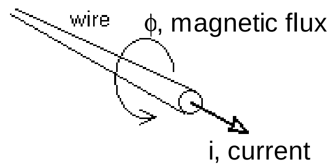
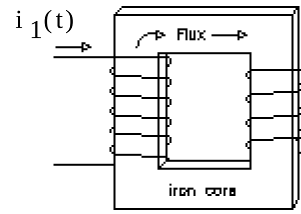
Electromagnetics basics

1. Electric currents produce magnetic fields.



Right-hand-rule

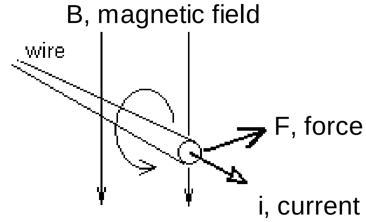
2. A fluctuating magnetic field passing through a coil of wire will induce a voltage in that coil. Basis of transformer secondary, and primary too (back EMF).



$$v_2(t) = N\cdot\frac{d}{dt}\phi$$

3. A wire with a current in the presence of another magnetic field feels a force.

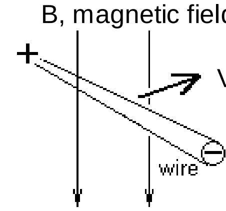
   (Basis of electric motors, also explains why generators resist the mechanical input)



F, force = $i\cdot B\cdot \ell$

ℓ = length of the wire

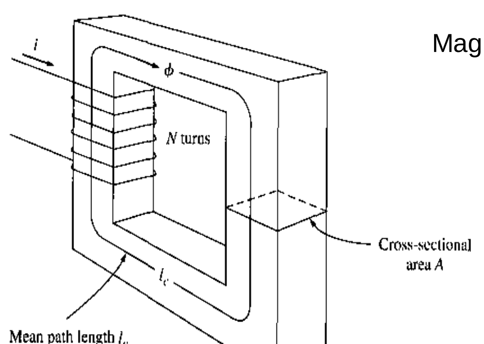
4. A voltage will be induced on wires moving in the presence of a magnetic field. This is very similar to 2. (Basis of electric generators)



1. Magnetic field from a current

Ampere's law: $\oint \mathbf{H}\cdot\mathbf{dl} = I_{net}$

For us...



(Chapman, Fig.1.3)

Magnetic field intensity: $H = \frac{N\cdot i}{\ell_c} \quad \left(\frac{A\cdot turns}{meter}\right)$

Ampere-turns: = $N\cdot i$
(like voltage)

Flux density: $B = \mu\cdot H = \frac{\mu\cdot N\cdot i}{\ell_c}$ (tesla, T)

Flux: $\phi = B\cdot A$ (weber) (Wb)
(like current)

Permeability of free space: $\mu_o := 4\cdot\pi\cdot 10^{-7}\cdot\frac{henry}{m}$

Relative permeability: μ_r

Permeability: $\mu = \mu_r\cdot\mu_o$

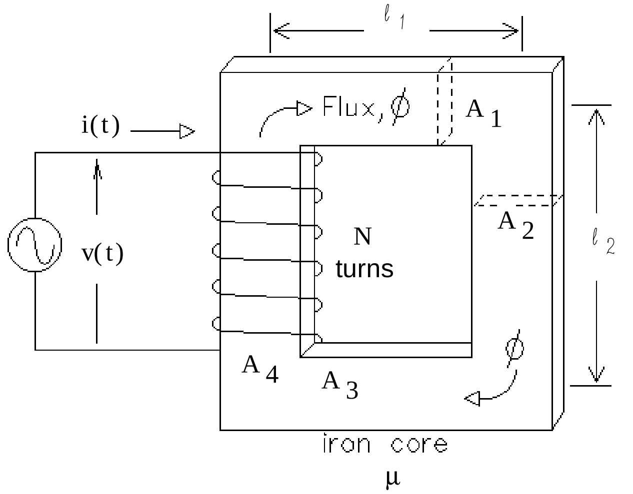
Reluctance of core: $\mathcal{R}_c = \frac{\ell_c}{\mu\cdot A_c} \quad \left(\frac{A\cdot turns}{Wb}\right)$
(like resistance)

Flux: $\phi = \frac{N\cdot i}{\mathcal{R}_c}$ (weber, Wb)

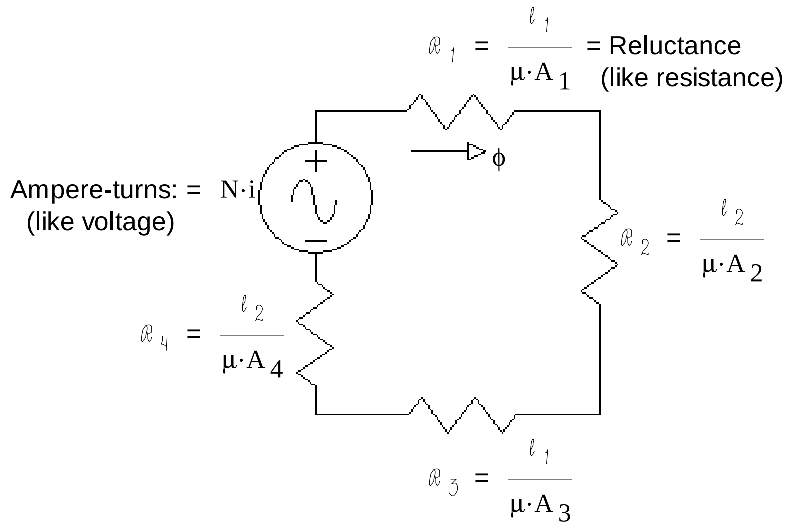
Inductance: $L = \frac{N^2}{\mathcal{R}_c} = N^2\cdot\left(\frac{\mu_r\cdot\mu_o\cdot A_c}{\ell_c}\right)$ (henry, H)

material	relative permeability μ_r
Mu-metal	20000
Permalloy	8000
Electrical steel	4000
ferrite (nickel zinc)	16 - 640
ferrite (manganese zinc)	>640
Steel	700
Nickel	100

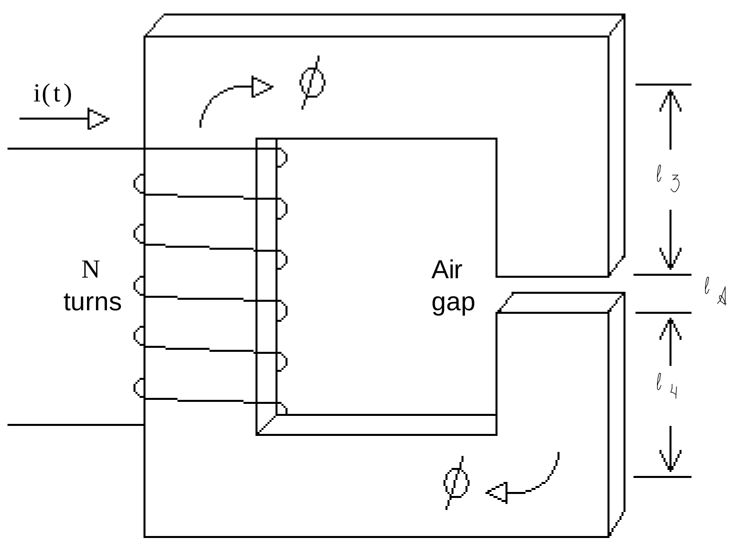
Magnetic "Circuits"

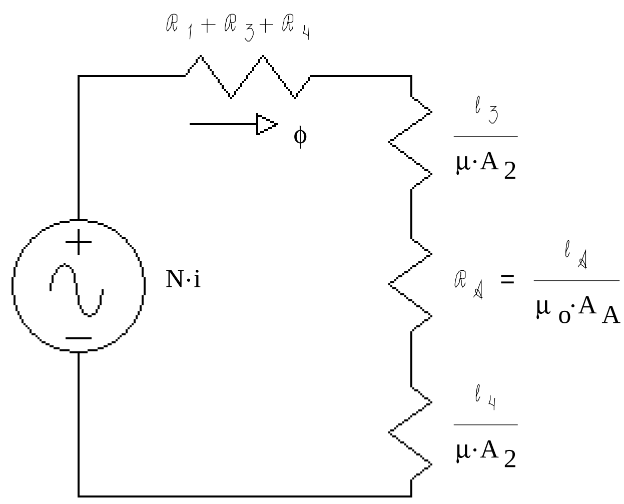


Similar to this electric circuit



magnetic flux = $\phi = \dfrac{N \cdot i}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}$ (weber) (Wb)
(like current)





$$\phi = \frac{N \cdot i}{\mathcal{R}_1 + \mathcal{R}_3 + \mathcal{R}_4 + \dfrac{\ell_3 + \ell_4}{\mu \cdot A_2} + \mathcal{R}_A} \quad \text{(weber) (Wb)}$$

Flux density: $B = \dfrac{\phi}{A} = \mu \cdot H$ (tesla, T)

Magnetic field intensity: $H = \dfrac{B}{\mu} = \dfrac{\phi}{A \cdot \mu}$ $\left(\dfrac{A \cdot turns}{meter}\right)$

$$v(t) = N \cdot \frac{d}{dt}\phi = N \cdot \frac{d}{dt} B \cdot A$$

$$= -N \cdot \frac{d}{dt}\phi = -N \cdot \frac{d}{dt} B \cdot A$$ often shown with a negative sign

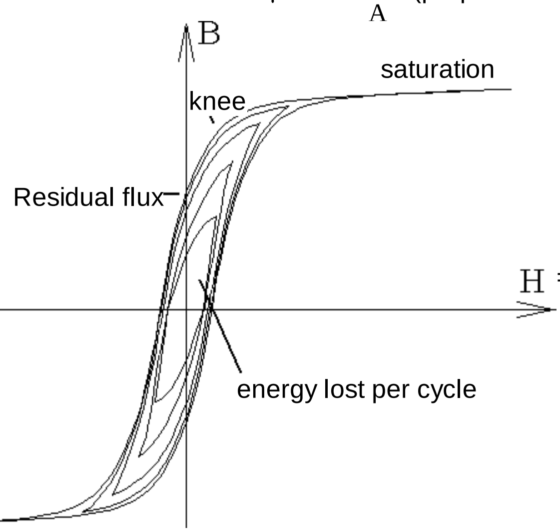
- indicates that this voltage tries to produce a current to oppose the change.

