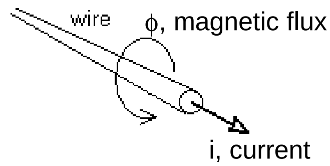
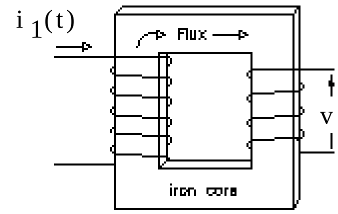
Electromagnetics basics

1. Electric currents produce magnetic fields.



Right-hand-rule

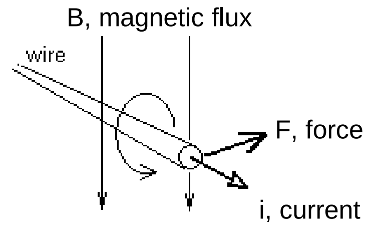
2. A fluctuating magnetic field passing through a coil of wire will induce a voltage in that coil. Basis of transformer secondary, and primary too (back EMF).



$$v_2(t) = N\cdot\frac{d}{dt}\phi$$

3. A wire with a current in the presence of another magnetic field feels a force.

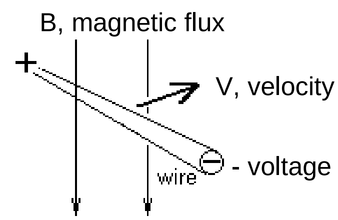
   (Basis of electric motors, also explains why generators resist the mechanical input)



F, force = $i\cdot B\cdot \ell$

ℓ = length of the wire

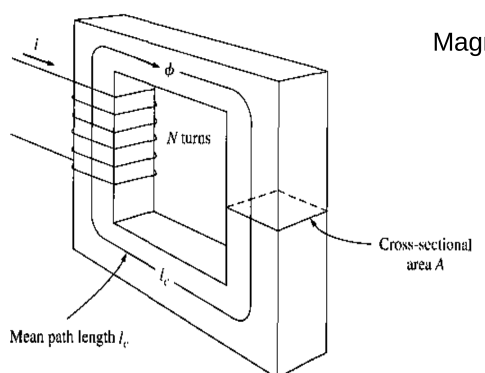
4. A voltage will be induced on wires moving in the presence of a magnetic field. This is very similar to 2. (Basis of electric generators)



1. Magnetic field from a current

Ampere's law: $\oint \mathbf{H}\cdot\mathbf{dl} = I_{net}$

For us...



(Chapman, Fig.1.3)

Magnetic field intensity: $H = \frac{N\cdot i}{\ell_c}$ $\left(\frac{A\cdot turns}{meter}\right)$

Ampere-turns: = $N\cdot i$
(like voltage)

Flux density: $B = \mu\cdot H = \frac{\mu\cdot N\cdot i}{\ell_c}$ (tesla, T)

Flux: $\phi = B\cdot A$ (weber) (Wb)
(like current)

Permeability of free space: $\mu_o := 4\cdot\pi\cdot 10^{-7}\cdot\frac{henry}{m}$

Relative permeability: μ_r

Permeability: $\mu = \mu_r\cdot\mu_o$

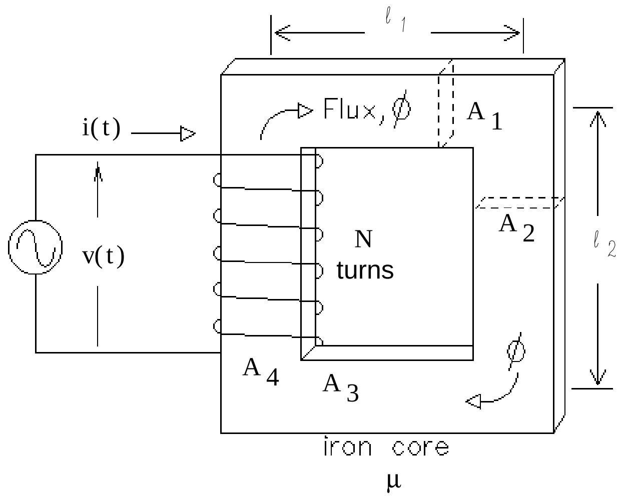
Reluctance of core: $\mathcal{R}_c = \frac{\ell_c}{\mu\cdot A_c}$ $\left(\frac{A\cdot turns}{Wb}\right)$
(like resistance)

Flux: $\phi = \frac{N\cdot i}{\mathcal{R}_c}$ (weber, Wb)

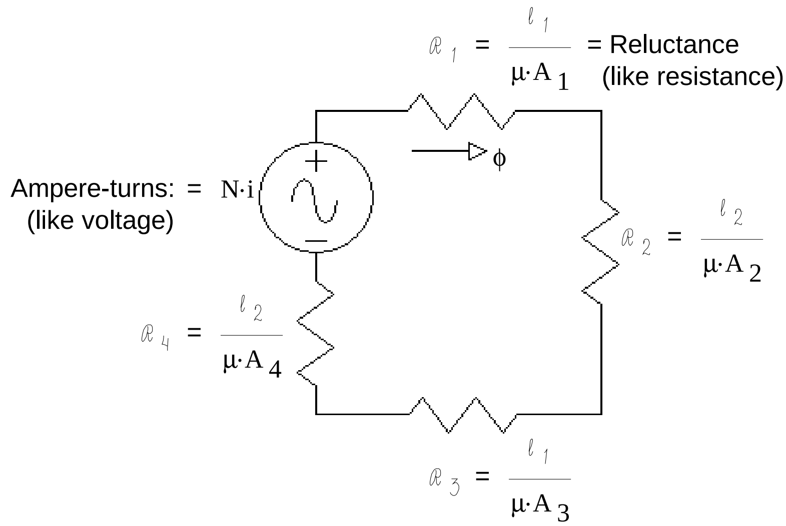
Inductance: $L = \frac{N^2}{\mathcal{R}_c} = N^2\cdot\left(\frac{\mu_r\cdot\mu_o\cdot A_c}{\ell_c}\right)$ (henry, H)

material	relative permeability μ_r
Mu-metal	20000
Permalloy	8000
Electrical steel	4000
ferrite (nickel zinc)	16 - 640
ferrite (manganese zinc)	>640
Steel	700
Nickel	100

