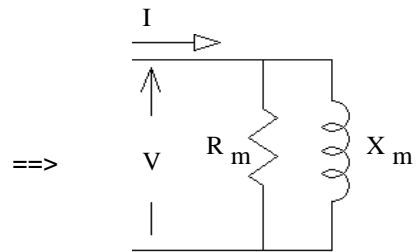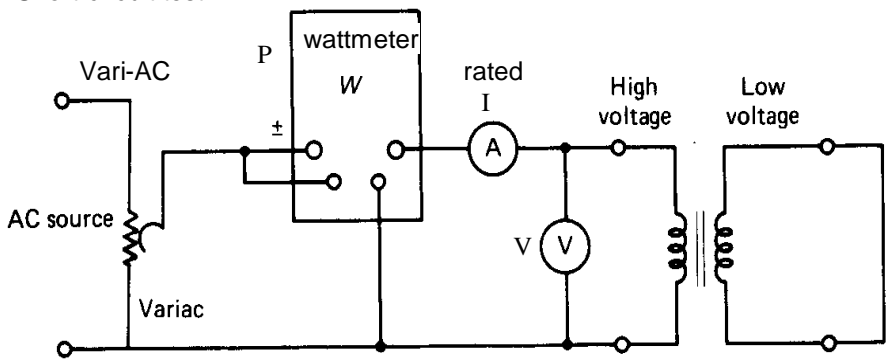
Open-circuit test



(Ryff, Fig.7.16)

$$S = I \cdot V \qquad Q = \sqrt{S^2 - P^2}$$

V = rated voltage

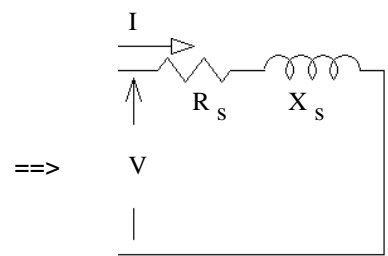$$R_m = \frac{V^2}{P} \qquad X_m = \frac{V^2}{Q}$$

Short-circuit test



(Ryff, Fig.7.17)

$$S = I \cdot V \qquad Q = \sqrt{S^2 - P^2}$$

I = rated current

$$R_s = \frac{P}{I^2} \qquad X_s = \frac{Q}{I^2}$$

## Other Transformers

Multi-tap transformers:  Many transformers have more than two connections to primary and/or the secondary. The extra connections are called "taps" and may allow you to select from several different voltages or get more than one voltage at the same time.
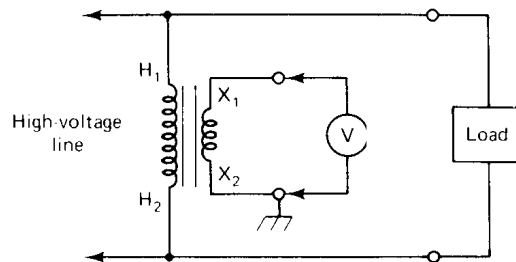
Isolation Transformers:  All transformers but auto transformers isolate the primary from the secondary. An Isolation transformer has a 1:1 turns ratio and is just for isolation.

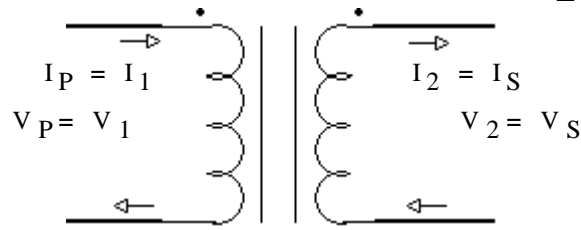Special Sensing Transformers
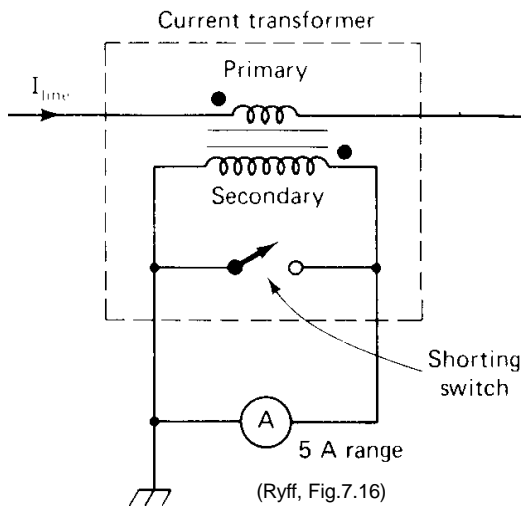


(Ryff, Fig.7.16)

CT   The secondary **must** always be shorted or nearly shorted!
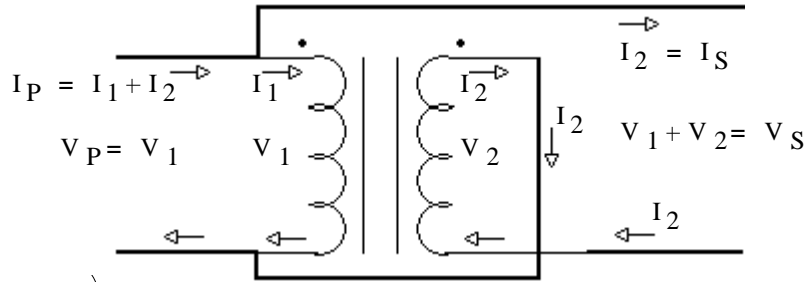


(Ryff, Fig.7.17)

"Potential" transformer,
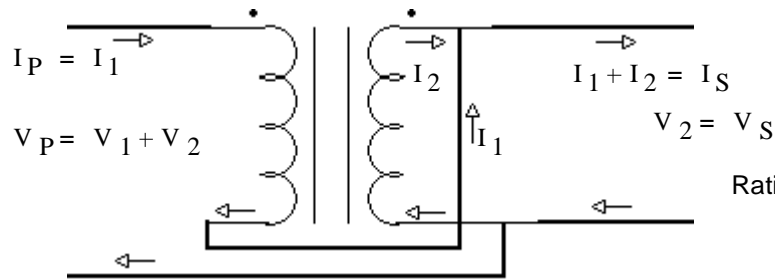for voltage monitoring

VT or PT

Regular winding connections

$I_P = I_1$

$V_P = V_1$

$I_2 = I_S$

$V_2 = V_S$

## Auto Transformer Connections          4 basic possibilities

$I_P = I_1 + I_2$          $I_1$          $I_2$          $I_2 = I_S$

$V_P = V_1$          $V_1$          $V_2$          $I_2$          $V_1 + V_2 = V_S$

$I_2$

Rating:

$\left(I_{1\_rated} + I_{2\_rated}\right) \cdot V_P$

$I_P = I_1$          $I_2$          $I_1 + I_2 = I_S$

$V_P = V_1 + V_2$          $I_1$          $V_2 = V_S$
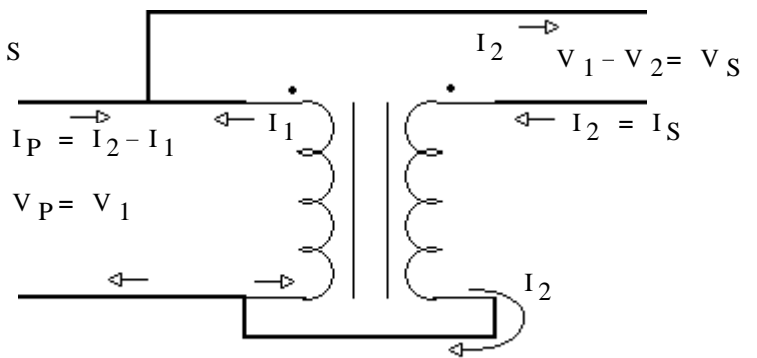
Rating:  $\left(I_{1\_rated} + I_{2\_rated}\right) \cdot V_S$

Subtraction connections                                        Actual currents are in the reverse directions

$I_2$          $V_1 - V_2 = V_S$                                        $I_2$          $V_1 - V_2 = V_S$

$I_P = I_2 - I_1$          $I_1$          $I_2$                    $I_P = I_2 - I_1$          $I_1$          $I_2 = I_S$

$V_P = V_1$          $I_2 = I_S$          $V_P = V_1$

$I_2$                                        $I_2$
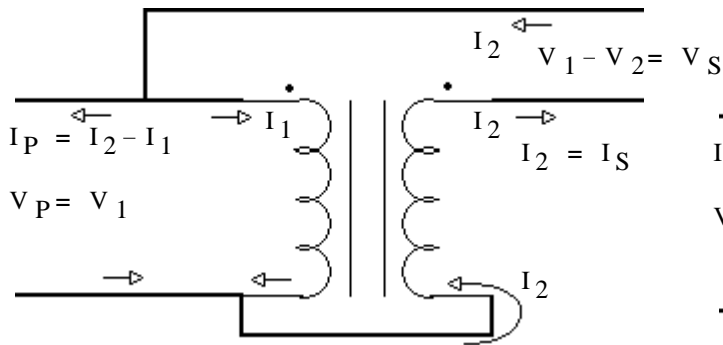
Rating:  $\left(I_{2\_rated} - I_{1\_rated}\right) \cdot V_P$          Power Flow Through Transformer

$I_1$          $I_2$          $I_2 - I_1 = I_S$

$V_P = V_1 - V_2$          $I_1$          $V_2 = V_S$

$I_1$
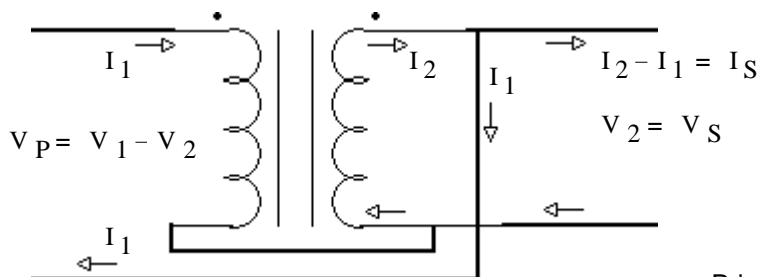
Rating:  $\left(I_{2\_rated} - I_{1\_rated}\right) \cdot V_S$

Primary and Secondary could be
swapped on any of these connections
for an additional 4 possibilities

**ECE 3600    Transformer  notes  p4**

Vari-AC type autotransformer

This adjustable autotransformer is wound on a toroidal core.