Non-ideal Ferrromagnetic materials (B-H curve)

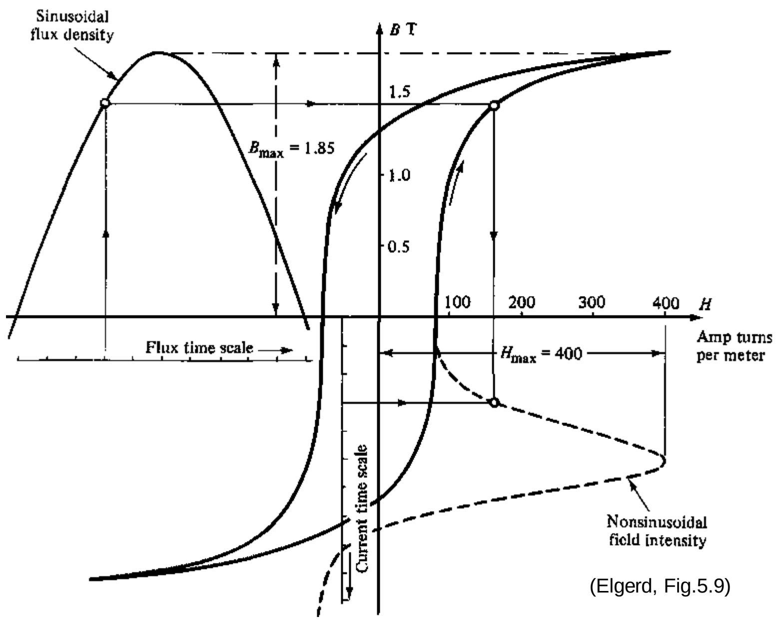
Magnetics are not really linear

The B-H or Hystereses curve

$B = \mu \cdot H = \frac{\phi}{A}$ (proportional to voltage) $v(t) = N \cdot \frac{d}{dt}\phi = -N \cdot \frac{d}{dt} B \cdot A$



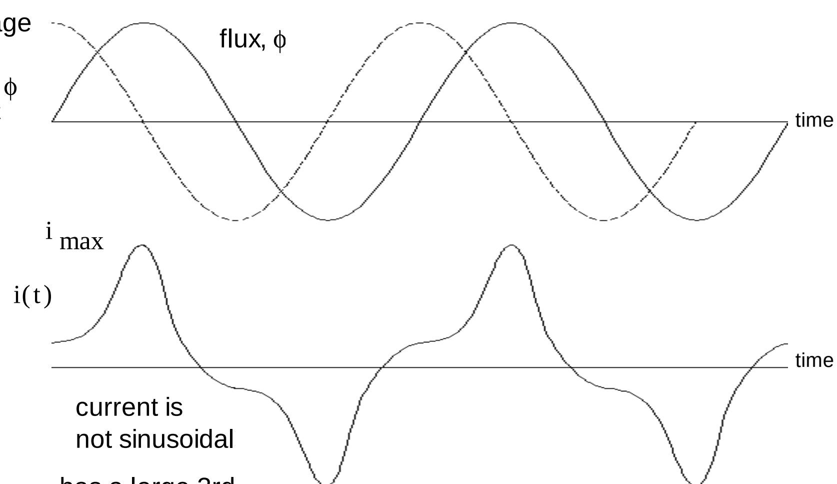
$H = \frac{N \cdot i}{l_c}$ (proportional to current)



(Elgerd, Fig.5.9)

voltage

$v(t) = N \cdot \frac{d}{dt}\phi$



current is
not sinusoidal

has a large 3rd
harmonic component

Sources: Electric Machinery and Power System Fundamentals, Stephen J. Chapman
Basic Electric Power Enineering, Ollie I. Elgerd

Transformer basics and ratings

A Transformer is two coils of wire that are magnetically coupled.

Transformers are only useful for AC, which is one of the big reasons electrical power is generated and distributed as AC.

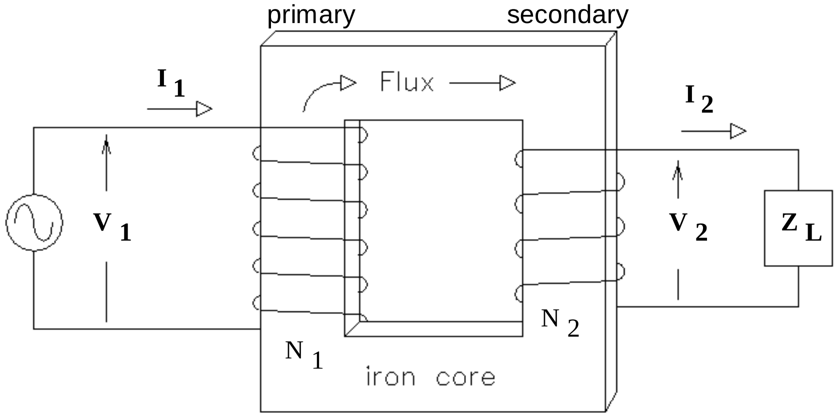
Transformer turns and turns ratios are rarely given, V_p/V_s is much more common where V_p/V_s is the rated primary over rated secondary voltages. You may take this to be the same as N_1/N_2 although in reality N_2 is usually a little bit bigger to make up for losses. Also common: $V_p : V_s$.

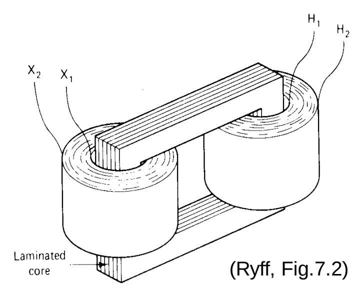
Transformers are rated in VA   Transformer Rating (VA) = (rated V) x (rated I) , on either side. (Both RMS)

Don't allow voltages over the rated V , regardless of the actual current.
Don't allow currents over the rated I , regardless of the actual voltage.

Ideal Transformers

Iron-core transformer





(Ryff, Fig.7.2)

Ideal: $P_1 = P_2$

power in = power out

Transformation of voltage and current

$$\frac{N_1}{N_2} = \frac{\mathbf{V_1}}{\mathbf{V_2}} = \frac{\mathbf{I_2}}{\mathbf{I_1}}$$

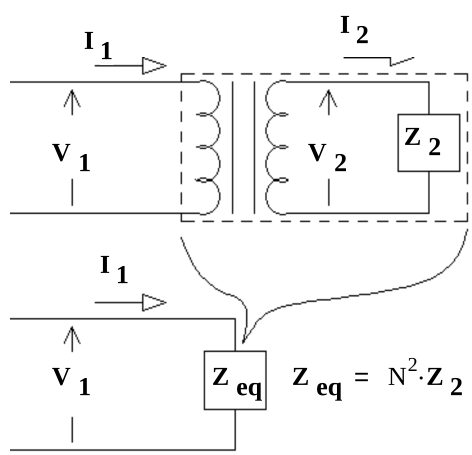
Turns ratio

Turns ratio as defined in Chapman text: $a = \frac{N_1}{N_2}$, same as $N = \frac{N_1}{N_2}$

Note: some other texts define the turns ratio as: $\frac{N_2}{N_1}$

Be careful how you and others use this term

Transformation of impedance



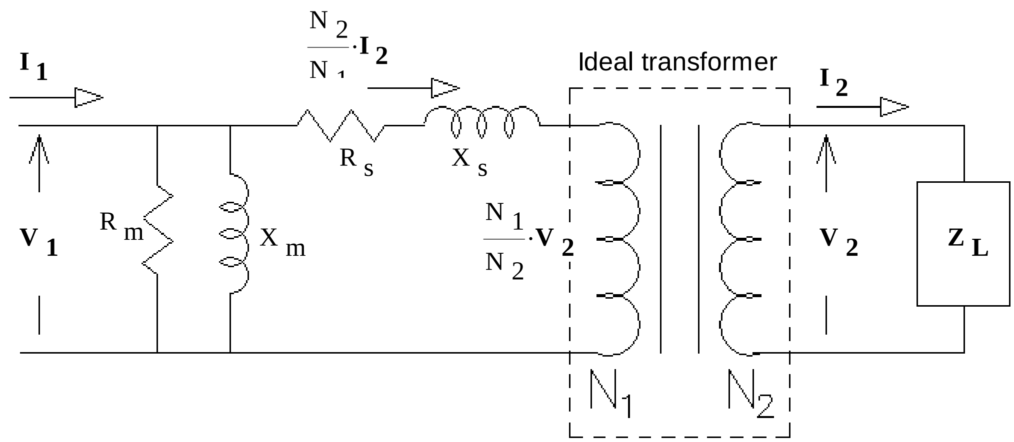
You can replace the entire transformer and load with ($\mathbf{Z_{eq}}$). This "impedance transformation" can be very handy.

$$\mathbf{Z_{eq}} = N^2 \cdot \mathbf{Z_2} = \left(\frac{N_1}{N_2}\right)^2 \cdot \mathbf{Z_2}$$

Transformers can be used for "impedance matching"

This also works the opposite way, to move an impedance from the primary to the secondary, multiply by: $\left(\frac{N_2}{N_1}\right)^2$

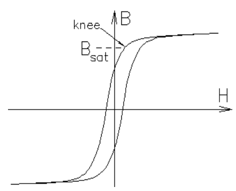
Model of non-ideal Transformer

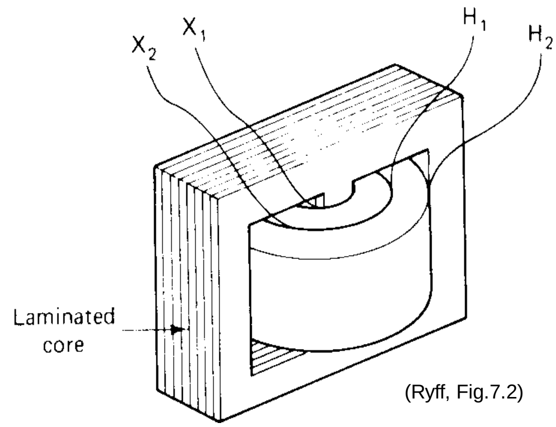


R_m - Core losses

Eddy-current losses - minimized by laminating the core and adding silicon to raise the resistivity

Hysteresis losses - caused by the B-H hysteresis curve





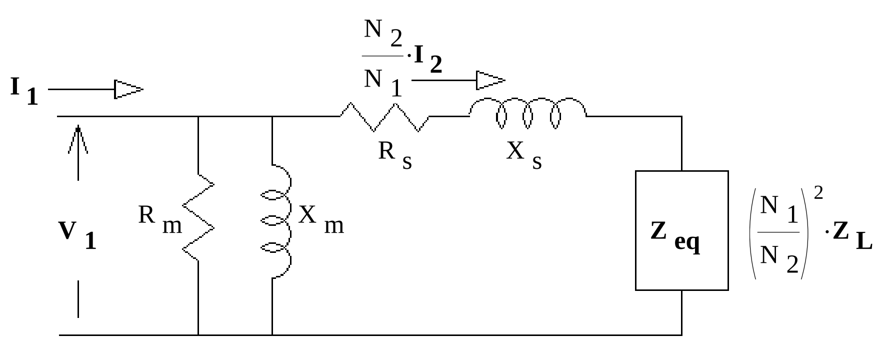
(Ryff, Fig.7.2)