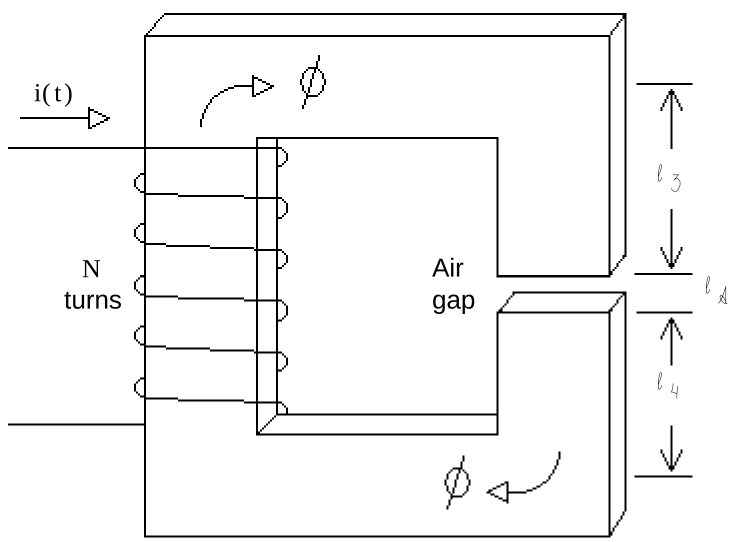
Magnetic "Circuits"

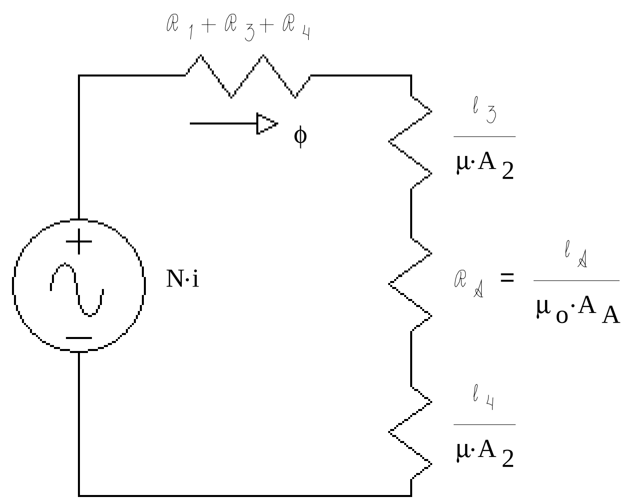


Similar to this electric circuit



magnetic flux = $\phi = \dfrac{N \cdot i}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}$ (weber) (Wb)
(like current)





$$\phi = \frac{N \cdot i}{\mathcal{R}_1 + \mathcal{R}_3 + \mathcal{R}_4 + \dfrac{\ell_3 + \ell_4}{\mu \cdot A_2} + \mathcal{R}_A} \quad \text{(weber) (Wb)}$$

Flux density: $B = \dfrac{\phi}{A} = \mu \cdot H$ (tesla, T)

Magnetic field intensity: $H = \dfrac{B}{\mu} = \dfrac{\phi}{A \cdot \mu}$ $\left(\dfrac{A \cdot \text{turns}}{\text{meter}}\right)$

$$v(t) = N \cdot \frac{d}{dt}\phi = N \cdot \frac{d}{dt} B \cdot A$$

$$= -N \cdot \frac{d}{dt}\phi = -N \cdot \frac{d}{dt} B \cdot A$$ often shown with a negative sign

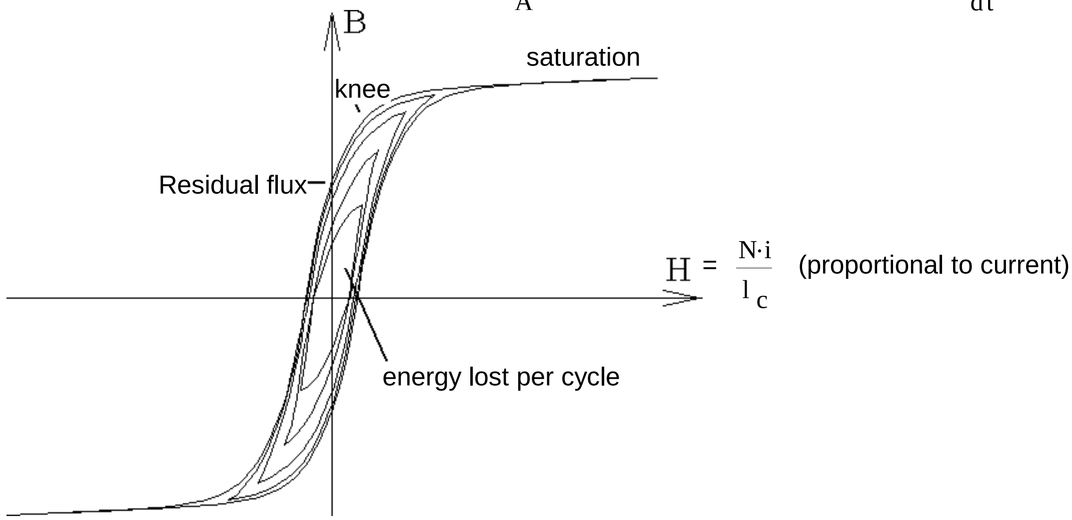
- indicates that this voltage tries to
produce a current to oppose the change.

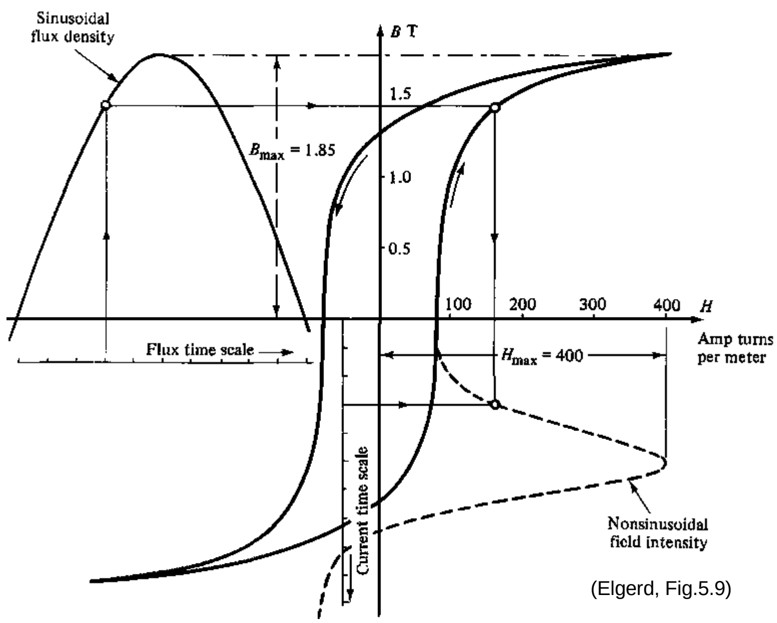
Non-ideal Ferrromagnetic materials (B-H curve)