$V_P$

moveable tap

$V_S$

showing only a few windings

Side view,

A

B

Top view

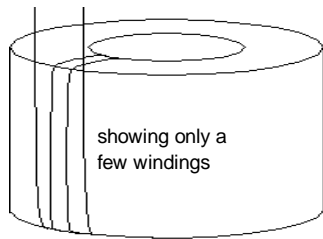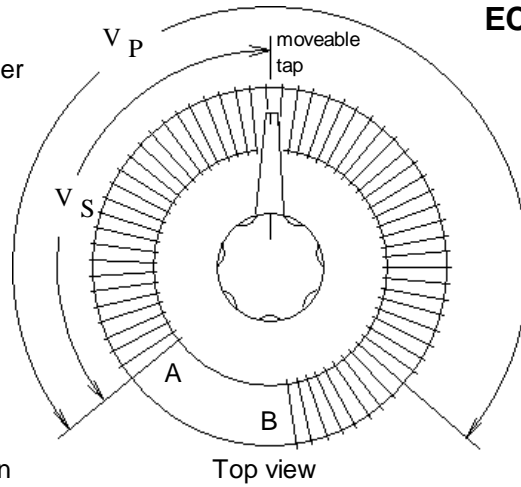If you cut the toroid open and straightened it out, you would get the views below.

step down

step up

$I_P = I_2$

moveable tap

B

$I_2$

$I_P$

$V_2$

B

$I_1 = I_P$

$I_S$

moveable tap

$V_1$

$I_1$

$V_S$

$V_P$

$V_P = V_1$

$I_2$

$V_2 = V_S$

A

A
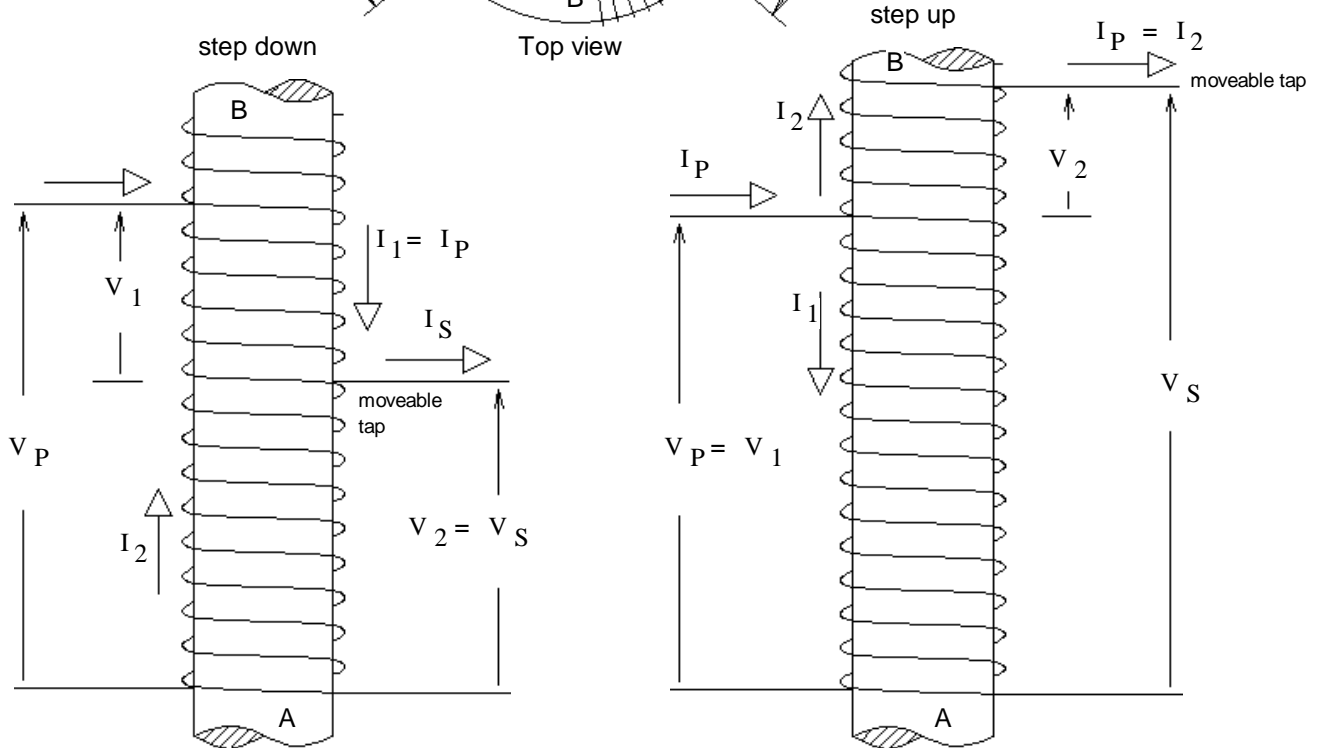
Vari-AC type autotransformer "Rating" ,   Based or the maximum winding current:  $I_{max}$

$I_2 = I_{max}$     $I_1 = I_{max}$          $I_2 = I_{max}$          $I_1 = I_{max}$

$\dfrac{S_{max}(V_S)}{V_P \cdot I_{max}}$

$\dfrac{V_P \cdot V_S}{\left| V_P - V_S \right|}$

$V_P \cdot I_{max}$

$V_S \cdot I_{max}$

$\dfrac{V_P \cdot V_S}{\left| V_P - V_S \right|}$

step down <-- | --> step up

$\dfrac{V_S}{V_P}$
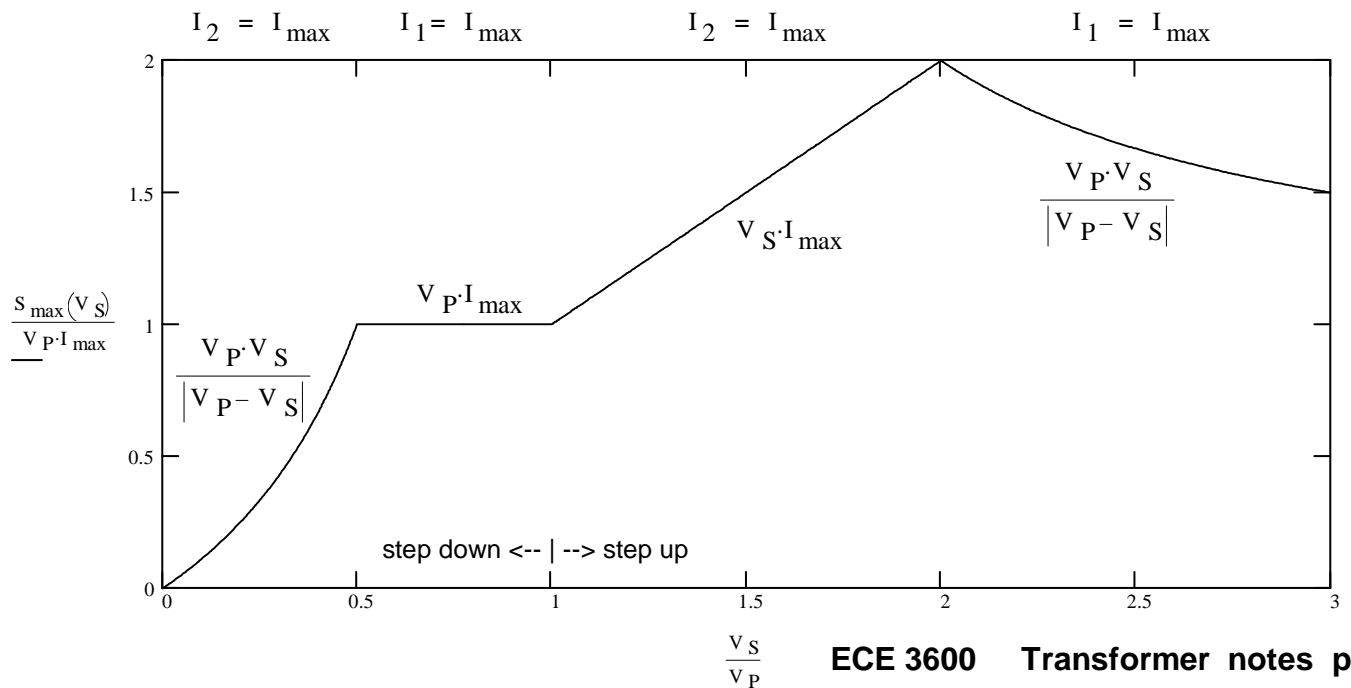
# 3-phase Transformer Connections