X_m - basic inductance caused by the need to magnetize the core

R_s - Winding resistance (copper) losses

X_s - Reactance caused by flux leakage (leakage reactance)

Move the load impedance to simplification the analysis



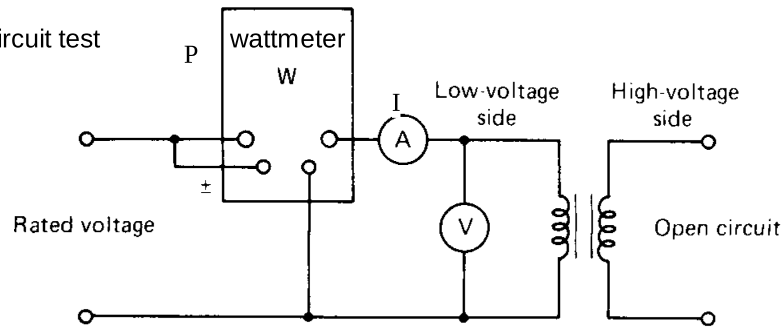
Typical calculations

Voltage regulation $\%VR = \frac{V_{no\_load} - V_{full\_load}}{V_{full\_load}} \cdot 100 \cdot \%$

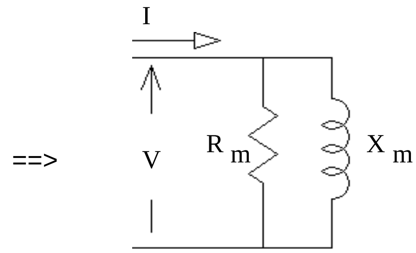
Efficiency $\eta = \frac{P_{out}}{P_{in}} \cdot 100 \cdot \%$

<u>Tests to find parameters</u>

Open-circuit test



(Ryff, Fig.7.16)

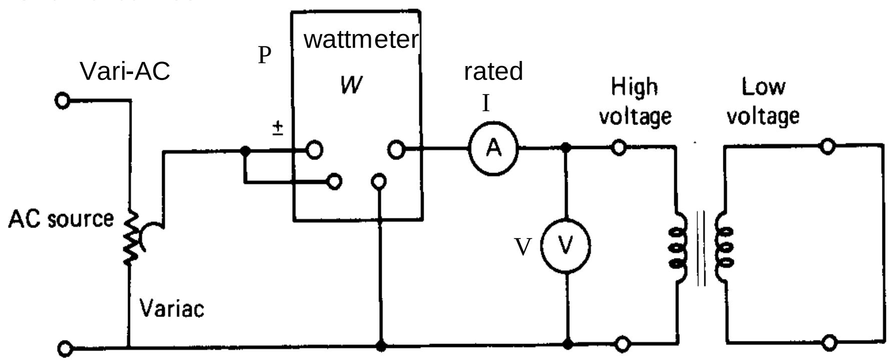


$S = I \cdot V \qquad Q = \sqrt{S^2 - P^2}$

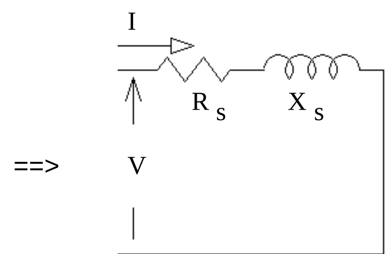
$R_m = \dfrac{V^2}{P} \qquad X_m = \dfrac{V^2}{Q}$

V = rated voltage

Short-circuit test



(Ryff, Fig.7.17)



$S = I \cdot V \qquad Q = \sqrt{S^2 - P^2}$

$R_s = \dfrac{P}{I^2} \qquad X_s = \dfrac{Q}{I^2}$

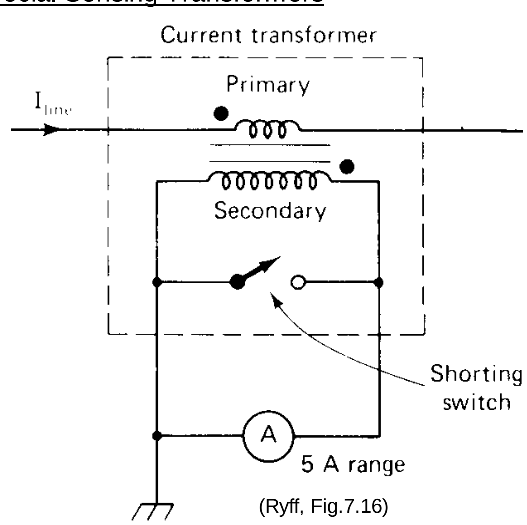
I = rated current

Other Transformers

Multi-tap transformers: Many transformers have more than two connections to primary and/or the secondary. The extra connections are called "taps" and may allow you to select from several different voltages or get more than one voltage at the same time.

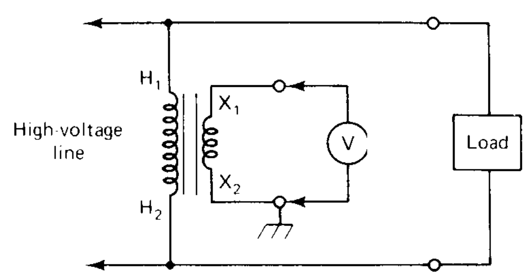
Isolation Transformers: All transformers but auto transformers isolate the primary from the secondary. An Isolation transformer has a 1:1 turns ratio and is just for isolation.

<u>Special Sensing Transformers</u>



(Ryff, Fig.7.16)

CT The secondary **<u>must</u>** always be shorted or nearly shorted!

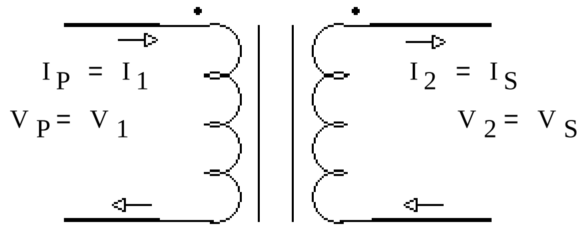


(Ryff, Fig.7.17)