Magnetics are not really linear

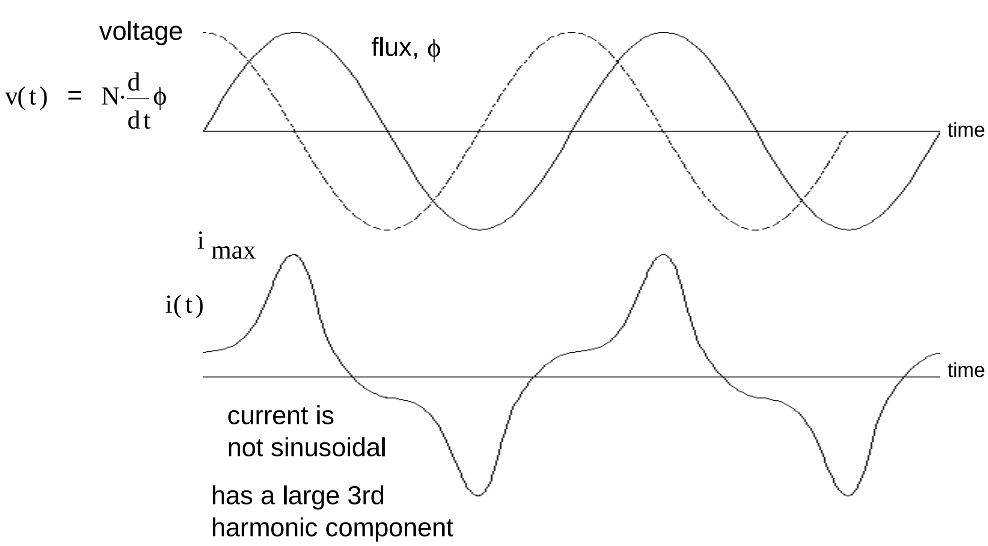
The B-H or Hystereses curve

$B = \mu \cdot H = \frac{\phi}{A}$ (proportional to voltage) $v(t) = N \cdot \frac{d}{dt}\phi = N \cdot \frac{d}{dt} B \cdot A$





(Elgerd, Fig.5.9)



Sources: Electric Machinery and Power System Fundamentals, Stephen J. Chapman
Basic Electric Power Enineering, Ollie I. Elgerd

Transformer basics and ratings

A Transformer is two coils of wire that are magnetically coupled.

Transformers are only useful for AC, which is one of the big reasons electrical power is generated and distributed as AC.

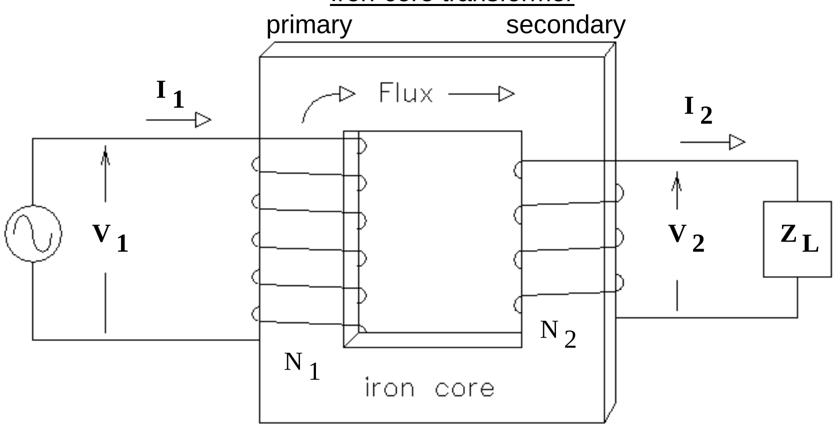
Transformer turns and turns ratios are rarely given, V_p/V_s is much more common where V_p/V_s is the rated primary voltage over rated secondary voltage. Ideally, you may take this to be the same as N_1/N_2 although in reality N_2 is usually a little bit bigger to make up for losses. Another commonway to show the same thing: $V_p : V_s$.

Transformers are rated in VA   Transformer Rating (VA) = (rated V) x (rated I) , on either side.

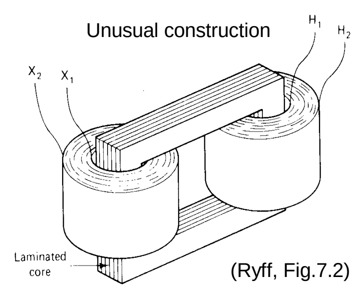
Don't allow voltages over the rated V, regardless of the actual current.
Don't allow steady-state currents over the rated I, regardless of the actual voltage.
Short-term inrush and startup currents may be higher as long as there's no overheating.

Ideal Transformers

Iron-core transformer



Unusual construction



(Ryff, Fig.7.2)

Ideal: $P_1 = P_2$

power in = power out

Transformation of voltage and current

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

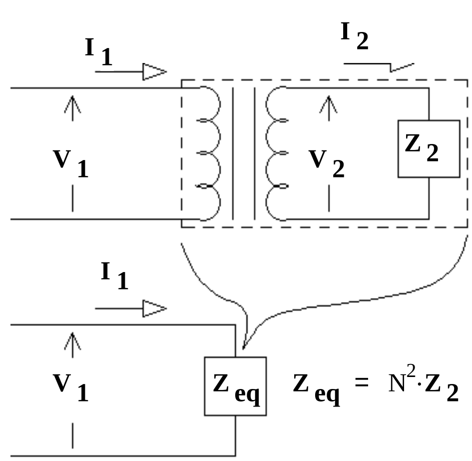
Turns ratio

Turns ratio as defined in Chapman text: $a = \frac{N_1}{N_2}$, same as $N = \frac{N_1}{N_2}$