Y - Y Connection

Multiple cores



(Elgerd, Fig.5.18)
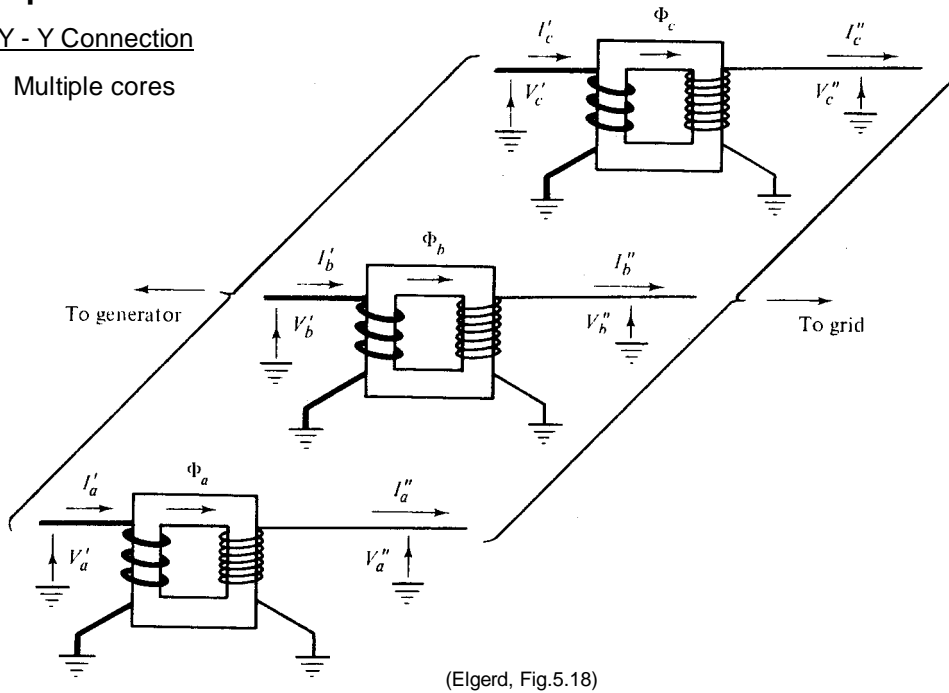
Symbol

Single-core (3-phase) transformer

Advantages: cheaper, less core loss

Disadvantage:  Individual transformers can limp along even if one winding fails, this cannot.
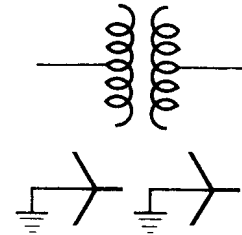


(Elgerd, Fig.5.20)



(Ryff, Fig.7.18)
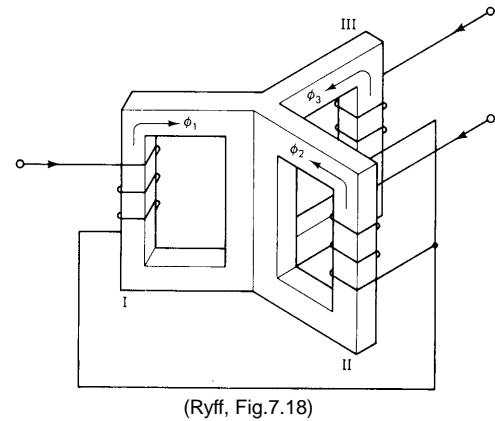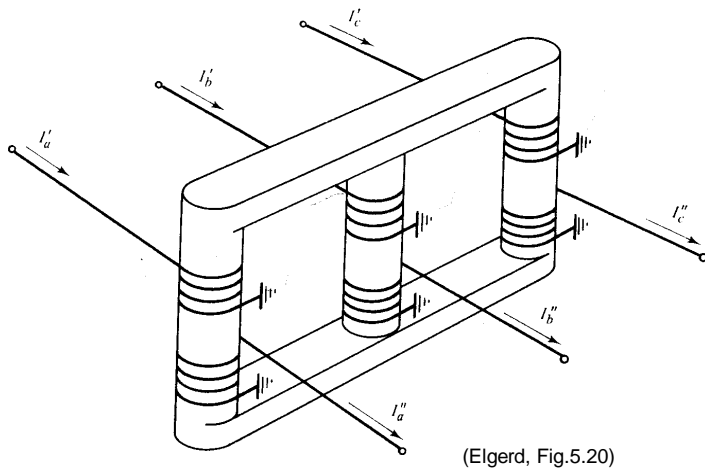
Third-harmonic currents (due to B - H non-linearity) add up to a significant neutral current.

Any Δ winding will allow the third-harmonic current to flow in a loop.



60 cps in phase $a$

60 cps in phase $b$

60 cps in phase $c$

180 cps in $a$, $b$ and $c$

time

(Elgerd, Fig.5.21)

To
22 kV
generator

To
340 kV
grid
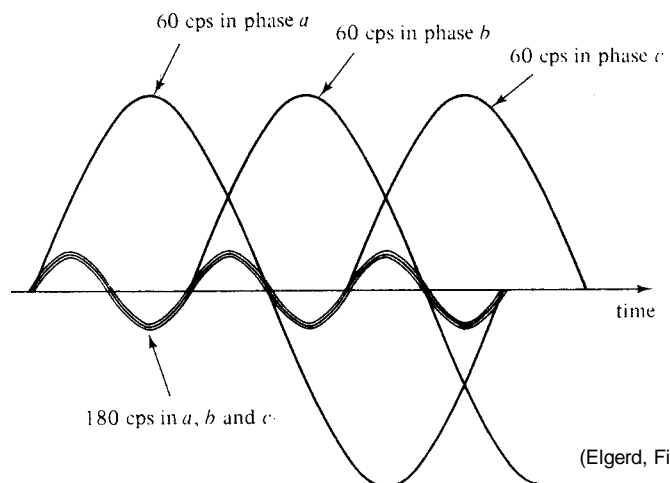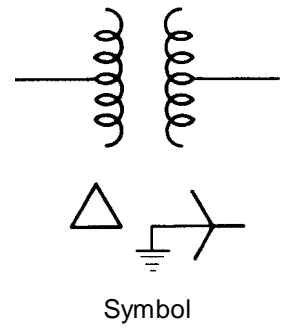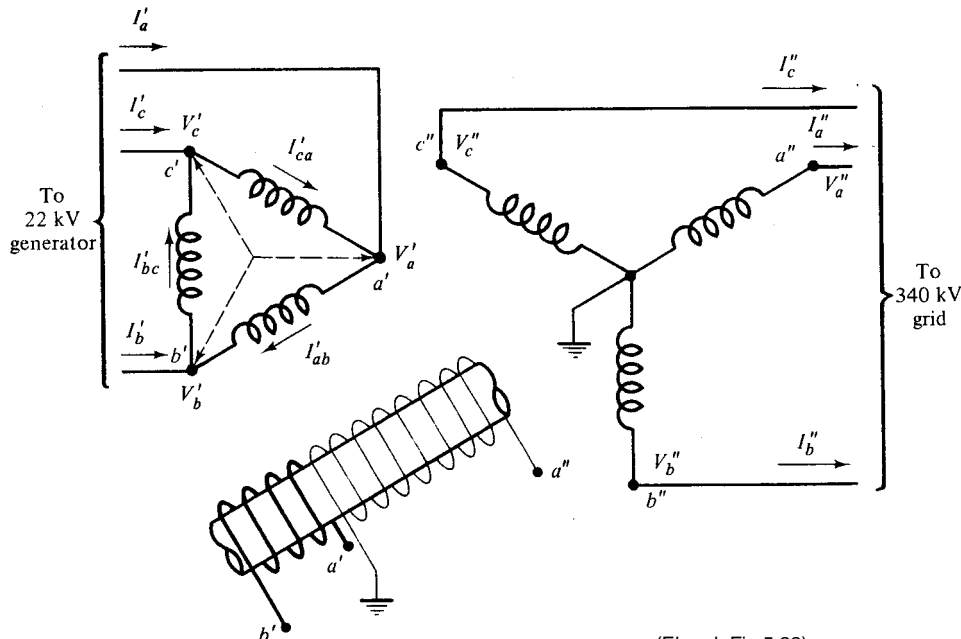
Symbol

Δ - Y is usually step-up

Y - Δ is usually step-down

(Elgerd, Fig.5.22)

Δ - Y and  Δ -Y always introduce a 30⁰ phase shift, + or - depending on how they are wired.



Alt 1

Δ - Y is usually step-up

30° phase shift

Alt 2

Different
connections

- 30⁰
phase
shift

30°

30°
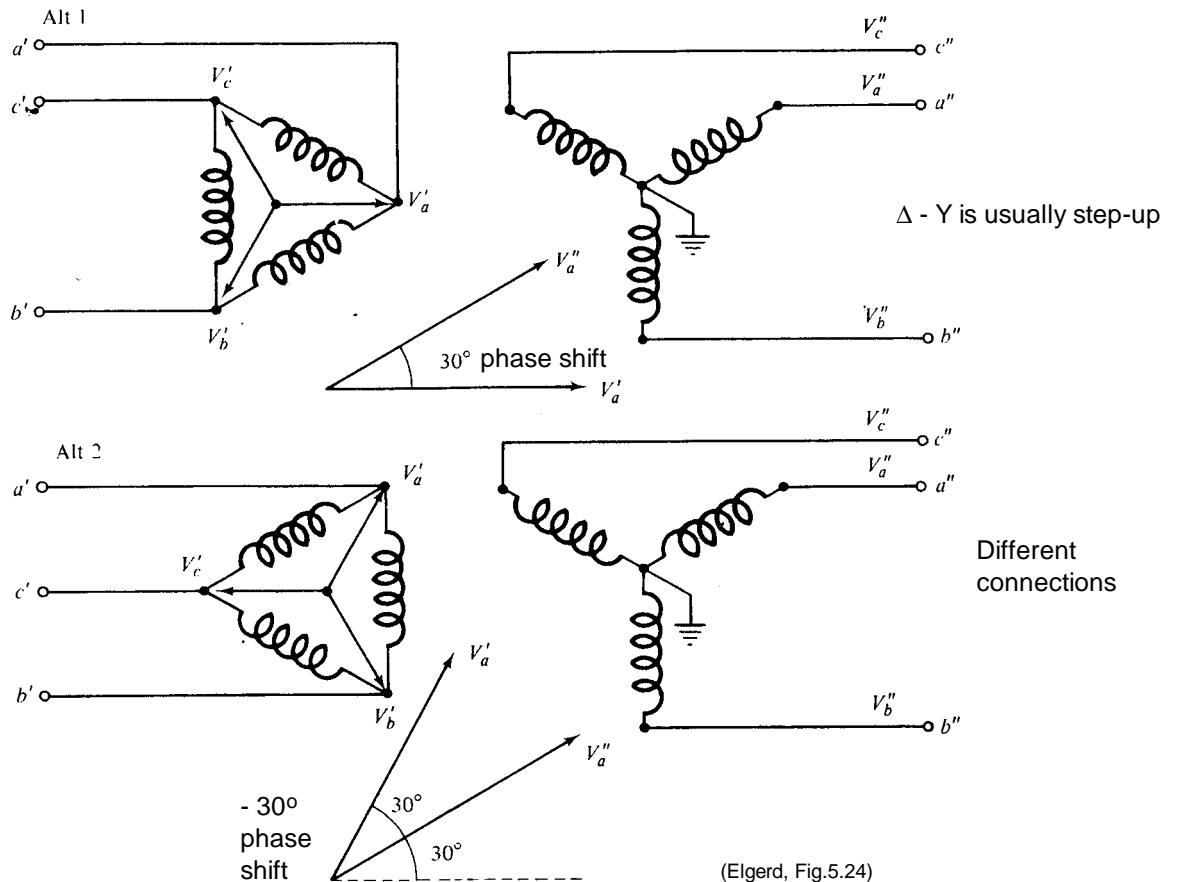
(Elgerd, Fig.5.24)