"Potential" transformer, for voltage monitoring

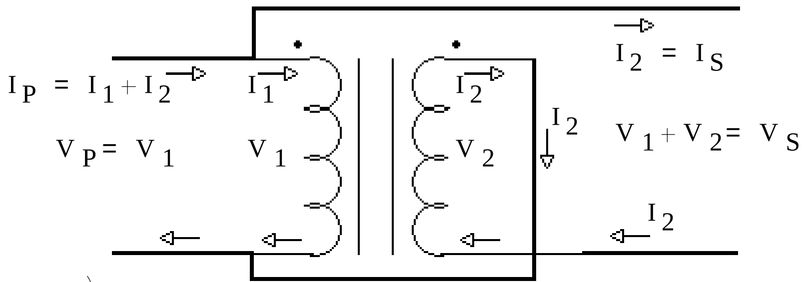
VT or PT

Regular winding connections

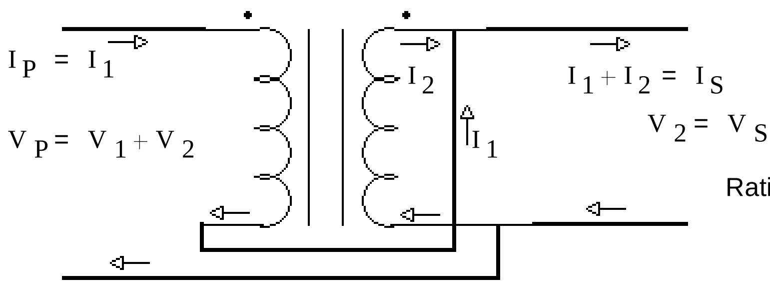


Auto Transformer Connections

4 basic possibilities

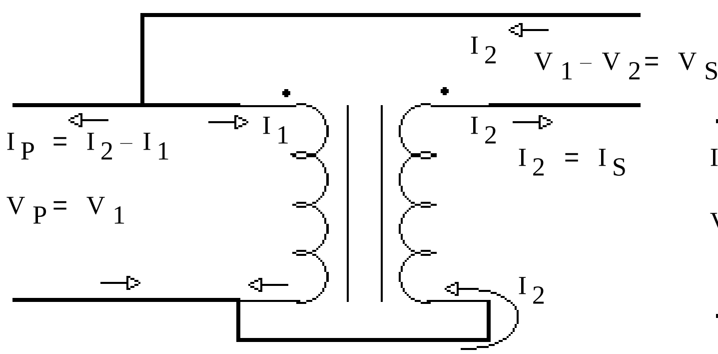


Rating:
$\left(I_{1\_rated} + I_{2\_rated}\right) \cdot V_P$

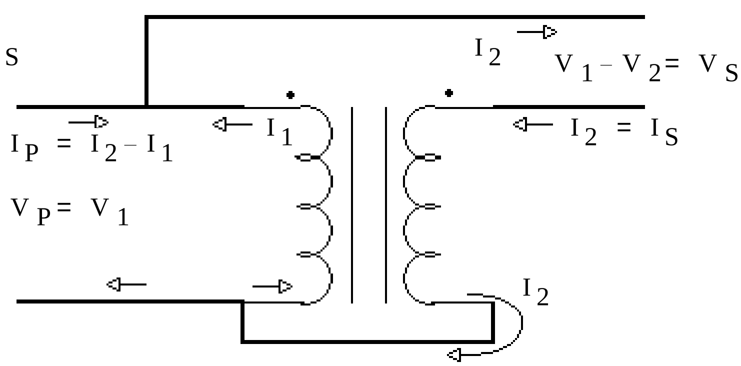


Rating: $\left(I_{1\_rated} + I_{2\_rated}\right) \cdot V_S$

Subtraction connections

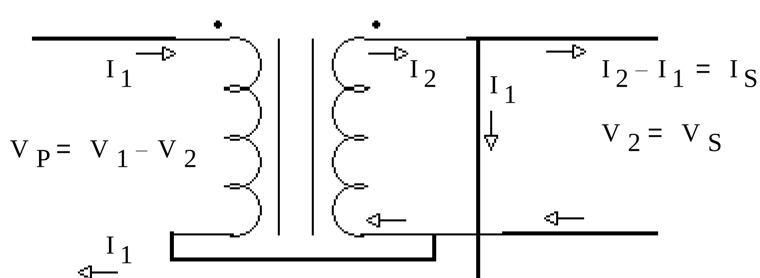


Actual currents are in the reverse directions



Rating: $\left(I_{2\_rated} - I_{1\_rated}\right) \cdot V_P$

Power Flow Through Transformer

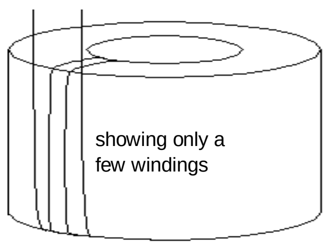


Rating: $\left(I_{2\_rated} - I_{1\_rated}\right) \cdot V_S$

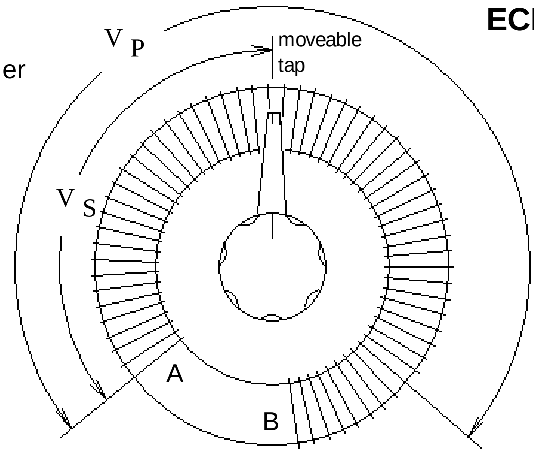
Primary and Secondary could be swapped on any of these connections for an additional 4 possibilities

Vari-AC type autotransformer

This adjustable autotransformer is wound on a toroidal core.



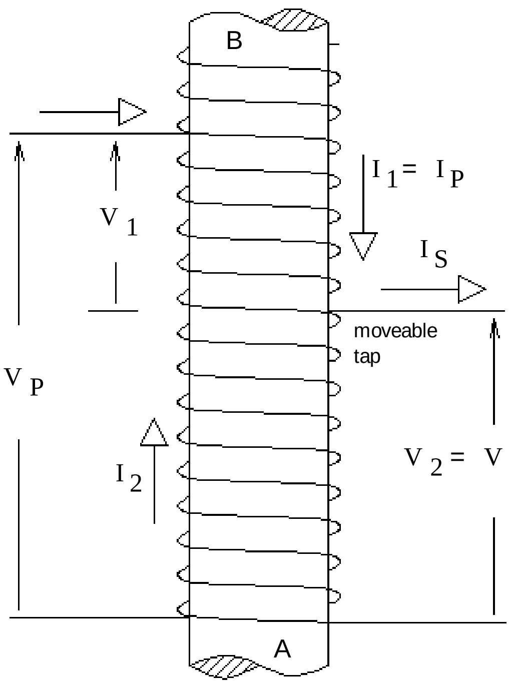
Side view,



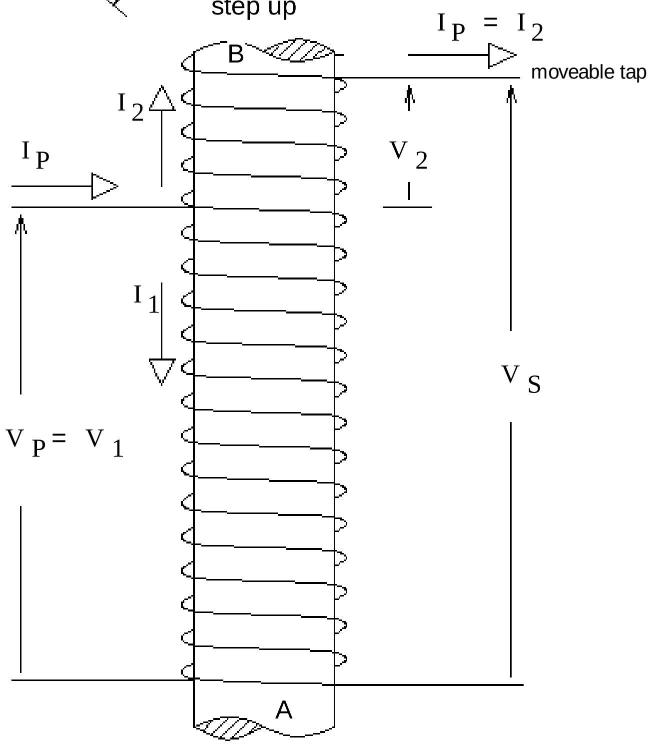
Top view

If you cut the toroid open and straightened it out, you would get the views below.

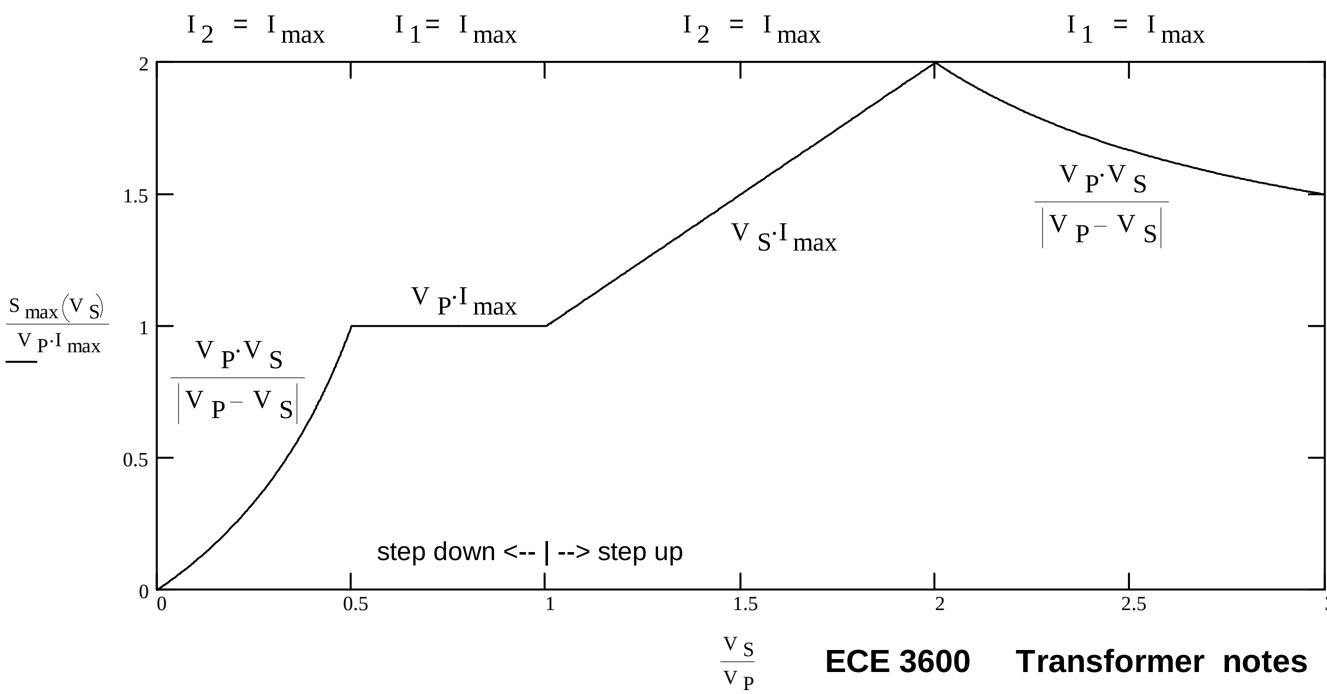
step down



step up



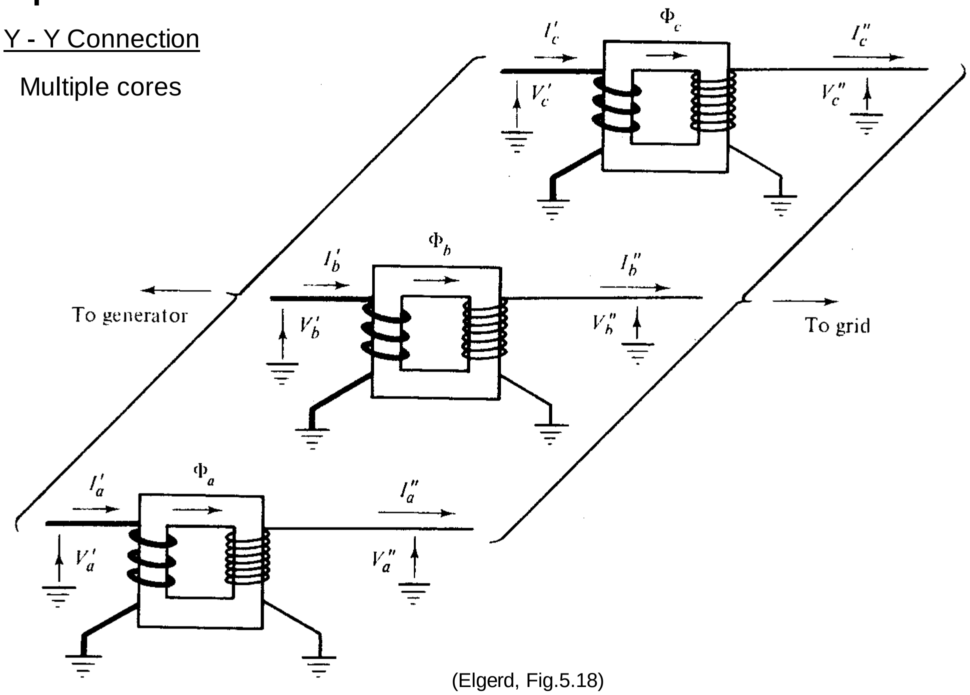
Vari-AC type autotransformer "Rating" , Based or the maximum winding current: I_{max}



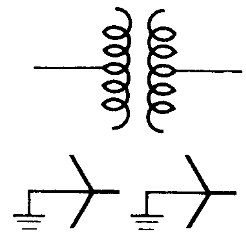
3-phase Transformer Connections

Y - Y Connection

Multiple cores



(Elgerd, Fig.5.18)

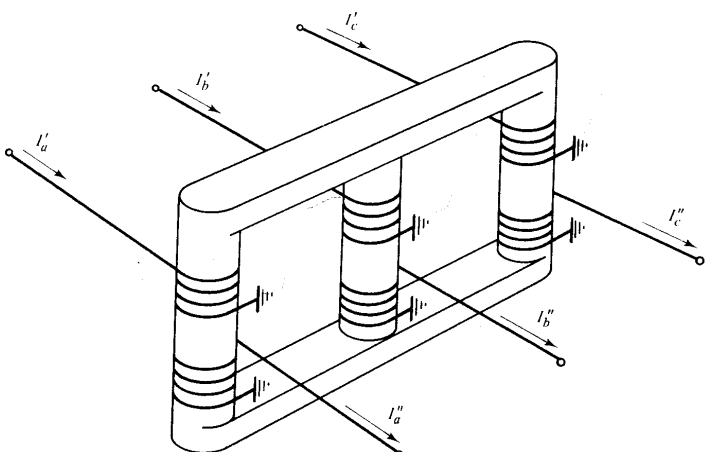


Symbol

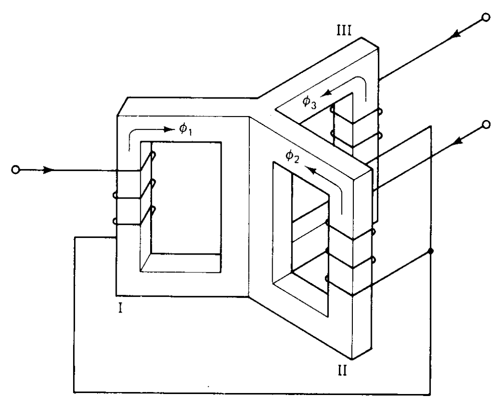
Single-core (3-phase) transformer

Advantages: cheaper, less core loss

Disadvantage: Individual transformers can limp along even if one winding fails, this cannot.