Note: some other texts define the turns ratio as: $\frac{N_2}{N_1}$

Be careful how you and others use this term

Transformation of impedance



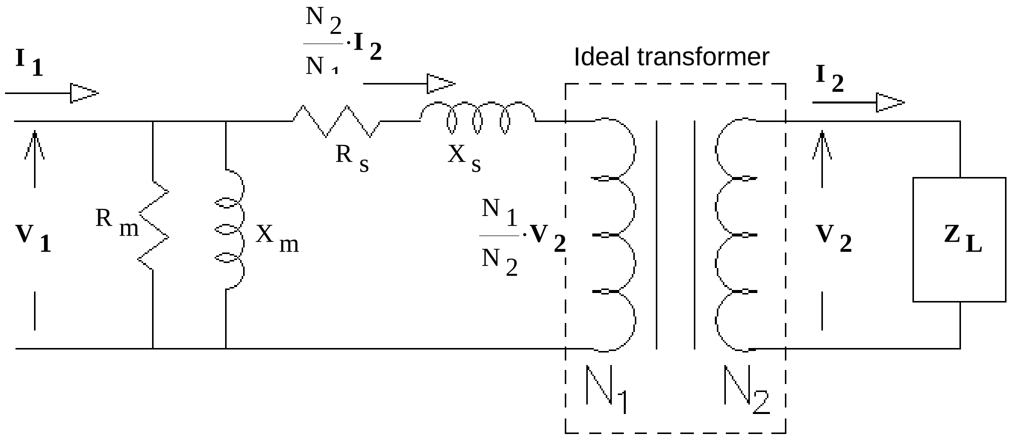
You can replace the entire transformer and load with (Z_{eq}). This "impedance transformation" can be very handy.

$$Z_{eq} = N^2 \cdot Z_2 = \left(\frac{N_1}{N_2}\right)^2 \cdot Z_2$$

Transformers can be used for "impedance matching"

This also works the opposite way, to move an impedance from the primary to the secondary, multiply by: $\left(\frac{N_2}{N_1}\right)^2$

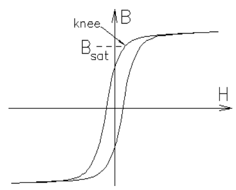
Model of non-ideal Transformer

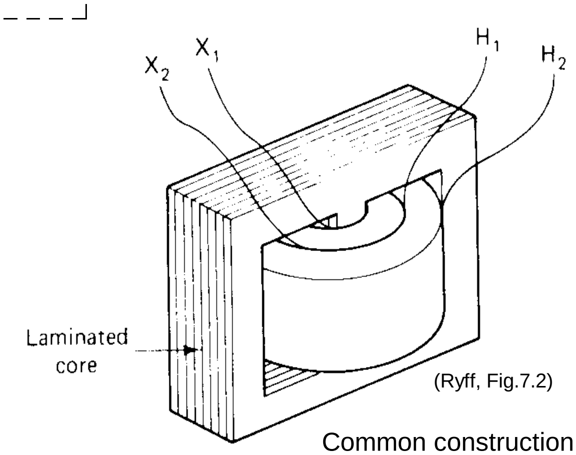


R_m - Core losses

Eddy-current losses - minimized by laminating the core and adding silicon to raise the resistivity

Hysteresis losses - caused by the B-H hysteresis curve





(Ryff, Fig.7.2)

Common construction


(Parts Express)

X_m - basic inductance caused by the need to magnetize the core

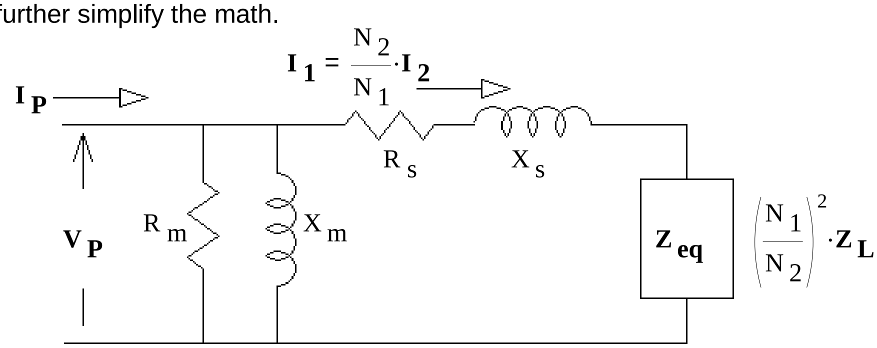
R_s - Winding resistance (copper) losses

X_s - Reactance caused by flux leakage (leakage reactance)

Actually, a more accurate model would have R_m and X_m to the right of R_s and X_s because the magnetization current still has to pass through the windings, but this model is simpler to work with and accurate enough.

Move the load impedance to further simplify the math.

Note:
I_P can be slightly larger than the rated current to makeup for losses.



Typical calculations

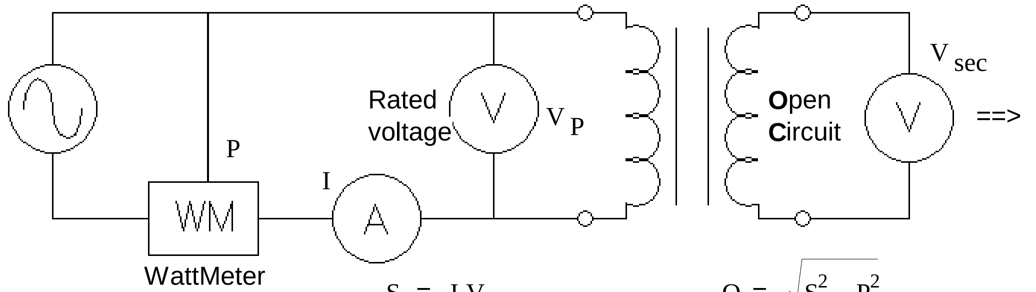
Voltage regulation $\%VR = \frac{V_{no\_load} - V_{full\_load}}{V_{full\_load}} \cdot 100 \cdot \%$

Note: A low $\%VR$ is good and a high $\%VR$ is bad, counter-intuitive.