Y - Δ  (usually step-down) would also give a -30⁰ phase shift

Single-core 3-phase transformers can also be used this way

$V_{an}$

$V_{an} + \Delta V_{an}$

$\Delta V_{an}$

Windings drawn
in parallel are on
the same core

Used to control the direction
of power flow on the network

$\Delta V_{an}$   $a$

Shifted  $V_{an}$ — Original  $V_{an}$

$n$

$V_{bn}$

$V_{bn} + \Delta V_{bn}$

$\Delta V_{bn}$

$\Delta V_{bn}$

$V_{cn}$

$V_{cn} + \Delta V_{cn}$

$c$

$b$

$\Delta V_{cn}$

$\rightarrow |\Delta V_{cn}| \leftarrow$

(Stevenson, Fig.9.15)

(Stevenson, Fig.9.14)
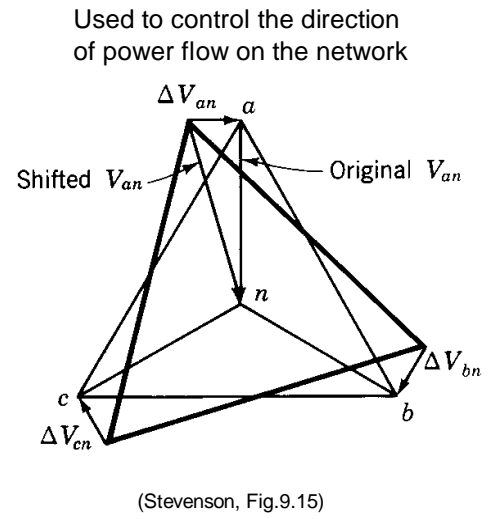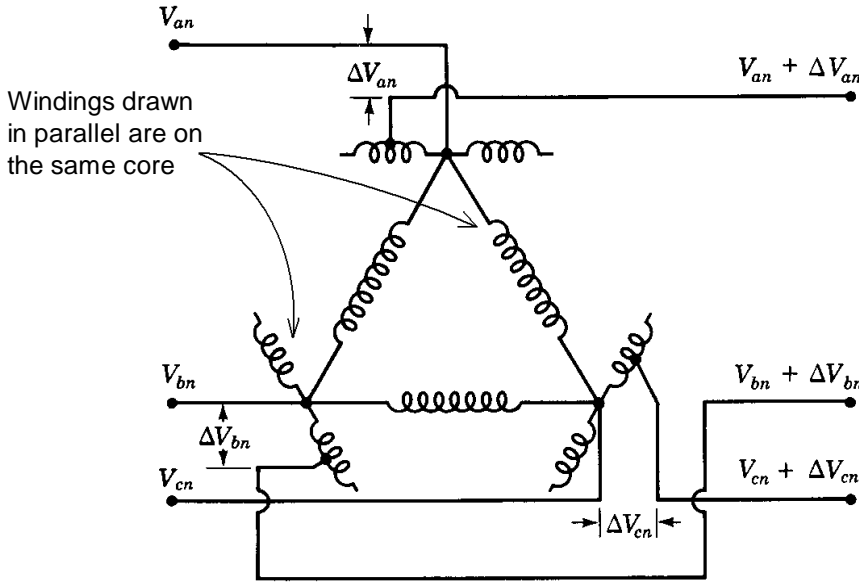
Voltage Regulating Transformers

$\leftarrow \Delta V_{an} \rightarrow$

$a$   $V_{an}$

$V_{an} + \Delta V_{an}$

$b$

$c$

Series
transformers

$n$

(Stevenson, Fig.9.13)

Off-Nominal Turns Ratio

Note the weird $I_2$ direction

$I_1$

$I_2$

$Z_s$

$V_1$

$1 : t$

$V_2$
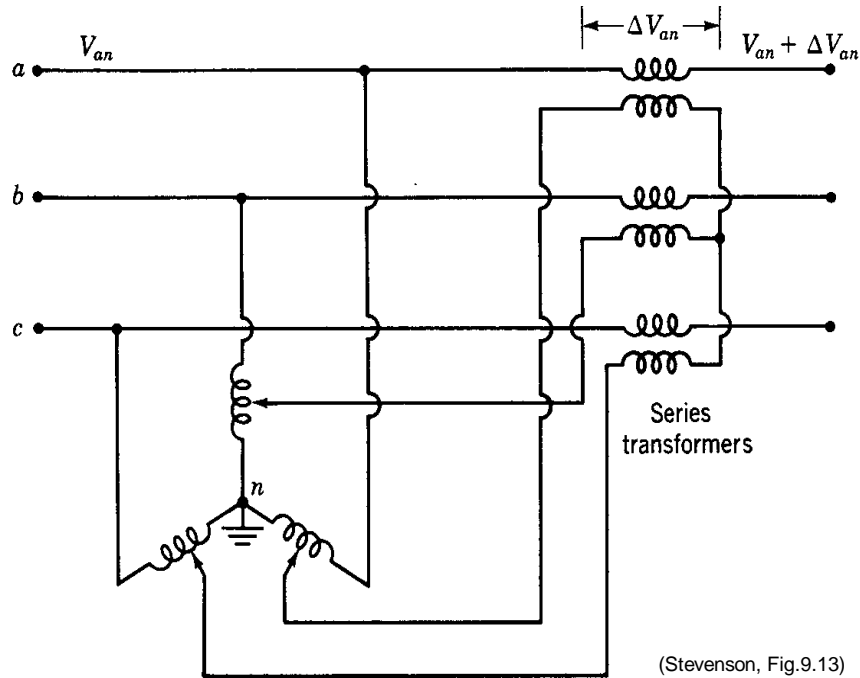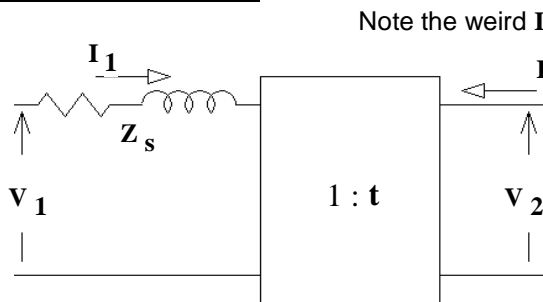
$$Z_s = \frac{1}{Y_s}$$

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{bmatrix} Y_s & -\dfrac{Y_s}{t} \\ -\dfrac{Y_s}{\bar{t}} & \dfrac{Y_s}{(|t|)^2} \end{bmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

If there is a phase shift, **t** will be complex

$\bar{t}$  = complex conjugate of  **t**

# Transformer Examples

**Ex1.** An ideal transformer has 330 turns on the primary winding and 36 turns on the secondary. If the primary is connected across a 110 V (rms) generator, what is the rms output voltage?

$$110 \cdot volt \cdot \frac{36}{330} = 12 \cdot volt$$

**Ex2** A transformer has $N_1 = 320$ turns and $N_2 = 1000$ turns. If the input voltage is $v(t) = (255 \text{ V})\cos(\omega t)$, what rms voltage is developed across the secondary coil?

$$\frac{255 \cdot volt}{\sqrt{2}} \cdot \frac{1000}{320} = 563 \cdot volt$$

**Ex3** A transformer is rated at 480V / 120V, 1.2kVA. Assume the transformer is ideal and all voltages and currents are RMS.



$$|\mathbf{Z}_L| = 20 \cdot \Omega$$
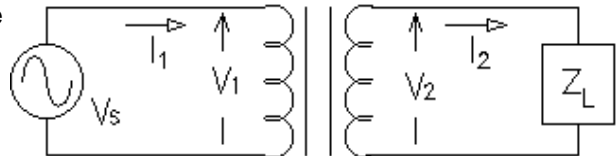$$pf := 75 \cdot \% \quad \text{lagging}$$
$$\mathbf{V}_L := 110 \cdot V$$

a) What is the current rating of the primary?

$$\frac{1.2 \cdot kVA}{480 \cdot V} = 2.5 \cdot A$$

b) What is the current rating of the secondary?

$$\frac{1.2 \cdot kVA}{120 \cdot V} = 10 \cdot A$$

c) The secondary has 100 turns of wire. How many turns does the primary have?

$$N_2 := 100 \qquad N_1 := \frac{480 \cdot V}{120 \cdot V} \cdot N_2 \qquad N_1 = 400 \quad \text{turns}$$

d) $\mathbf{V}_L := 110 \cdot V$ How big is the source voltage ($|\mathbf{V}_S|$)?