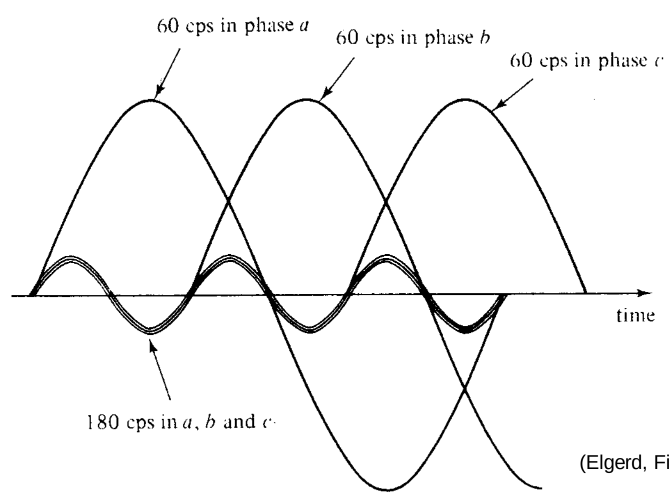
(Elgerd, Fig.5.20)



(Ryff, Fig.7.18)

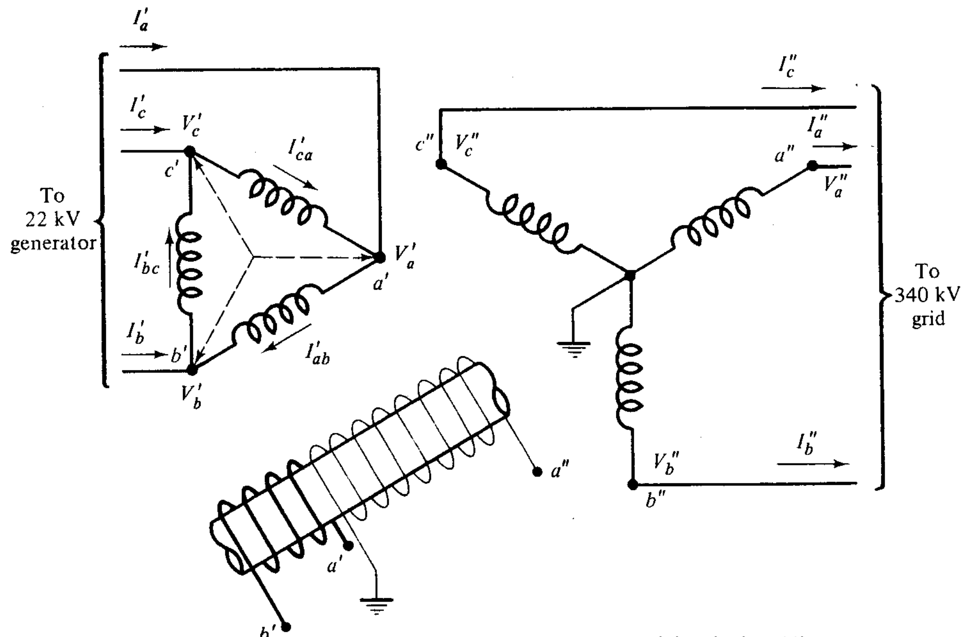
Third-harmonic currents (due to B - H non-linearity) add up to a significant neutral current.

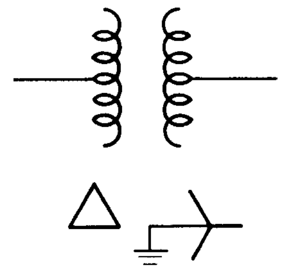
Any Δ winding will allow the third-harmonic current to flow in a loop.



(Elgerd, Fig.5.21)

Δ - Y Connections, Y - Δ are the opposite





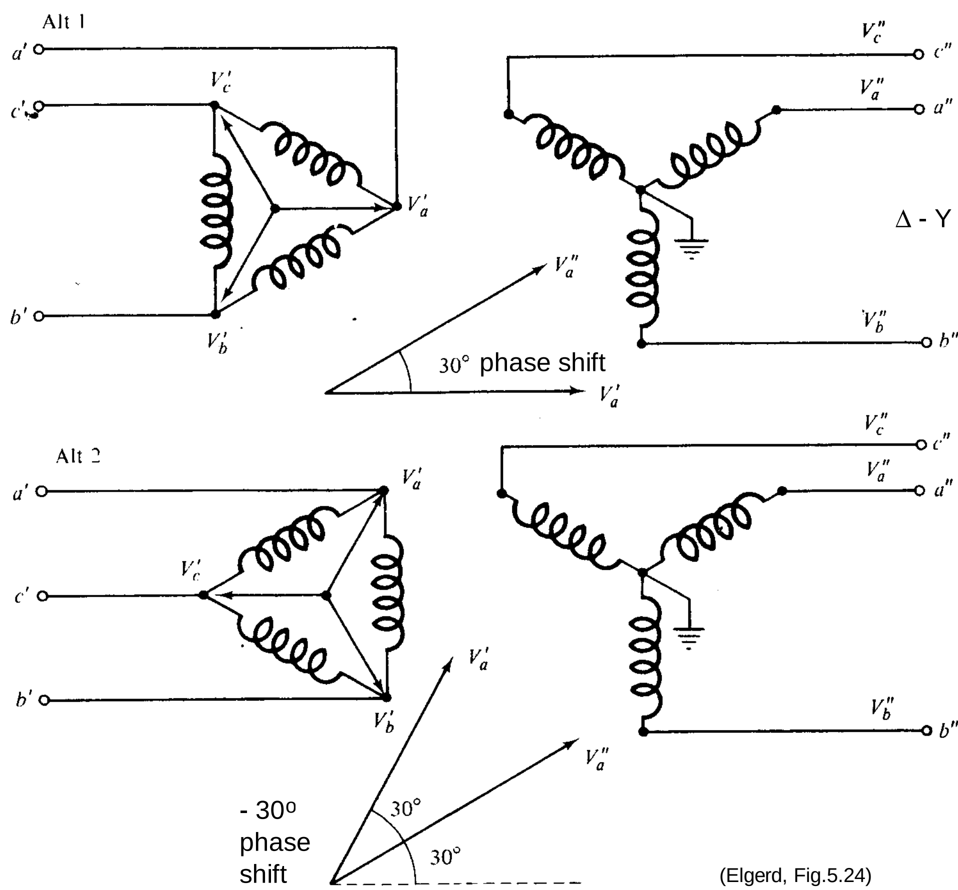
Symbol

Δ - Y is usually step-up

Y - Δ is usually step-down

(Elgerd, Fig.5.22)

Δ - Y and Δ -Y always introduce a 30^o phase shift, + or - depending on how they are wired.



Δ - Y is usually step-up

Different connections

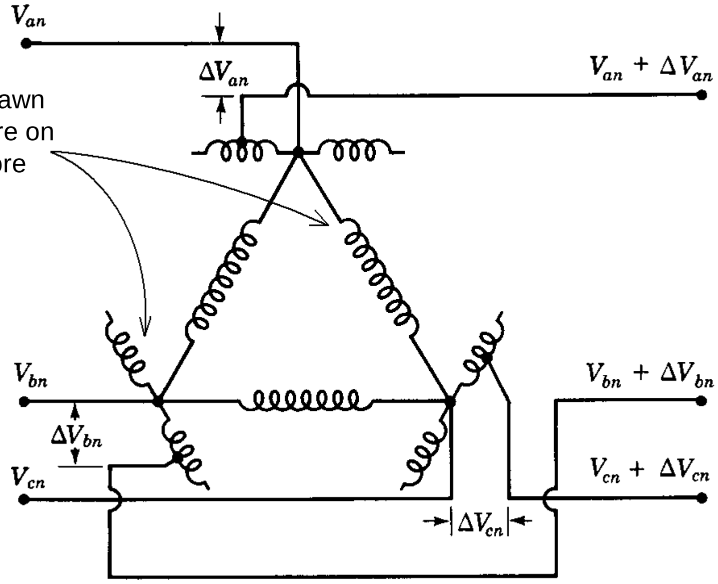
(Elgerd, Fig.5.24)