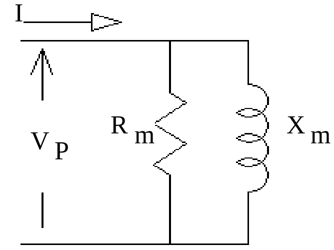
Efficiency $\eta = \frac{P_{out}}{P_{in}} \cdot 100 \cdot \%$

Tests to find parameters ECE 3600 Transformer notes p3

Open-**C**ircuit test



Model reduces to



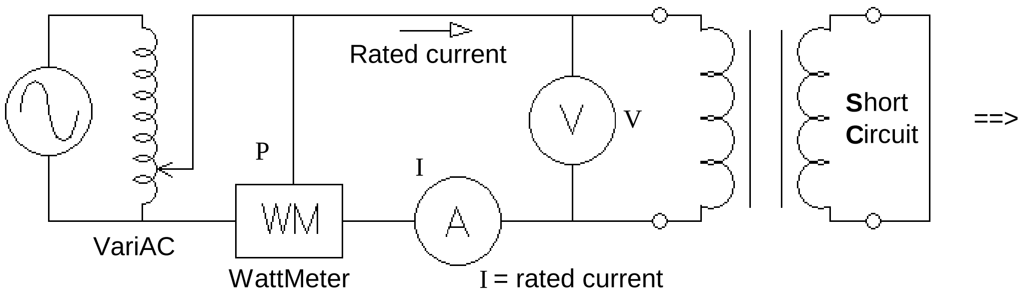
$S = I \cdot V_P$ $\qquad Q = \sqrt{S^2 - P^2}$ $\qquad R_m = \dfrac{V_P^{\,2}}{P}$ $\qquad X_m = \dfrac{V_P^{\,2}}{Q}$

V_P = rated voltage

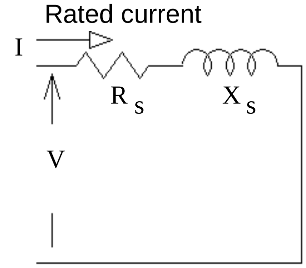
I = "Magnitization current"

Turns ratio to use in the model: $\dfrac{N_1}{N_2} = \dfrac{V_P}{V_{sec}}$

Short-**C**ircuit test



Rated current



I = rated current

$S = I \cdot V$ $\qquad Q = \sqrt{S^2 - P^2}$ $\qquad R_s = \dfrac{P}{I^2}$ $\qquad X_s = \dfrac{Q}{I^2}$

Determining $\dfrac{N_1}{N_2}$ if you're working from transformer ratings and parameters.

Manufacturers of transformers are well aware of R_s and X_s and how they reduce the output voltage, so they add a few windings (1 - 5%) to the secondary in order to make up for the loss. This lowers the effective turns ratio of the ideal transformer in the model by the same 1 - 5%.

If you're given transformer ratings as V_{Prated}/V_{Srated}, S_{rated} along with R_s and X_s , what turn ratio (N) would the manufacturer actually use for the transformer?

The following calculations are based on: $V_P = V_{Prated}$ $\quad V_S = V_{Srated}$ $\quad P_{out} = S_{rated}$ and $pf = 1$

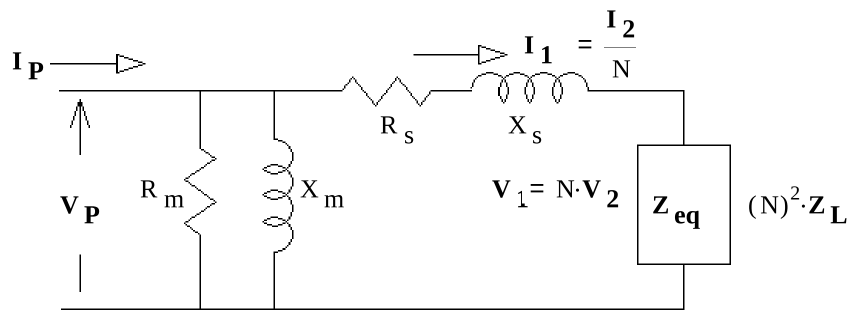
Then: $R_L = \dfrac{V_{Srated}^{\,2}}{S_{rated}}$ define: $R_x = \dfrac{V_{Prated}^{\,2}}{S_{rated}} - 2 \cdot R_s$ for ease of calculation below

R_L , referred to primary side = $R_{eq} = \dfrac{R_x + \sqrt{R_x^{\,2} - 4 \cdot \left(R_s^{\,2} + X_s^{\,2}\right)}}{2}$ and, $N = \sqrt{\dfrac{R_{eq}}{R_L}}$

Finding R_{eq} required lots of messy algebra, which I'm skipping here.

Just use the calculations above as formulas if you're not given a value for N along with the other parameters.

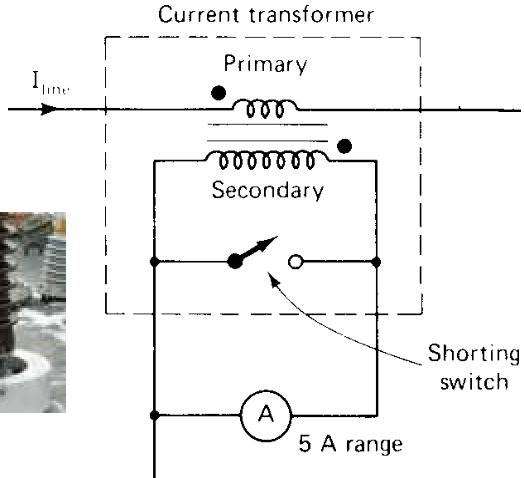
The model becomes:



Special Sensing Transformers

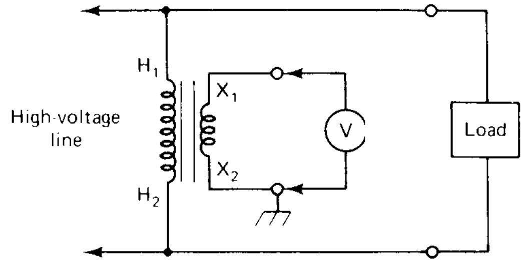
(IET Wiley)

CT



(Ryff, Fig.7.16)

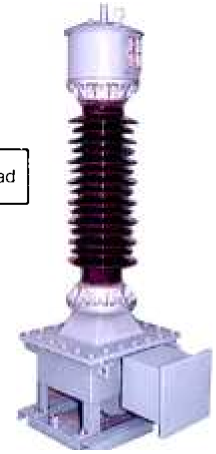
The secondary **must** always be shorted or nearly shorted!