$$\mathbf{V}_S := \frac{N_1}{N_2} \cdot \mathbf{V}_L \qquad\qquad \mathbf{V}_S = 440 \cdot V$$

e) The secondary load ($Z_L$) has a magnitude of 20 Ω at a power factor of 75%. Find the secondary current, $I_2$ (magnitude **and angle**).

$$pf := 75 \cdot \%$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_L}{20 \cdot \Omega} = 5.5 \cdot A \qquad pf = 0.75 \qquad a\cos(pf) = 41.4 \cdot deg \qquad \mathbf{I}_2 = 5.5A \underline{/-41.4^\circ}$$

f) Find the primary current, $\mathbf{I}_1$ (magnitude **and angle**).

$$\mathbf{I}_1 = \frac{100}{400} \cdot 5.5 \cdot A = 1.375 \cdot A \qquad a\cos(pf) = 41.4 \cdot deg \qquad \mathbf{I}_1 = 1.375A \underline{/-41.4^\circ}$$

Transformer is ideal, so angle is exactly the same as the load.

g) How much average power does the load dissipate?

$$P_L = |\mathbf{V}_2| \cdot |\mathbf{I}_2| \cdot pf = 110 \cdot V \cdot 5.5 \cdot A \cdot 75 \cdot \% = 453.8 \cdot watt$$

h) How much average power does the power source ($\mathbf{V}_S$) supply?
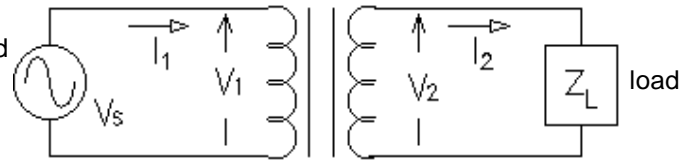
$$P_S = P_L = 454 \cdot watt$$

i) What is the load as seen by $\mathbf{V}_S$? (magnitude **and angle**)

$$\left(\frac{400}{100}\right)^2 \cdot 20 \cdot \Omega = 320 \cdot \Omega \qquad a\cos(pf) = 41.4 \cdot deg \qquad \mathbf{Z}_{eq} = 320\Omega \underline{/41.4^\circ}$$

# Transformer Examples  p2

Ex4  A transformer is rated at $480\text{V}/240\text{V}$, $1.2\text{kVA}$.
Assume the transformer is ideal and all voltages and
currents are RMS.

How much power does the load consume?



$$\mathbf{V_L} := \mathbf{V_S} \cdot \left(\frac{240}{480}\right) \qquad \left|\mathbf{V_L}\right| = 220 \cdot \text{V}$$

$$\left|\mathbf{V_S}\right| = 440 \cdot \text{V} \qquad \left|\mathbf{Z_L}\right| = 16 \cdot \Omega$$
$$\text{pf} := 80 \cdot \% \quad \text{lagging}$$

$$P_L := \frac{\left(\left|\mathbf{V_L}\right|\right)^2}{\left|\mathbf{Z_L}\right|} \cdot \text{pf} \qquad P_L = 2.42 \cdot 10^3 \ \cdot \text{W}$$

Ex5  The transformer shown in the circuit below is ideal.  It is rated at $220/110$ V,  $200$ VA,  $60$ Hz

Find the following:

All values are RMS unless specified otherwise.

a) The primary current (magnitude).

$$\left|\mathbf{I_1}\right| = ?$$

$$V_S := 120 \cdot V \qquad R_1 := 40 \cdot \Omega \qquad Z_L := (15 + 20 \cdot j) \cdot \Omega$$



$$R_1 = 40 \cdot \Omega$$



$$Z_{eq} = \left(\frac{N_1}{N_2}\right)^2 \cdot Z_L$$

$$Z_{eq} := \left(\frac{220}{110}\right)^2 \cdot Z_L$$

$$Z_{eq} = 60 + 80j \ \cdot \Omega$$
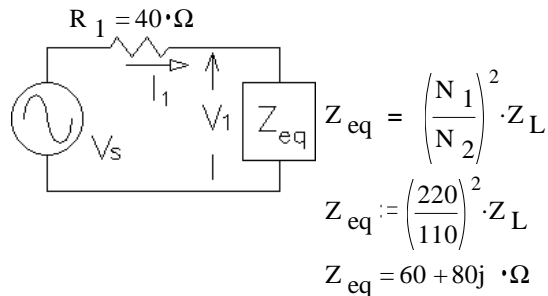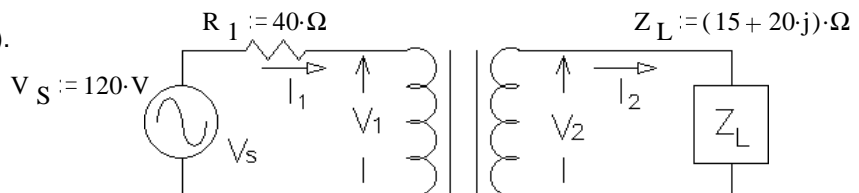
$$R_1 + Z_{eq} = 100 + 80j \ \cdot \Omega$$

$$I_1 := \frac{V_S}{\sqrt{100^2 + 80^2} \cdot \Omega} \qquad I_1 = 0.937 \cdot A$$

b) The primary voltage (magnitude).

$$\left|\mathbf{V_1}\right| = ?$$

$$V_1 := I_1 \cdot \sqrt{60^2 + 80^2} \cdot \Omega \qquad V_1 = 93.7 \cdot V$$

c) The secondary voltage (magnitude).

$$\left|\mathbf{V_2}\right| = ?$$

$$V_2 = \frac{110}{220} \cdot V_1 = 46.85 \cdot V$$

d) The power supplied by the source.

$$P_S = ?$$

$$P_S = I_1^2 \cdot 100 \cdot \Omega = 87.8 \cdot W$$

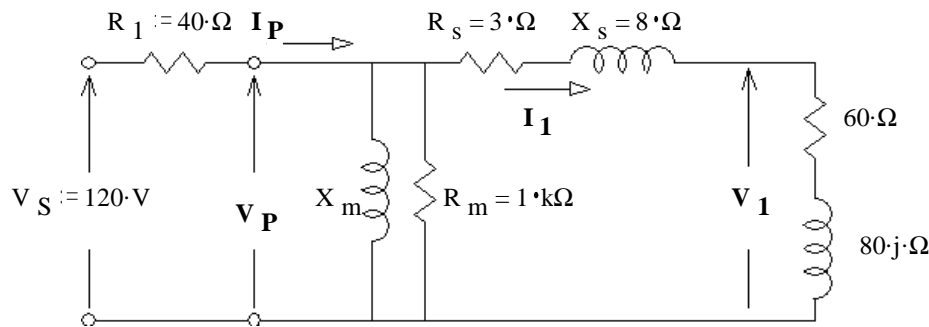e) Is this transformer operating within its ratings?  Show  your evidence.

$$I_{1max} = \frac{200 \cdot VA}{220 \cdot V} = 0.909 \cdot A \quad < \quad I_1 = 0.937 \cdot A$$

NO                            **Transformer  Examples  p2**

## Transformer Examples  p3

Ex6   Repeat Ex5 with a nonideal transformer whose characteristics are shown below.

$$R_m := 1 \cdot k\Omega \qquad X_m := 400 \cdot \Omega \qquad\qquad R_s := 3 \cdot \Omega \qquad\qquad X_s := 8 \cdot \Omega$$



$$Z_{eq} := \left(\frac{220}{110}\right)^2 \cdot (15 + 20 \cdot j) \cdot \Omega$$

$$Z_{eq} = 60 + 80j \quad \cdot \Omega$$

Find the following:

a) The primary current (magnitude).

$$\left| I_P \right| = ?$$

$$R_s + X_s \cdot j + Z_{eq} = 63 + 88j \quad \cdot \Omega$$

$$\cfrac{1}{\cfrac{1}{X_m \cdot j} + \cfrac{1}{R_m} + \cfrac{1}{(63 + 88 \cdot j) \cdot \Omega}} = 45.255 + 71.041j \quad \cdot \Omega$$

$$R_1 + (45.255 + 71.041 \cdot j) \cdot \Omega = 85.255 + 71.041j \quad \cdot \Omega$$

$$I_P := \frac{120 \cdot V}{(85.255 + 71.041 \cdot j) \cdot \Omega} \qquad\qquad \begin{aligned} I_P &= 0.831 - 0.692j \quad \cdot A \\ \left| I_P \right| &= 1.081 \cdot A \end{aligned}$$

b) The primary voltage (magnitude).

$$\left| V_P \right| = ? \qquad\qquad V_P := I_P \cdot (45.255 + 71.041 \cdot j) \cdot \Omega \qquad \begin{aligned} V_P &= 86.771 + 27.689j \quad \cdot V \\ \left| V_P \right| &= 91.082 \cdot V \end{aligned}$$

c) The secondary voltage (magnitude).

$$\left| V_2 \right| = ? \qquad\qquad I_1 := \frac{V_P}{(63 + 88 \cdot j) \cdot \Omega} \qquad\qquad I_1 = 0.675 - 0.503j \quad \cdot A$$

$$V_1 := I_1 \cdot (60 + 80 \cdot j) \cdot \Omega \qquad\qquad \begin{aligned} V_1 &= 80.723 + 23.8j \quad \cdot V \\ \left| V_1 \right| &= 84.158 \cdot V \\ \left| V_2 \right| &= \frac{110}{220} \cdot 84.16 \cdot V = 42.08 \cdot V \end{aligned}$$

d) The power supplied by the source.

$$P_S = ? \qquad\qquad P_S = 120 \cdot V \cdot Re\left(I_P\right) = 99.69 \cdot W$$

e) Is this transformer operating within its ratings?  Show  your evidence.

$$I_{1max} = \frac{200 \cdot VA}{220 \cdot V} = 0.909 \cdot A \quad < \quad \left| I_P \right| = 1.081 \cdot A \qquad NO$$

f)  Find the efficiency, assuming that the only useful output is from $Z_L$.