Y - Δ (usually step-down) would also give a -30^o phase shift

Single-core 3-phase transformers can also be used this way

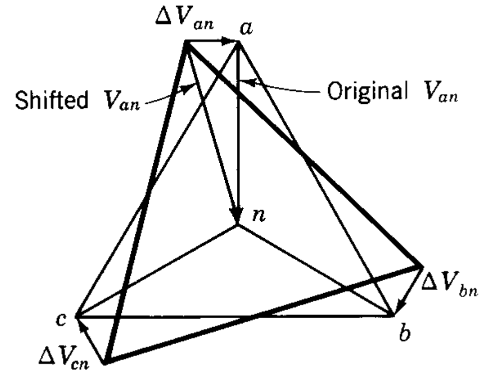
Phase-Shifting Transformers

Windings drawn in parallel are on the same core



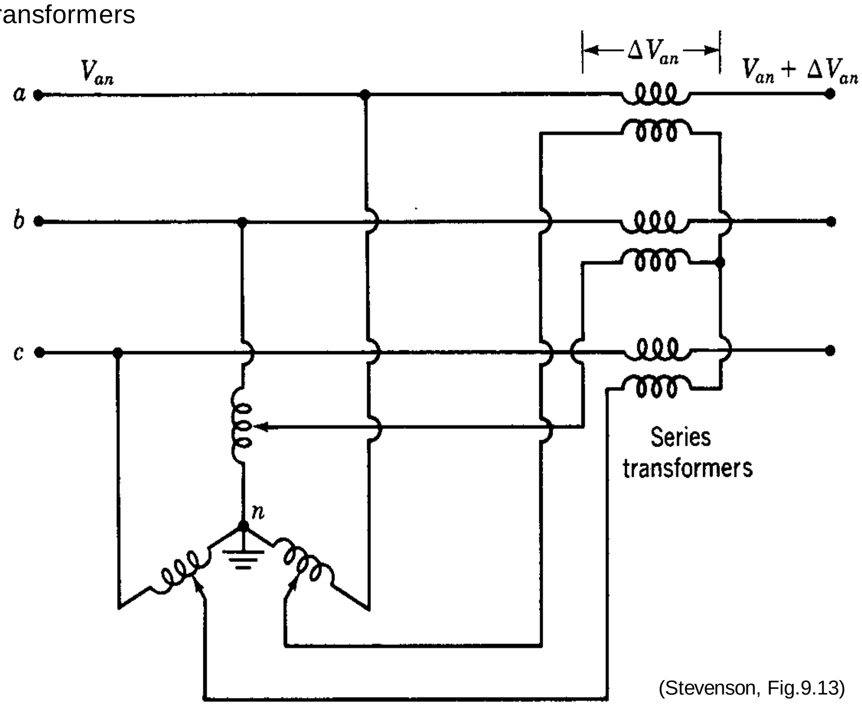
(Stevenson, Fig.9.14)

Used to control the direction of power flow on the network



(Stevenson, Fig.9.15)

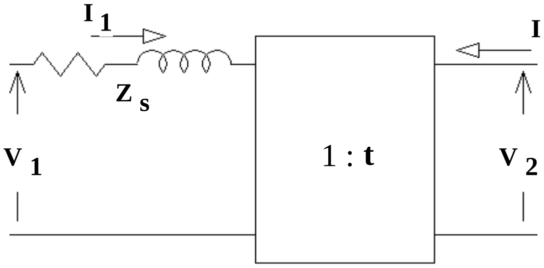
Voltage Regulating Transformers



(Stevenson, Fig.9.13)

Off-Nominal Turns Ratio

Note the weird $\mathbf{I_2}$ direction



$$Z_s = \frac{1}{Y_s}$$

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{bmatrix} Y_s & -\frac{Y_s}{t} \\ -\frac{Y_s}{\bar{t}} & \frac{Y_s}{(|t|)^2} \end{bmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

If there is a phase shift, $\mathbf{t}$ will be complex

$\bar{\mathbf{t}}$ = complex conjugate of $\mathbf{t}$

Transformer Examples

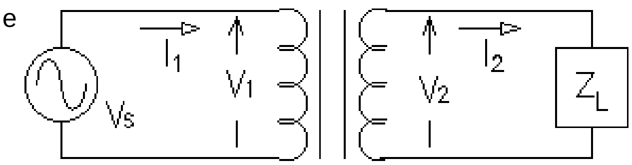
Ex1. An ideal transformer has 330 turns on the primary winding and 36 turns on the secondary. If the primary is connected across a 110 V (rms) generator, what is the rms output voltage?

$$110\cdot\text{volt}\cdot\frac{36}{330} = 12\cdot\text{volt}$$

Ex2 A transformer has $N_1 = 320$ turns and $N_2 = 1000$ turns. If the input voltage is $v(t) = (255\text{ V})\cos(\omega t)$, what rms voltage is developed across the secondary coil?

$$\frac{255\cdot\text{volt}}{\sqrt{2}}\cdot\frac{1000}{320} = 563\cdot\text{volt}$$

Ex3 A transformer is rated at 480V / 120V, 1.2kVA. Assume the transformer is ideal and all voltages and currents are RMS.



$|\mathbf{Z}_L| = 20\cdot\Omega$

$pf := 75\cdot\%$ lagging

$\mathbf{V}_L := 110\cdot V$

a) What is the current rating of the primary?

$$\frac{1.2\cdot kVA}{480\cdot V} = 2.5\cdot A$$

b) What is the current rating of the secondary?

$$\frac{1.2\cdot kVA}{120\cdot V} = 10\cdot A$$

c) The secondary has 100 turns of wire. How many turns does the primary have?

$$N_2 := 100 \qquad N_1 := \frac{480\cdot V}{120\cdot V}\cdot N_2 \qquad N_1 = 400 \text{ turns}$$

d) $\mathbf{V}_L := 110\cdot V$ How big is the source voltage ($|\mathbf{V}_S|$)?

$$\mathbf{V}_S := \frac{N_1}{N_2}\cdot\mathbf{V}_L \qquad \mathbf{V}_S = 440\cdot V$$

e) The secondary load (Z_L) has a magnitude of 20 Ω at a power factor of 75%. Find the secondary current, $\mathbf{I_2}$ (magnitude **and <u>angle</u>**).

$pf := 75\cdot\%$

$$\mathbf{I}_2 = \frac{\mathbf{V}_L}{20\cdot\Omega} = 5.5\cdot A \qquad pf = 0.75 \qquad \text{acos}(pf) = 41.4\cdot deg \qquad \mathbf{I_2} = 5.5A\ \underline{/-41.4^o}$$

f) Find the primary current, $\mathbf{I_1}$ (magnitude **and <u>angle</u>**).

$$\mathbf{I}_1 = \frac{100}{400}\cdot 5.5\cdot A = 1.375\cdot A \qquad \text{acos}(pf) = 41.4\cdot deg \qquad \mathbf{I_1} = 1.375A\ \underline{/-41.4^o}$$

Transformer is ideal, so angle is exactly the same as the load.

g) How much average power does the load dissipate?

$$P_L = |\mathbf{V}_2|\cdot|\mathbf{I}_2|\cdot pf = 110\cdot V\cdot 5.5\cdot A\cdot 75\cdot\% = 453.8\cdot\text{watt}$$

h) How much average power does the power source ($\mathbf{V_S}$) supply?

$$P_S = P_L = 454\cdot\text{watt}$$

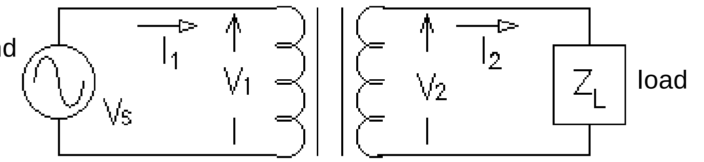
i) What is the load as seen by $\mathbf{V_S}$? (magnitude **and <u>angle</u>**)

$$\left(\frac{400}{100}\right)^2\cdot 20\cdot\Omega = 320\cdot\Omega \qquad \text{acos}(pf) = 41.4\cdot deg \qquad \mathbf{Z_{eq}} = 320\Omega\ \underline{/41.4^o}$$

Transformer Examples p2

Ex4 A transformer is rated at 480V/240V, 1.2kVA.
Assume the transformer is ideal and all voltages and currents are RMS.

How much power does the load consume?



$|\mathbf{V_S}| = 440 \cdot V$ $\quad$ $|\mathbf{Z_L}| = 16 \cdot \Omega$

$pf := 80 \cdot \%$ lagging

$$\mathbf{V_L} := \mathbf{V_S} \cdot \left(\frac{240}{480}\right) \qquad |\mathbf{V_L}| = 220 \cdot V$$

$$P_L := \frac{\left(|\mathbf{V_L}|\right)^2}{|\mathbf{Z_L}|} \cdot pf \qquad P_L = 2.42 \cdot 10^3 \cdot W$$

Ex5 The transformer shown in the circuit below is ideal. It is rated at 220/110 V, 200 VA, 60 Hz

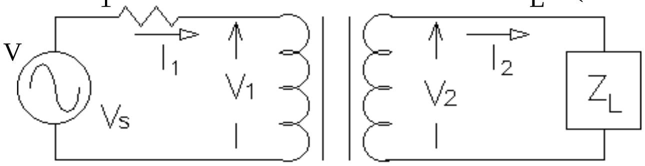
Find the following:

All values are RMS unless specified otherwise.

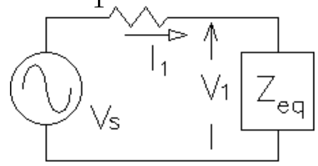
a) The primary current (magnitude).

$|\mathbf{I_1}| = ?$

$R_1 := 40 \cdot \Omega$ $\quad$ $Z_L := (15 + 20 \cdot j) \cdot \Omega$ $\quad$ $V_S := 120 \cdot V$



$R_1 = 40 \cdot \Omega$



$$Z_{eq} = \left(\frac{N_1}{N_2}\right)^2 \cdot Z_L$$

$$Z_{eq} := \left(\frac{220}{110}\right)^2 \cdot Z_L$$

$$Z_{eq} = 60 + 80j \cdot \Omega$$

$$R_1 + Z_{eq} = 100 + 80j \cdot \Omega$$

$$I_1 := \frac{V_S}{\sqrt{100^2 + 80^2} \cdot \Omega} \qquad I_1 = 0.937 \cdot A$$

b) The primary voltage (magnitude).

$|\mathbf{V_1}| = ?$

$$V_1 := I_1 \cdot \sqrt{60^2 + 80^2} \cdot \Omega \qquad V_1 = 93.7 \cdot V$$

c) The secondary voltage (magnitude).

$|\mathbf{V_2}| = ?$

$$V_2 = \frac{110}{220} \cdot V_1 = 46.85 \cdot V$$

d) The power supplied by the source.

$P_S = ?$

$$P_S = I_1^2 \cdot 100 \cdot \Omega = 87.8 \cdot W$$

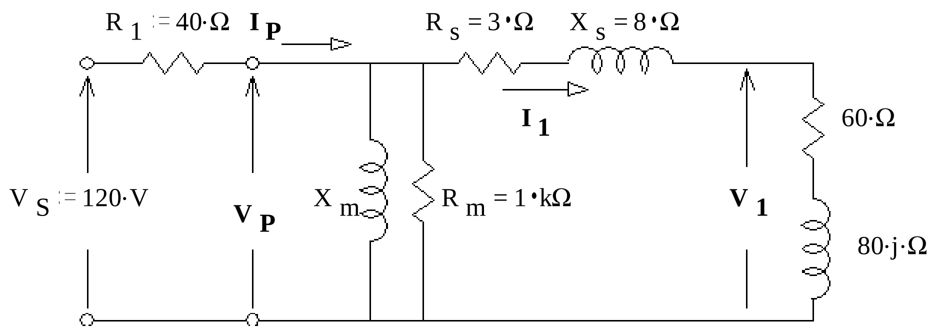
e) Is this transformer operating within its ratings? Show your evidence.

$$I_{1max} = \frac{200 \cdot VA}{220 \cdot V} = 0.909 \cdot A \quad < \quad I_1 = 0.937 \cdot A$$

NO

Ex6 Repeat Ex5 with a nonideal transformer whose characteristics are shown below.