(Ryff, Fig.7.17)

"Potential" transformer, for voltage monitoring

VT or PT



base grounded

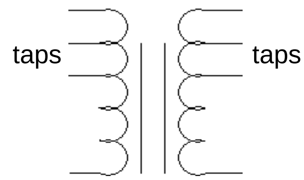
(powerlinedelhi)

Many newer PTs are actually voltage dividers, not transformers.

Other Transformers

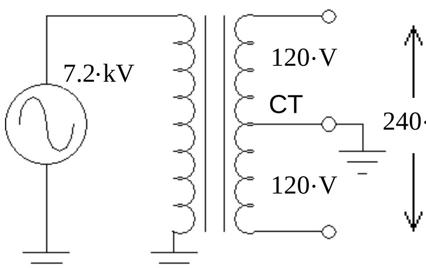
Multi-tap transformers

Many transformers have more than two connections to primary and/or the secondary. The extra connections are called "taps" and may allow you to select from several different voltages or get more than one voltage at the same time.

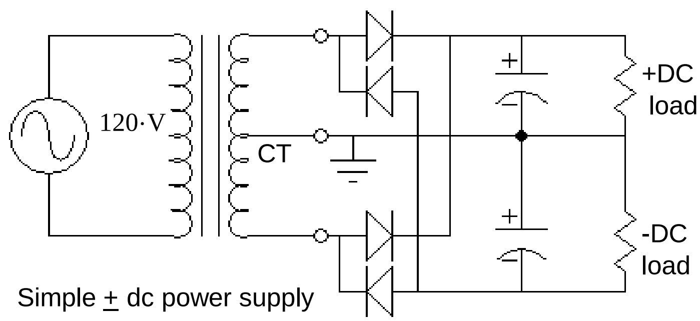


A center tap is very common.

(Wikipedia)

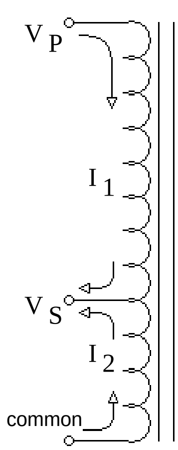


Typical tranformer for residential distribution



Simple $\pm$ dc power supply

Autotransformers



step-down

Single-winding transformers where the primary and secondary share windings. For step-down, the secondary is some fraction of the primary. For step-up, the primary is some fraction of the secondary.

Because of the way the currents flow within the windings, the current of the low-voltage side is greater than any current within the windings. Less current meas that autotransformers can be economical.

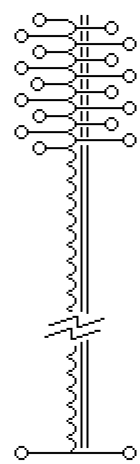
A variAC is an adjustable autotransformer.

Normal transformers can also be wired as autotransformers. More info to come.

Load tap changing

Multiple taps near the top of the transformer can be used to boost or buck (reduce) the voltage a bit. Transformers like this are often used in substations for voltage regulation. Typically, they can adjust the voltage $\pm$ 10% in 33 steps (0.625% per step). Those that can change taps while under load are called "Load tap changing". They can either be regular transformers or autotransformers, the latter are usually just called "voltage regulators". Most can be set up to work automatically.

The tap changing circuitry is not shown at right. It can be rather tricky in that it can not short two taps together nor can it open the circuit during switching.



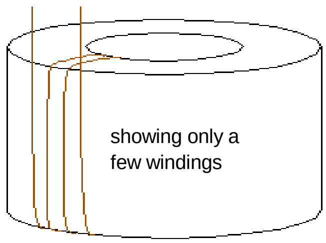
Voltage Regulator

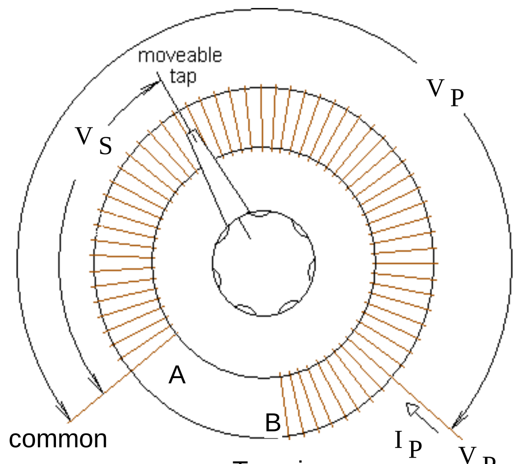
Isolation Transformers

All transformers (except autotransformers) isolate the primary from the secondary. An Isolation transformer has a 1:1 turns ratio and is just for isolation.

VariAC-type Autotransformer

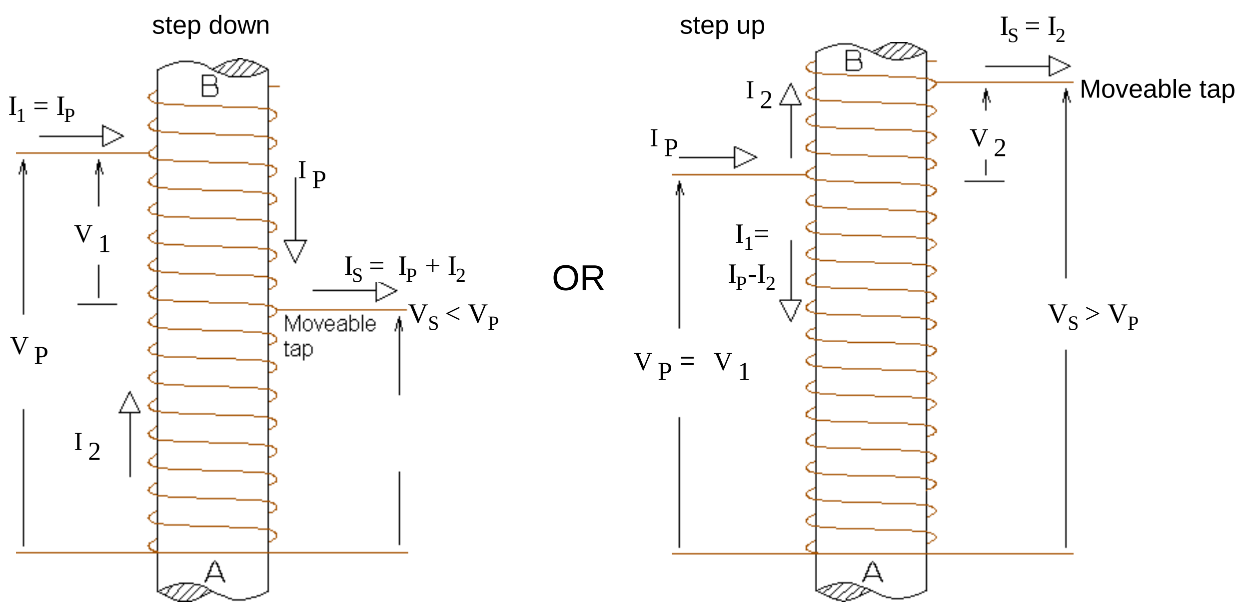
This adjustable autotransformer is wound on a toroidal core.