$$\eta = \frac{\left(\left| I_1 \right|\right)^2 \cdot 60 \cdot \Omega}{99.69 \cdot W} \cdot 100 \cdot \% = 42.63 \cdot \%$$

# Transformer Examples  p4

Ex7  Find the voltage regulation and full-load efficiency of the transformer whith the following ratings and characteristics.
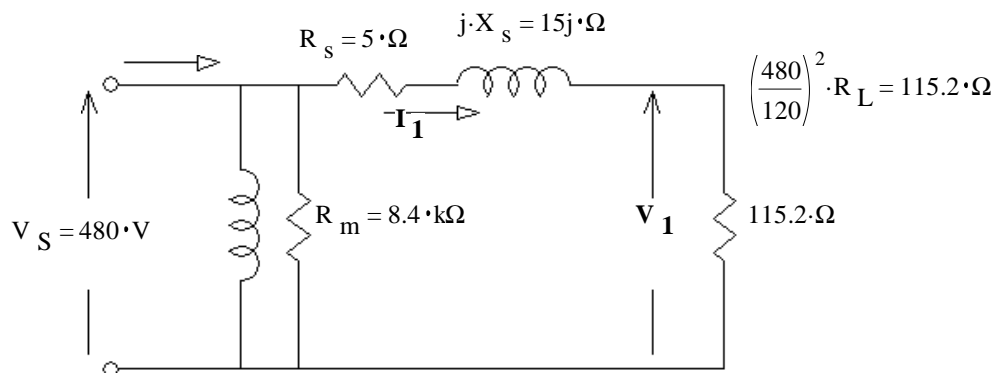
Rated at 480/120 V, 2 kVA, 60 Hz     $R_m := 8.4 \cdot k\Omega$     $X_m := 2 \cdot k\Omega$     $R_s := 5 \cdot \Omega$     $X_s := 15 \cdot \Omega$

No load:



$0 \cdot A$

$V_S := 480 \cdot V$     $V_1 := 480 \cdot V$     $V_2 := 120 \cdot V$

$V_{NL1} := 480$
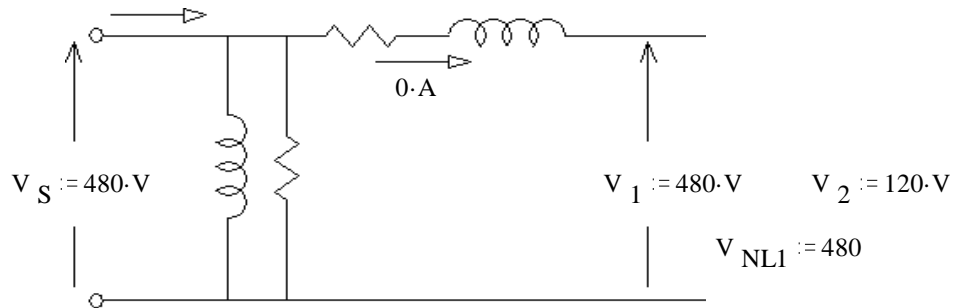
Full load:     Roughly:     $R_L := \dfrac{(120 \cdot V)^2}{2000 \cdot W}$     $R_L = 7.2 \cdot \Omega$

$R_s = 5 \cdot \Omega$     $j \cdot X_s = 15j \cdot \Omega$

$\left(\dfrac{480}{120}\right)^2 \cdot R_L = 115.2 \cdot \Omega$



$V_S = 480 \cdot V$     $R_m = 8.4 \cdot k\Omega$     $V_1$     $115.2 \cdot \Omega$

$I_1 := \dfrac{V_S}{(120.2 + 15j) \cdot \Omega}$     $I_1 = 3.932 - 0.491j \cdot A$

$V_1 := I_1 \cdot 115.2 \cdot \Omega$     $V_1 = 452.979 - 56.528j \cdot V$

$V_{FL1} := |V_1|$     $V_{FL1} = 456.493 \cdot V$

Voltage regulation:     $\%VR = \dfrac{V_{no\_load} - V_{full\_load}}{V_{full\_load}} \cdot 100 \cdot \% = \dfrac{480 - 456.493}{456.493} \cdot 100 \cdot \% = 5.149 \cdot \%$

Efficiency     $\eta = \dfrac{P_{out}}{P_{in}} \cdot 100 \cdot \% = \dfrac{P_{out}}{P_{out} + P_{losses}} \cdot 100 \cdot \%$

$P_{out} := (|I_1|)^2 \cdot 115.2 \cdot \Omega$     $P_{out} = 1.809 \cdot kW$

$P_{losses} := (|I_1|)^2 \cdot 5 \cdot \Omega + \dfrac{(480 \cdot V)^2}{8400 \cdot \Omega}$     $P_{losses} = 0.106 \cdot kW$

$\eta = \dfrac{P_{out}}{P_{out} + P_{losses}} \cdot 100 \cdot \% = 0.945$

# Transformer Examples p5

Ex8  A 500/100-V, 2.5-kVA transformer is subjected to an OC test and a SC test with the results below.

a) Draw a model of this transformer and find the values of all the elements of the model, including the turns ratio.

During the open-circuit test:  $I_{OC} := 0.5 \cdot A$  $P_{OC} := 150 \cdot W$

During the short-circuit test:  $V_{SC} := 22 \cdot V$  $P_{SC} := 70 \cdot W$

$S_{rated} := 2.5 \cdot kVA$  $V_{rated} := 500 \cdot V$  $I_{rated} := \dfrac{S_{rated}}{V_{rated}}$  $I_{rated} = 5 \cdot A$
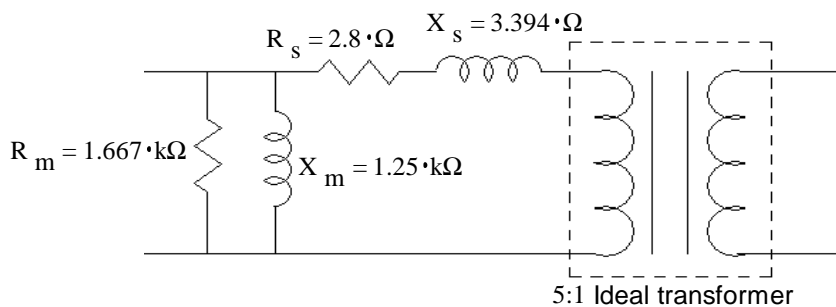
Open-circuit test used to find $R_m$ and $X_m$

$V_{OC} := V_{rated}$  $R_m := \dfrac{V_{OC}^2}{P_{OC}}$  $R_m = 1.667 \cdot k\Omega$

$Q_{OC} := \sqrt{(V_{OC} \cdot I_{OC})^2 - P_{OC}^2}$  $Q_{OC} = 200 \cdot VAR$  $X_m := \dfrac{V_{OC}^2}{Q_{OC}}$  $X_m = 1.25 \cdot k\Omega$

Short-circuit test used to find $R_s$ and $X_s$

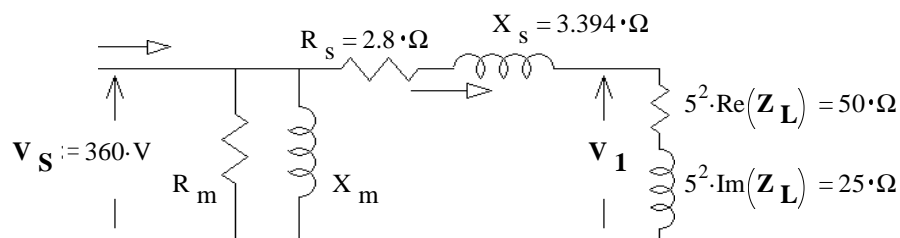$I_{SC} := I_{rated}$  $R_s := \dfrac{P_{SC}}{I_{SC}^2}$  $R_s = 2.8 \cdot \Omega$

$Q_{SC} := \sqrt{(V_{SC} \cdot I_{SC})^2 - P_{SC}^2}$  $Q_{SC} = 0.085 \cdot kVAR$  $X_s := \dfrac{Q_{SC}}{I_{SC}^2}$  $X_s = 3.394 \cdot \Omega$



5:1 Ideal transformer

b) The transformer is connected to a primary source voltage of 360V and loaded with $Z_L := (2 + 1 \cdot j) \cdot \Omega$. (you may add these parts to your drawing if you wish.)

Find the secondary voltage. Magnitude only.  $\left| V_2 \right| = ?$



$5^2 \cdot \text{Re}(Z_L) = 50 \cdot \Omega$

$5^2 \cdot \text{Im}(Z_L) = 25 \cdot \Omega$

$$|\mathbf{V}_1| \;=\; V_1 := \mathbf{V}_S \cdot \frac{\sqrt{(50 \cdot \Omega)^2 + (25 \cdot \Omega)^2}}{\sqrt{\left(R_s + 50 \cdot \Omega\right)^2 + \left(X_s + 25 \cdot \Omega\right)^2}} \qquad\qquad V_1 = 335.687 \cdot V$$

$$|\mathbf{V}_2| \;=\; V_2 = \frac{V_1}{5} = 67.14 \cdot V$$

c) Is this transformer operating within its ratings?  Show all evidence and calculate needed to to determine this.

$$|\mathbf{I}_1| \;=\; I_1 := \frac{\mathbf{V}_S}{\sqrt{\left(R_s + 50 \cdot \Omega\right)^2 + \left(X_s + 25 \cdot \Omega\right)^2}}$$

$$I_1 = 6.005 \cdot A \quad > \quad I_{rated} = 5 \cdot A$$

## NO !

Ex9

a) You have a 250/100-V, 500-VA transformer.
Show the necessary connections to use
this transformer to transform 350 V to 250 V.
Also show the 350-V source and the load.

b) Connected this way, determine the maximum
power that could be converted from 350 V to
250 V without overloading the transformer.
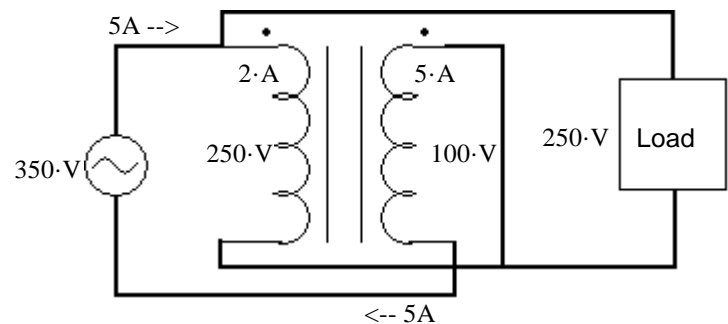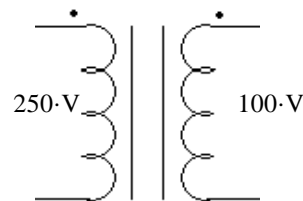




ratings:  $\dfrac{500 \cdot VA}{250 \cdot V} = 2 \cdot A \qquad \dfrac{500 \cdot VA}{100 \cdot V} = 5 \cdot A$

new VA rating and maximum power:   $5 \cdot A \cdot 350 \cdot V = 1.75 \cdot kW$

c) besides the right impedance magnitude, what other characteristic must the load posses in order to actually
use this much power?