$R_m := 1 \cdot k\Omega$ $X_m := 400 \cdot \Omega$ $R_s := 3 \cdot \Omega$ $X_s := 8 \cdot \Omega$



$$Z_{eq} := \left(\frac{220}{110}\right)^2 \cdot (15 + 20 \cdot j) \cdot \Omega$$

$$Z_{eq} = 60 + 80j \cdot \Omega$$

Find the following:

a) The primary current (magnitude).

$|\mathbf{I_P}| = ?$

$$R_s + X_s \cdot j + Z_{eq} = 63 + 88j \cdot \Omega$$

$$\frac{1}{\frac{1}{X_m \cdot j} + \frac{1}{R_m} + \frac{1}{(63 + 88 \cdot j) \cdot \Omega}} = 45.255 + 71.041j \cdot \Omega$$

$$R_1 + (45.255 + 71.041 \cdot j) \cdot \Omega = 85.255 + 71.041j \cdot \Omega$$

$$\mathbf{I_P} := \frac{120 \cdot V}{(85.255 + 71.041 \cdot j) \cdot \Omega} \qquad \mathbf{I_P} = 0.831 - 0.692j \cdot A$$

$$|\mathbf{I_P}| = 1.081 \cdot A$$

b) The primary voltage (magnitude).

$|\mathbf{V_P}| = ?$

$$\mathbf{V_P} := \mathbf{I_P} \cdot (45.255 + 71.041 \cdot j) \cdot \Omega \qquad \mathbf{V_P} = 86.771 + 27.689j \cdot V$$

$$|\mathbf{V_P}| = 91.082 \cdot V$$

c) The secondary voltage (magnitude).

$|\mathbf{V_2}| = ?$

$$\mathbf{I_1} := \frac{\mathbf{V_P}}{(63 + 88 \cdot j) \cdot \Omega} \qquad \mathbf{I_1} = 0.675 - 0.503j \cdot A$$

$$\mathbf{V_1} := \mathbf{I_1} \cdot (60 + 80 \cdot j) \cdot \Omega \qquad \mathbf{V_1} = 80.723 + 23.8j \cdot V$$

$$|\mathbf{V_1}| = 84.158 \cdot V$$

$$|\mathbf{V_2}| = \frac{110}{220} \cdot 84.16 \cdot V = 42.08 \cdot V$$

d) The power supplied by the source.

$P_S = ?$

$$P_S = 120 \cdot V \cdot \mathrm{Re}(\mathbf{I_P}) = 99.69 \cdot W$$

e) Is this transformer operating within its ratings? Show your evidence.

$$I_{1max} = \frac{200 \cdot VA}{220 \cdot V} = 0.909 \cdot A \quad < \quad |\mathbf{I_P}| = 1.081 \cdot A \qquad \text{NO}$$

f) Find the efficiency, assuming that the only useful output is from $\mathbf{Z_L}$.

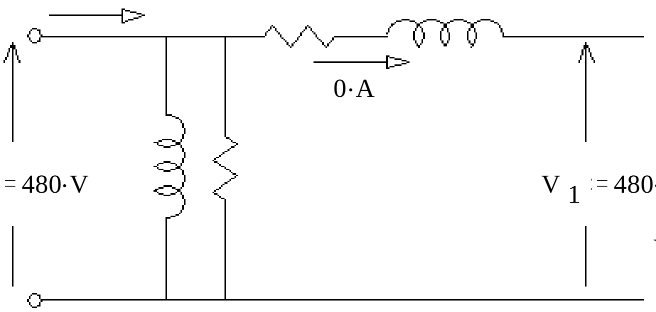
$$\eta = \frac{\left(|\mathbf{I_1}|\right)^2 \cdot 60 \cdot \Omega}{99.69 \cdot W} \cdot 100 \cdot \% = 42.63 \cdot \%$$

Transformer Examples p4

Ex7 Find the voltage regulation and full-load efficiency of the transformer whith the following ratings and characteristics.

Rated at 480/120 V, 2 kVA, 60 Hz $\quad R_m := 8.4\cdot k\Omega \quad X_m := 2\cdot k\Omega \quad R_s := 5\cdot\Omega \quad X_s := 15\cdot\Omega$

No load:

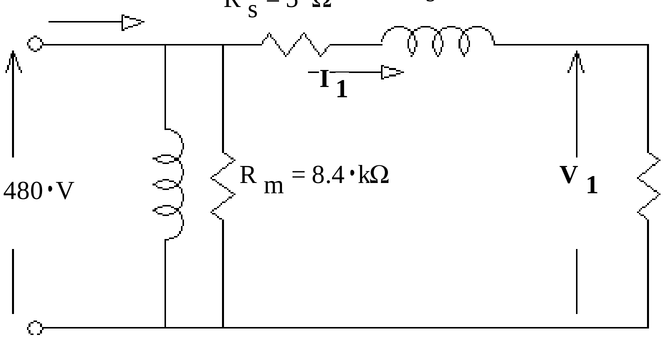


$V_S := 480\cdot V \qquad V_1 := 480\cdot V \qquad V_2 := 120\cdot V$

$V_{NL1} := 480$

Full load: Roughly: $R_L := \frac{(120\cdot V)^2}{2000\cdot W} \qquad R_L = 7.2\cdot\Omega$

$R_s = 5\cdot\Omega \qquad j\cdot X_s = 15j\cdot\Omega \qquad \left(\frac{480}{120}\right)^2\cdot R_L = 115.2\cdot\Omega$



$V_S = 480\cdot V$

$\mathbf{I_1} := \frac{V_S}{(120.2 + 15j)\cdot\Omega} \qquad \mathbf{I_1} = 3.932 - 0.491j \cdot A$

$\mathbf{V_1} := \mathbf{I_1}\cdot 115.2\cdot\Omega \qquad \mathbf{V_1} = 452.979 - 56.528j \cdot V$

$V_{FL1} := |\mathbf{V_1}| \qquad V_{FL1} = 456.493\cdot V$

Voltage regulation: $\%VR = \frac{V_{no\_load} - V_{full\_load}}{V_{full\_load}}\cdot 100\cdot\% = \frac{480 - 456.493}{456.493}\cdot 100\cdot\% = 5.149\cdot\%$

Efficiency $\eta = \frac{P_{out}}{P_{in}}\cdot 100\cdot\% = \frac{P_{out}}{P_{out} + P_{losses}}\cdot 100\cdot\%$

$P_{out} := \left(|\mathbf{I_1}|\right)^2\cdot 115.2\cdot\Omega \qquad P_{out} = 1.809\cdot kW$

$P_{losses} := \left(|\mathbf{I_1}|\right)^2\cdot 5.\cdot\Omega + \frac{(480\cdot V)^2}{8400\cdot\Omega} \qquad P_{losses} = 0.106\cdot kW$

$\eta = \frac{P_{out}}{P_{out} + P_{losses}}\cdot 100\cdot\% = 0.945$

Ex8 A 500/100-V, 2.5-kVA transformer is subjected to an OC test and a SC test with the results below.

a) Draw a model of this transformer and find the values of all the elements of the model, including the turns ratio.

During the open-circuit test: $I_{OC} := 0.5 \cdot A$ $\quad P_{OC} := 150 \cdot W$

During the short-circuit test: $V_{SC} := 22 \cdot V$ $\quad P_{SC} := 70 \cdot W$

$S_{rated} := 2.5 \cdot kVA$ $\quad V_{rated} := 500 \cdot V$ $\quad I_{rated} := \dfrac{S_{rated}}{V_{rated}}$ $\quad I_{rated} = 5 \cdot A$

Open-circuit test used to find R_m and X_m

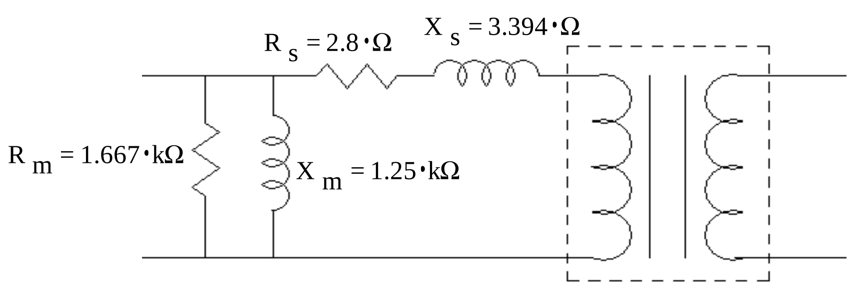
$V_{OC} := V_{rated}$ $\quad R_m := \dfrac{V_{OC}^2}{P_{OC}}$ $\quad R_m = 1.667 \cdot k\Omega$

$Q_{OC} := \sqrt{\left(V_{OC} \cdot I_{OC}\right)^2 - P_{OC}^2}$ $\quad Q_{OC} = 200 \cdot VAR$ $\quad X_m := \dfrac{V_{OC}^2}{Q_{OC}}$ $\quad X_m = 1.25 \cdot k\Omega$

Short-circuit test used to find R_s and X_s

$I_{SC} := I_{rated}$ $\quad R_s := \dfrac{P_{SC}}{I_{SC}^2}$ $\quad R_s = 2.8 \cdot \Omega$

$Q_{SC} := \sqrt{\left(V_{SC} \cdot I_{SC}\right)^2 - P_{SC}^2}$ $\quad Q_{SC} = 0.085 \cdot kVAR$ $\quad X_s := \dfrac{Q_{SC}}{I_{SC}^2}$ $\quad X_s = 3.394 \cdot \Omega$

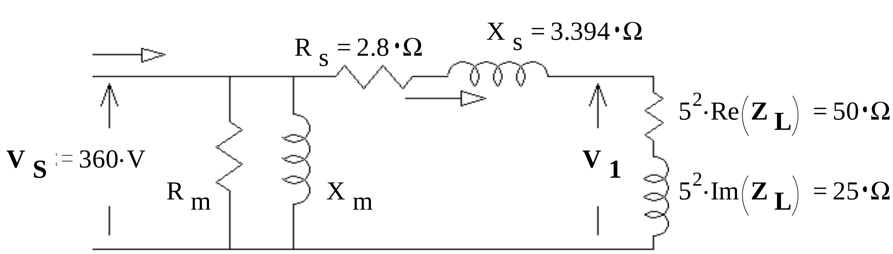


5:1 Ideal transformer

b) The transformer is connected to a primary source voltage of 360V and loaded with $\mathbf{Z_L} := (2 + 1 \cdot j) \cdot \Omega$.