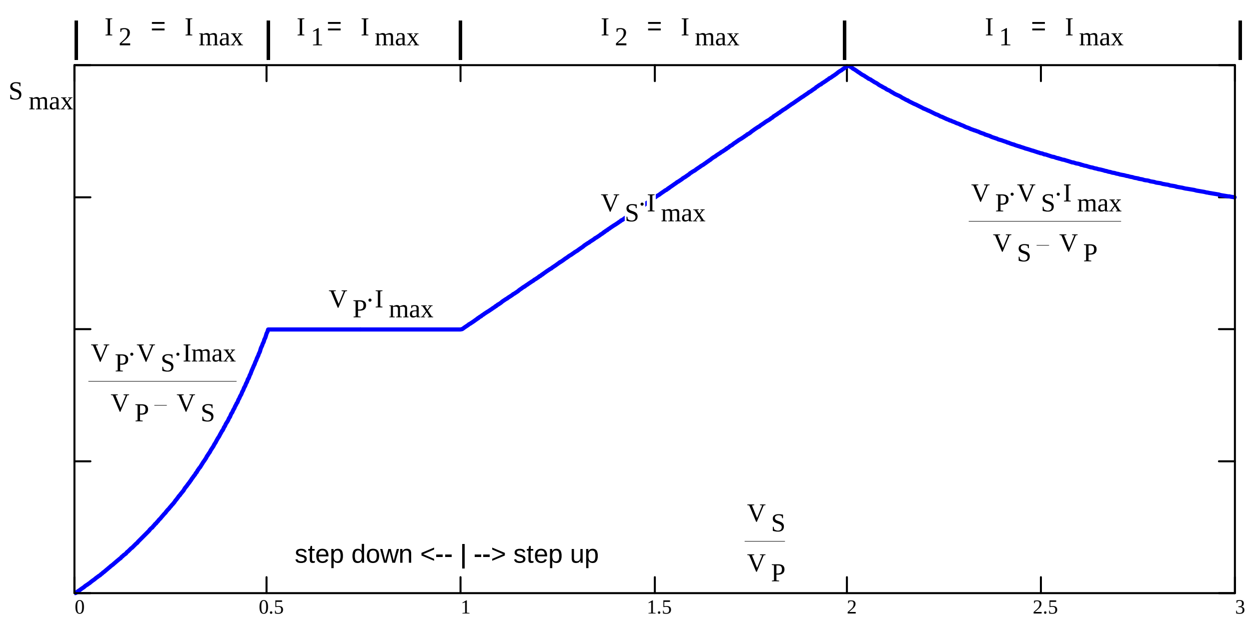


(Staco Energy)

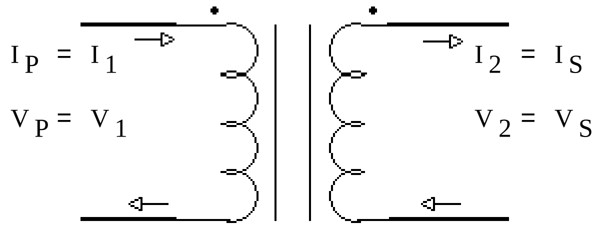
If you cut the toroid open and straightened it out, you would get the views below.



Vari-AC type autotransformer "Rating" , Based or the maximum winding current: I_{max}



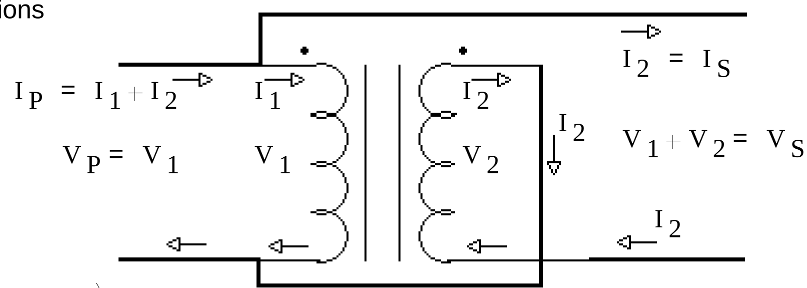
Regular winding connections



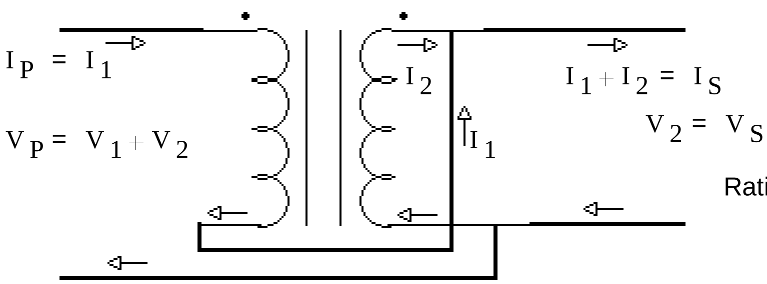
Auto Transformer Connections

4 basic possibilities

Addition connections



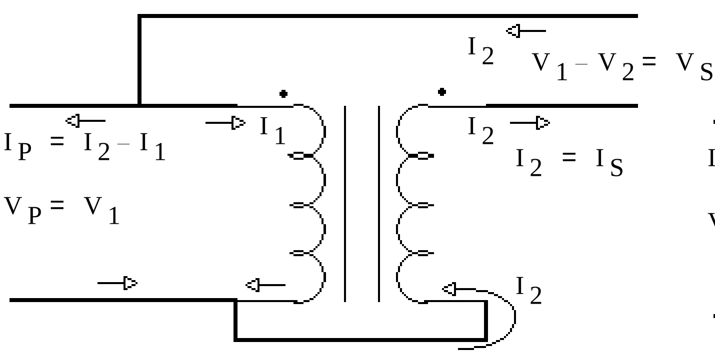
Rating:
$(I_{1\_rated} + I_{2\_rated}) \cdot V_P$