Load must be purely resistive (power factor is 1).

d) Could this transformer also be used to transform 280 V to 200 V ?  If yes, what is the maximum power that could
be transformed?

Same connections as above     Maximum power:     $5 \cdot A \cdot 280 \cdot V = 1.4 \cdot kW$

**Transformer  Examples   p6**

# ECE 3600   homework  # 6                              Due:  Fri, 9/16/22

1. Textbook problem 1-7  (p49)

2. Textbook Example 1-2 (p20) with

| | |
|---|---|
| Mean magnetic length: | $l_c := 50 \cdot cm$ |
| Air gap length: | $l_a := 0.06 \cdot cm$ |
| Core cross-sectional area: | $A_c := 16 \cdot cm^2$ |

**Answers**          1.  $0.0054 \cdot Wb$          2.  $727 \cdot mA$

# ECE 3600   homework  # 7                              Due:  Tue, 9/20                    b
## Ideal transformers

1. A step-up transformer is designed to have an output voltage of $2200$ V (rms) when the primary is connected across a $240$ V (rms) source.

   a) If there are $150$ turns on the primary winding, how many turns are required on the secondary?

   b) If a load resistor across the secondary draws a current of $1.2$ A, what is the current in the primary, assuming ideal conditions?

2. An ideal transformer has a turns ratio ($N = N_1/N_2$) of $1.5$ . It is desired to operate a $200$ $\Omega$ resistive load at $150$ V (rms).

   a) Find the secondary and primary currents.

   b) Find the source voltage ($\mathbf{V_1}$).

   c) Find the power dissipated in the load resistor and the power delivered to the primary from the source.

   d) Find the impedance the source sees looking into the primary winding by calculating $\mathbf{Z_{eq}} = N^2\,\mathbf{Z_L}$ and again by calculating $\mathbf{V_1} / \mathbf{I_1}$.

3. The transformer shown in the circuit below is ideal.
   It is rated at 120/30 V, 80 VA, 60 Hz
   Find the following:

   a) $I_1$ = ?

   b) $V_2$ = ?

   $R_1 := 60 \cdot \Omega$

   $V_s := 120 \cdot V$

   $R_2 := 15 \cdot \Omega$
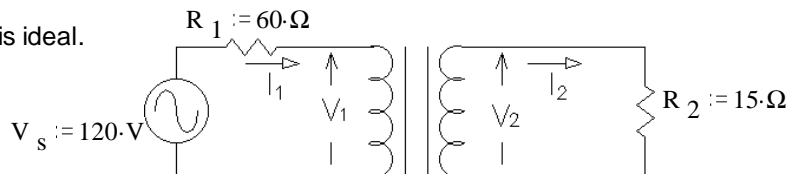
4. An ideal transformer has a rating of $500/125$ V, $10$ kVA, $60$ Hz.  It is loaded with an impedance of $5\Omega$ at 80% pf (0.80). The source voltage applied to the primary winding is $440$ V (rms).  Find:

   a) the load voltage                    b) the load current

   c) the kVA delivered to load          d) the power delivered to load

   e) the primary current                f) the power factor of primary

   g) the impedance the source sees looking into primary.

5. An ideal transformer is rated to deliver $400$ kVA at $460$ V to a customer.

   a) How much current can the transformer supply to the customer?

   b) If the customer's load is purely resistive (i.e. if the $pf = 1$), what is the maximum power the customer can receive?

   c) If the customer's power factor is $0.8$ (lagging), what is the maximum usable power the customer can receive?

   d) What is the maximum power if the power factor is $0.7$ (lagging)?

   e) If the customer requires $300$ kW to operate, what is the minimum allowable power factor given the rating of this transformer?

**Answers**

1. a) 1375 turns    b) 11 A        2.  a) 0.75 A,  0.50 A        b) 225 V        c) 112.5 W        d) 450Ω

3. a) 0.4·A        b) 24V        4. a) 110 V        b) 22 A        c) 2.42 kVA        d) 1.94 kW        e) 5.5 A        f) 0.80        g) 80Ω /36.9° Ω

5. a) 870·A    b) 400·kW    c) 320·kW        d) 280·kW    e) 0.75                ECE 3600   homework  #  6 & 7

# ECE 3600   homework # 8                    Due: Fri, 9/23/22

1. The city of Murray, Utah. distributes power to neighborhoods with a 12.47-kV three-phase system. (12.47 kV is the line voltage.) Each group of houses is served from one phase and ground, and transformed to 240/120 V by a pole transformer, as shown.
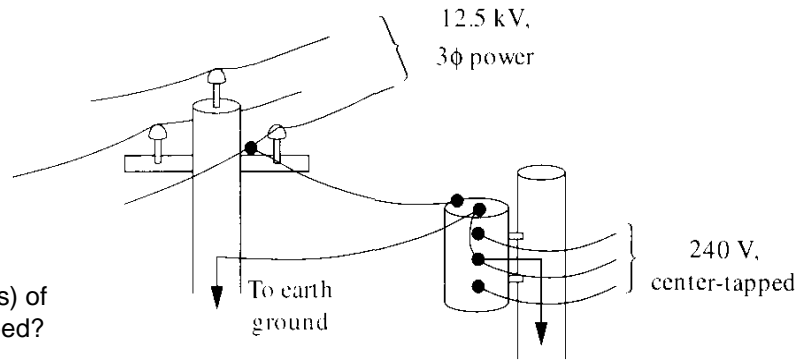
   a) What is the turns ratio (primary/secondary turns) of the pole transformer to give 240 V, center-tapped?

   b) When a 1500-W toaster-oven is turned on, how much does the current increase in the high-voltage wire? Assume the power factor is unity and the transformer is 100% efficient.

   c) Repeat b) for a clothes drier that draws 15 A.

2. A single-phase transformer is designed to operate at 60 Hz. Its voltage ratings are: primary, 500 V; secondary, 200 V. The maximum permissible load is 30 kVA.

   a)  What will be the magnitudes of primary and secondary currents when the device is full-loaded?

   b) Loading is accomplished by an impedance connected across the 200-V terminals. How many ohms will correspond to full-load of the transformer? (Use IT model.)

3. 5.3 The 30-kVA transformer of the problem above is made subject to a SC test. One winding is short-circuited and the other winding is fed from a 60-Hz voltage source. The voltage is raised until rated current is circulated in the windings. This occurs when the applied voltage equals 5.11% of rated winding voltage. The transformer consumes 290 W during the test.

   a) Compute the series impedance $\mathbf{Z_s} = R_s + jX_s$ of the transformer referred to primary and secondary sides.

   b) Compute the core flux during the SC test. (Express its magnitude in percent of normal operating flux.)

   c) Why is it permissible to assume that all of the 290 W constitute ohmic losses in R, and no part of it is core loss?

4. 5.5 The transformer in Exercise 5.3 is fed from a 500-V source. A load impedance of $\quad Z_L := (1.03 + j \cdot 0.72) \cdot \Omega$ is connected across the secondary.
   a) Find the currents in both windings and the secondary voltage by use of the IT model.

   b) Same as in part a) but now include the transformer impedance in your analysis. Take note of the change in your answers.

   c)  Is the transformer current-overloaded?

5. 5.6 The same 30-kVA transformer is made subject to an OC test. It is fed from a 500-V source with the secondary open. The transformer consumes 230 W and draws 0.8 A.

   a) Find $R_m$ and $X_m$ .

   b) Based upon the SC and OC test data compute the efficiency of the transformer when loaded with the $\mathbf{Z_L}$ impedance above.

## Answers

1. a) 30                  b)  208·mA

2. a) 60·A              150·A              b)  1.333·Ω

3. a) $0.0806 + 0.418 \cdot j \cdot \Omega$              $0.0129 + 0.0669 \cdot j \cdot \Omega$        b) 5.11% of normal rated value.

   c) Because the core flux is 5.11% of normal rated value, the core losses (which are approximately proportional to the square of the flux) are negligible.

4. a) 63.66·A          159.2·A          b) 61.23·A          153.1·A          c) Yes, a little

5. a) 1.09·kΩ          0.76·kΩ          b) 97.8·%                    **ECE 3600   homework # 8**

1. 5.7 A 500/200-V, 30-kVA transformer is reconnected as a 700/500-V autotransformer. Compute the new kVA rating of the device.