(you may add these parts to your drawing if you wish.)

Find the secondary voltage. Magnitude only. $\left|\mathbf{V_2}\right| = ?$



$$|\mathbf{V_1}| = V_1 := \mathbf{V_S} \cdot \frac{\sqrt{(50\cdot\Omega)^2 + (25\cdot\Omega)^2}}{\sqrt{(R_s + 50\cdot\Omega)^2 + (X_s + 25\cdot\Omega)^2}} \qquad V_1 = 335.687 \cdot V$$

$$|\mathbf{V_2}| = V_2 = \frac{V_1}{5} = 67.14 \cdot V$$

c) Is this transformer operating within its ratings? Show all evidence and calculate needed to to determine this.

$$|\mathbf{I_1}| = I_1 := \frac{\mathbf{V_S}}{\sqrt{(R_s + 50\cdot\Omega)^2 + (X_s + 25\cdot\Omega)^2}}$$

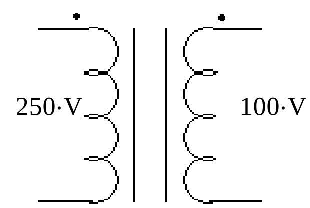
$$I_1 = 6.005 \cdot A \quad > \quad I_{rated} = 5 \cdot A$$

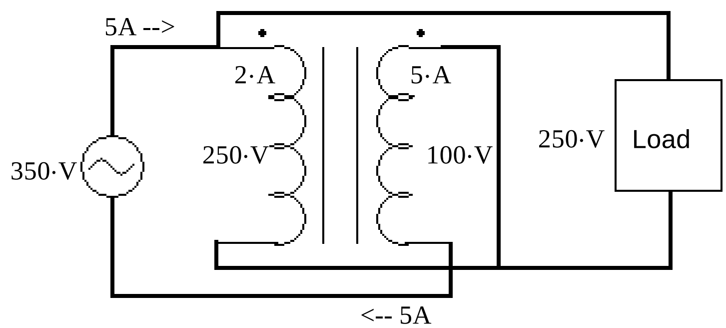
NO !

Ex9

a) You have a 250/100-V, 500-VA transformer. Show the necessary connections to use this transformer to transform 350 V to 250 V. Also show the 350-V source and the load.

b) Connected this way, determine the maximum power that could be converted from 350 V to 250 V without overloading the transformer.





ratings: $\frac{500\cdot VA}{250\cdot V} = 2 \cdot A \qquad \frac{500\cdot VA}{100\cdot V} = 5 \cdot A$

new VA rating and maximum power: $5\cdot A\cdot 350\cdot V = 1.75 \cdot kW$

c) besides the right impedance magnitude, what other characteristic must the load posses in order to actually use this much power?

Load must be purely resistive (power factor is 1).

d) Could this transformer also be used to transform 280 V to 200 V ? If yes, what is the maximum power that could be transformed?

Same connections as above Maximum power: $5\cdot A\cdot 280\cdot V = 1.4 \cdot kW$

ECE 3600 homework # 6 Due: Fri, 9/18/20

b

1. Textbook problem 1-7 (p49)
2. Textbook Example 1-2 (p20) with

Mean magnetic length:	$l_c := 50 \cdot cm$
Air gap length:	$l_a := 0.06 \cdot cm$
Core cross-sectional area:	$A_c := 16 \cdot cm^2$

Answers 1. $0.0054 \cdot Wb$ 2. $727 \cdot mA$

ECE 3600 homework # 7 Due: Tue, 9/22/20

b

Ideal transformers

1. A step-up transformer is designed to have an output voltage of 2200 V (rms) when the primary is connected across a 240 V (rms) source.

   a) If there are 150 turns on the primary winding, how many turns are required on the secondary?

   b) If a load resistor across the secondary draws a current of 1.2 A, what is the current in the primary, assuming ideal conditions?

2. An ideal transformer has a turns ratio ($N = N_1/N_2$) of 1.5 . It is desired to operate a 200 Ω resistive load at 150 V (rms).

   a) Find the secondary and primary currents.

   b) Find the source voltage ($\mathbf{V_1}$).

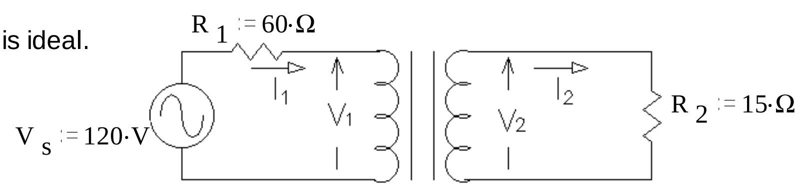
   c) Find the power dissipated in the load resistor and the power delivered to the primary from the source.

   d) Find the impedance the source sees looking into the primary winding by calculating $\mathbf{Z_{eq}} = N^2 \mathbf{Z_L}$ and again by calculating $\mathbf{V_1} / \mathbf{I_1}$.

3. The transformer shown in the circuit below is ideal. It is rated at 120/30 V, 80 VA, 60 Hz
   Find the following:

   a) I_1 = ?

   b) V_2 = ?



4. An ideal transformer has a rating of 500/125 V, 10 kVA, 60 Hz. It is loaded with an impedance of 5Ω at 80% pf (0.80). The source voltage applied to the primary winding is 440 V (rms). Find:

   a) the load voltage
   b) the load current
   c) the kVA delivered to load
   d) the power delivered to load
   e) the primary current
   f) the power factor of primary
   g) the impedance the source sees looking into primary.

5. An ideal transformer is rated to deliver 400 kVA at 460 V to a customer.

   a) How much current can the transformer supply to the customer?

   b) If the customer's load is purely resistive (i.e. if the pf = 1), what is the maximum power the customer can receive?

   c) If the customer's power factor is 0.8 (lagging), what is the maximum usable power the customer can receive?

   d) What is the maximum power if the power factor is 0.7 (lagging)?

   e) If the customer requires 300 kW to operate, what is the minimum allowable power factor given the rating of this transformer?

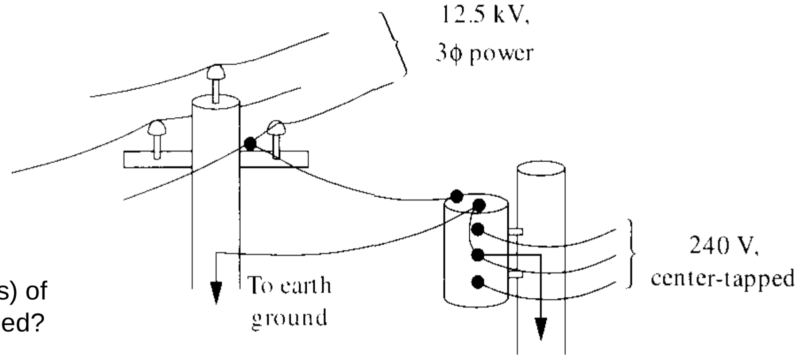
Answers

1. a) 1375 turns b) 11 A 2. a) 0.75 A, 0.50 A b) 225 V c) 112.5 W d) 450Ω

3. a) $0.4 \cdot A$ b) 24V 4. a) 110 V b) 22 A c) 2.42 kVA d) 1.94 kW e) 5.5 A f) 0.80 g) 80Ω /36.9º Ω

5. a) $870 \cdot A$ b) $400 \cdot kW$ c) $320 \cdot kW$ d) $280 \cdot kW$ e) 0.75

ECE 3600 homework # 8 Due: Fri, 9/25/20



1. The city of Murray, Utah. distributes power to neighborhoods with a 12.47-kV three-phase system. (12.47 kV is the line voltage.) Each group of houses is served from one phase and ground, and transformed to 240/120 V by a pole transformer, as shown.

   a) What is the turns ratio (primary/secondary turns) of the pole transformer to give 240 V, center-tapped?

   b) When a 1500-W toaster-oven is turned on, how much does the current increase in the high-voltage wire? Assume the power factor is unity and the transformer is 100% efficient.

   c) Repeat b) for a clothes drier that draws 15 A.

2. A single-phase transformer is designed to operate at 60 Hz. Its voltage ratings are: primary, 500 V; secondary, 200 V. The maximum permissible load is 30 kVA.

   a) What will be the magnitudes of primary and secondary currents when the device is full-loaded?

   b) Loading is accomplished by an impedance connected across the 200-V terminals. How many ohms will correspond to full-load of the transformer? (Use IT model.)

3. 5.3 The 30-kVA transformer of the problem above is made subject to a SC test. One winding is short-circuited and the other winding is fed from a 60-Hz voltage source. The voltage is raised until rated current is circulated in the windings. This occurs when the applied voltage equals 5.11% of rated winding voltage. The transformer consumes 290 W during the test.

   a) Compute the series impedance $\mathbf{Z_s} = R_s + jX_s$ of the transformer referred to primary and secondary sides.

   b) Compute the core flux during the SC test. (Express its magnitude in percent of normal operating flux.)

   c) Why is it permissible to assume that all of the 290 W constitute ohmic losses in R, and no part of it is core loss?

4. 5.5 The transformer in Exercise 5.3 is fed from a 500-V source. A load impedance of $Z_L := (1.03 + j\cdot 0.72)\cdot\Omega$ is connected across the secondary.

   a) Find the currents in both windings and the secondary voltage by use of the IT model.

   b) Same as in part a) but now include the transformer impedance in your analysis. Take note of the change in your answers.

   c) Is the transformer current-overloaded?

5. 5.6 The same 30-kVA transformer is made subject to an OC test. It is fed from a 500-V source with the secondary open. The transformer consumes 230 W and draws 0.8 A.

   a) Find R_m and X_m.

   b) Based upon the SC and OC test data compute the efficiency of the transformer when loaded with the $\mathbf{Z_L}$ impedance above.

Answers

1. a) 30 b) 208·mA

2. a) 60·A 150·A b) 1.333·Ω

3. a) $0.0806 + 0.418\cdot j\cdot\Omega$ $0.0129 + 0.0669\cdot j\cdot\Omega$ b) 5.11% of normal rated value.

   c) Because the core flux is 5.11% of normal rated value, the core losses (which are approximately proportional to the square of the flux) are negligible.

4. a) 63.66·A 159.2·A b) 61.23·A 153.1·A c) Yes, a little

5. a) 1.09·kΩ 0.76·kΩ b) 97.8·%