2. Show connections to the following 100/40-V, 200-VA transformers to get the voltage ratios desired. Compute the new VA rating of each connection.
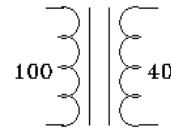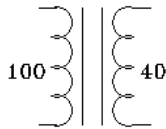
   a)  140/40 V                                              b)  140/100 V
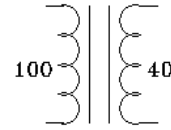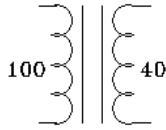
   c)  60/40 V                                               d)  60/100 V

3. 5.8 The terminals of a 500/200-V transformer can be interconnected in four different ways, two of which will result in a 700/500-V autotransformer.  Assume that you have interconnected the windings in the wrong way, but that you believe that you did it the right way.  In other words, you think that you have a 700/500-V autotransformer when in fact you have something else.  As you now connect the "700-V terminals" of your device to a 700-V source, you expect to obtain 500-V between what you presume to the "500-V terminals."  To your surprise you get an entirely different voltage.

   a) What voltage do you get?

   b) What will happen to your transformer with this kind of treatment?

4. a) Draw a per-phase drawing of for the balanced 3-phase, 60-Hz system shown.  You may neglect phase issues introduced by Y-Δ and Δ-Y connections.  You may need to modify the turns ratio of the transformer to reflect Y-Δ and Δ-Y connections.  Be sure to show values of the source, passive components and turns ratio on your drawing.



b) Find $\dfrac{V_1}{V_2}$ incuding phase angle

**ECE 3600   homework # 9   p1**
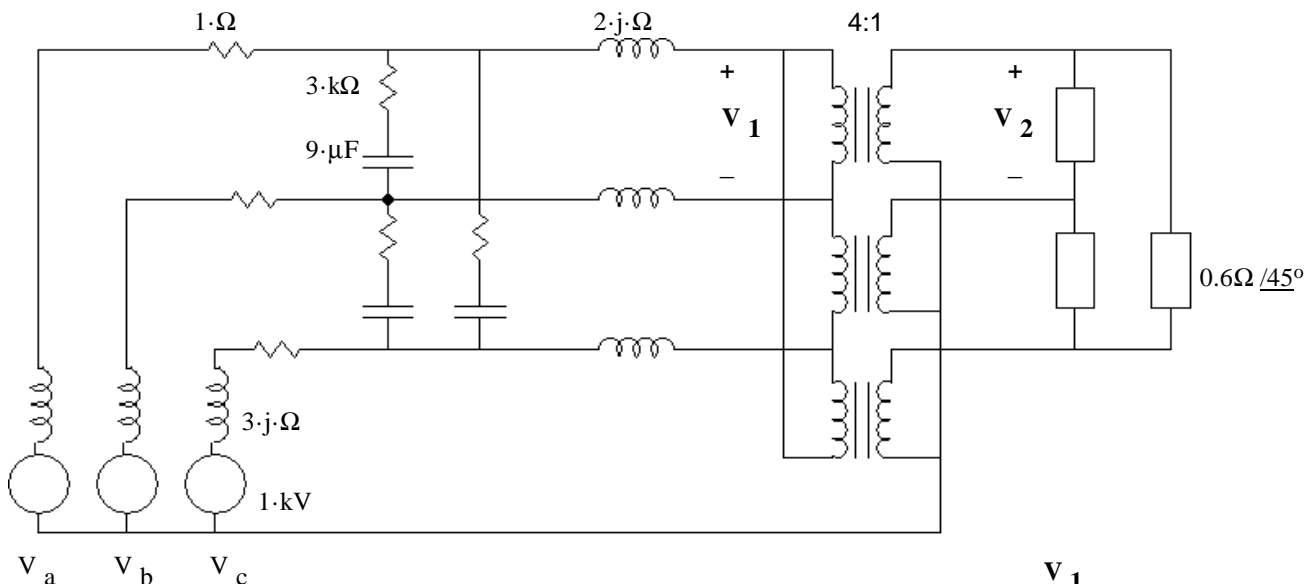
It is easy to see how to transform three-phase power with the use of three single-phase transformers, but there are two ways to transform three-phase power using only two single-phase transformers. The next two problems investigate these methods. In them, we will transform $480$ V three phase to $240$ V three phase; hence, the transformers have a turns ratio of 2:1. Hint: In both figures, the geometric orientation hints of the phasor relationships.

5. The configuration shown is called the "open-delta" or "V" connection, for obvious reasons. Identical 2:1 transformers are used.
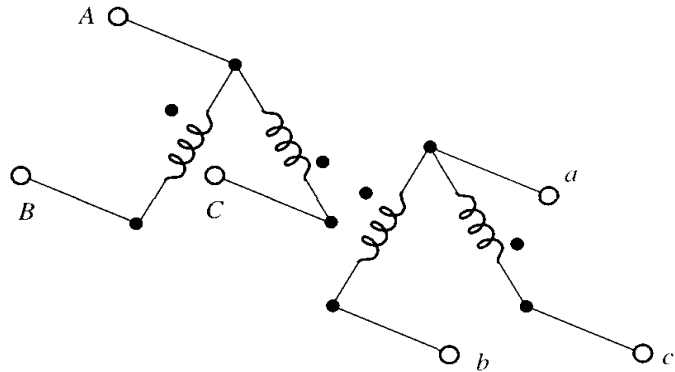
   a) Show that if ABC is $480$-V balanced three phase, abc is $240$-V balanced three-phase. Consider the ABC voltages to be a three-phase set and prove the abc set is three-phase.

   b) If the load is $30$ kVA, find the required kVA rating of the transformers to avoid overload. [You can solve this independent of part a)]

6. 1.22. The configuration shown is called the "T" connection. For this connection, the 2:1 transformers are not identical but have different voltage and kVA ratings. The bottom transformer is center-tapped so as to have equal, in-phase voltages for each half.
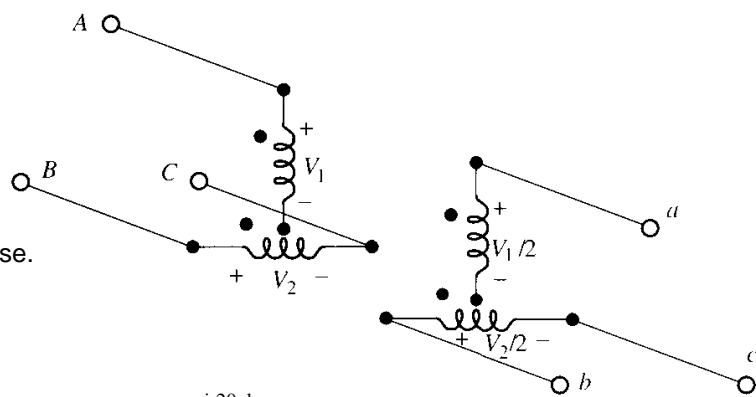
   a) Find the voltages $V_1$ and $V_2$ to make this transform 480-V to 240-V balanced three-phase.

   b) If the load is $30$ kVA, find the required kVA rating of each transformer to avoid overload.
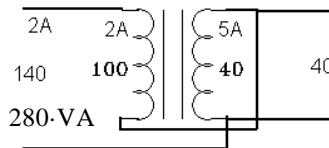
7. A phase-shifting transformer has a complex turns ratio of $t := 4 \cdot e^{j \cdot 20 \cdot \deg} = 4 \underline{/20^o}$

   It has a series impedance of $\mathbf{Z}_S := (0.05 + j \cdot 0.6) \cdot \Omega$
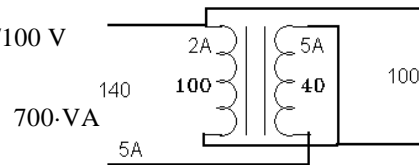
   Find the admittance matrix of this tranformer (see the last page of the transformer notes).
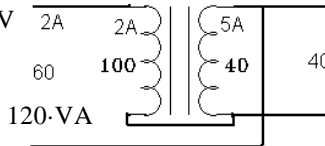
**Answers**

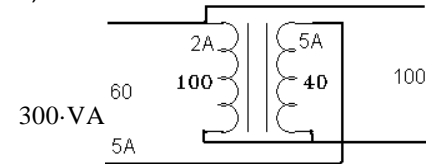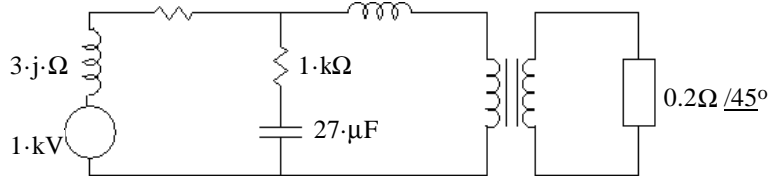1. $105 \cdot kVA$     2. a) $140/40$ V

b) $140/100$ V

3. a) $1167 \cdot V$

   b) The smoke gets out

c) $60/40$ V

d) $60/100$ V

4.     $1 \cdot \Omega$     $2 \cdot j \cdot \Omega$     $2.31:1$
$3 \cdot j \cdot \Omega$     $1 \cdot k\Omega$     $0.2\Omega \underline{/45^o}$
$1 \cdot kV$     $27 \cdot \mu F$     b) $2.309 \underline{/ -30^o}$

5. a) Calculate $\mathbf{V}_{bc}$ from the other two voltages and show that it has the correct magnitude and correct phase angle.

   b) $17.3 \cdot kVA$ per transformer,   $34.6 \cdot kVA$ for both

6. a) $415 \cdot 7 \cdot V$       $480 \cdot V$           b) $15 \cdot kVA$         $17.3 \cdot kVA$         $32.3 \cdot kVA$ for both

7. $\begin{pmatrix} 0.138 - 1.655 \cdot j & 0.109 + 0.401 \cdot j \\ -0.174 + 0.377 \cdot j & 8.621 \cdot 10^{-3} - 0.103 \cdot j \end{pmatrix} \cdot \dfrac{1}{\Omega}$