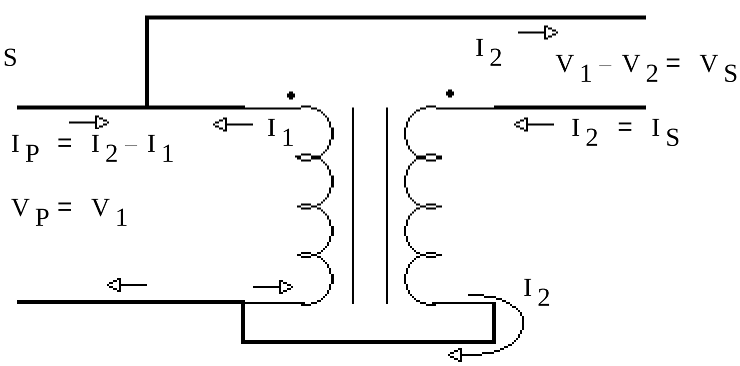


Rating: $(I_{1\_rated} + I_{2\_rated}) \cdot V_S$

Subtraction connections

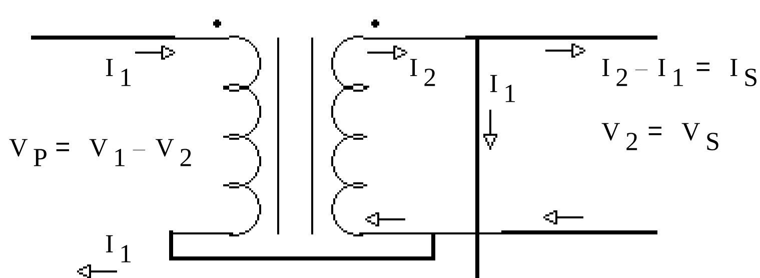
Currents I_1 and I_2 are flowing reverse of normal.





Rating: $(I_{2\_rated} - I_{1\_rated}) \cdot V_P$

Power Flow Through Transformer



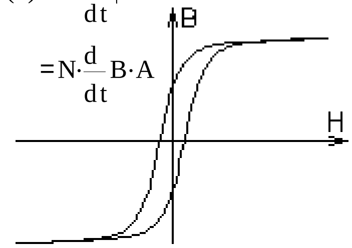
Rating: $(I_{2\_rated} - I_{1\_rated}) \cdot V_S$

Primary and Secondary could be swapped on any of these connections for an additional 4 possibilities

Inrush current

When a transformer is de-energized (switched off) its core may remain partially magnetized. When it is then re-energized (switched on) it may take several cycles before the B and the H re-center around the 0,0 point of the B-H plot. That can result in pushing the core far into saturation with large peaks of magnetic field intensity (H). H is directly proportional to current, so there are correspondingly large peaks of current. This inrush current is not sinusoidal and usually has a large DC component. Since it is dependent on where in the voltage cycle the transformer was de-energized it will be different each time the transformer is re-energized.

$$v(t) = N\cdot\frac{d}{dt}\phi$$
$$= N\cdot\frac{d}{dt}B\cdot A$$



Normal inrush currents can be just as large as abnormal short-circuit currents, yet protection devices (breakers and fuses) should not trip or blow-- a difficult protection problem.

Any device with a magnetic core will experience similar inrush currents.

Cooling and Oil-Immersion

High-voltage transformers are almost universally immersed in oil. That is, the core and windings are in a big enclosure filled with oil. Oil is a much better electrical insulator than air and also has much better thermal conductivity. Typically, it's mineral oil, but other, more expensive, oils and chemicals are also used to reduce fire and/or environmental hazards. PCBs are no longer used. Although PCB reduced the fire risk, it's highly toxic and stays in the environment a long time.

Core losses in a transformer will cause it to heat up even if it's not loaded. I^2R losses increase the heating under loaded conditions. Small transformers may just be air-cooled, but larger transformers require more cooling. Large oil-filled transformers typically cool that oil in radiators with fins next to the transformer. Those fins often have fans for forced-air cooling and the oil may be pumped through the transformer for forced-oil cooling. Transformers often have a tank to accommodate the thermal expansion of the oil. A bladder or inert gas inside the tank prevents contact with air.

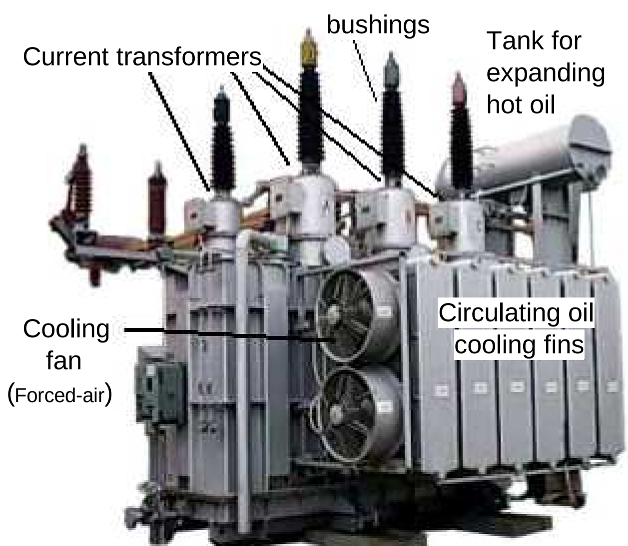
Cooling Types:

- AA Dry-type, Air cooled
- AFA Dry-type, Forced-Air cooled
- OA Oil Immersed, Air-cooled
- OA/FA Oil Immersed, Air / Forced-Air cooled
- OA/FA/FOA Oil Immersed, Air / Forced-Air / Forced-Oil and air cooled

Dissolved Gas Analysis

Analysis of the oil can reveal information about the health of the transformer. The simple version: Oxygen and Nitrogen indicate the oil has had contact with air. Carbon monoxide and dioxide indicate insulation degradation. Hydrogen indicates corona discharge. Methane, ethane, ethylene, and acetylene all indicate increasing levels of electrical faults and/or overheating with acetylene being the worst, indicating arcing. The oil is also checked for water, even a little of which is very bad. Regular maintenance includes filtering and drying the oil.

Large 3-phase Substation Transformer

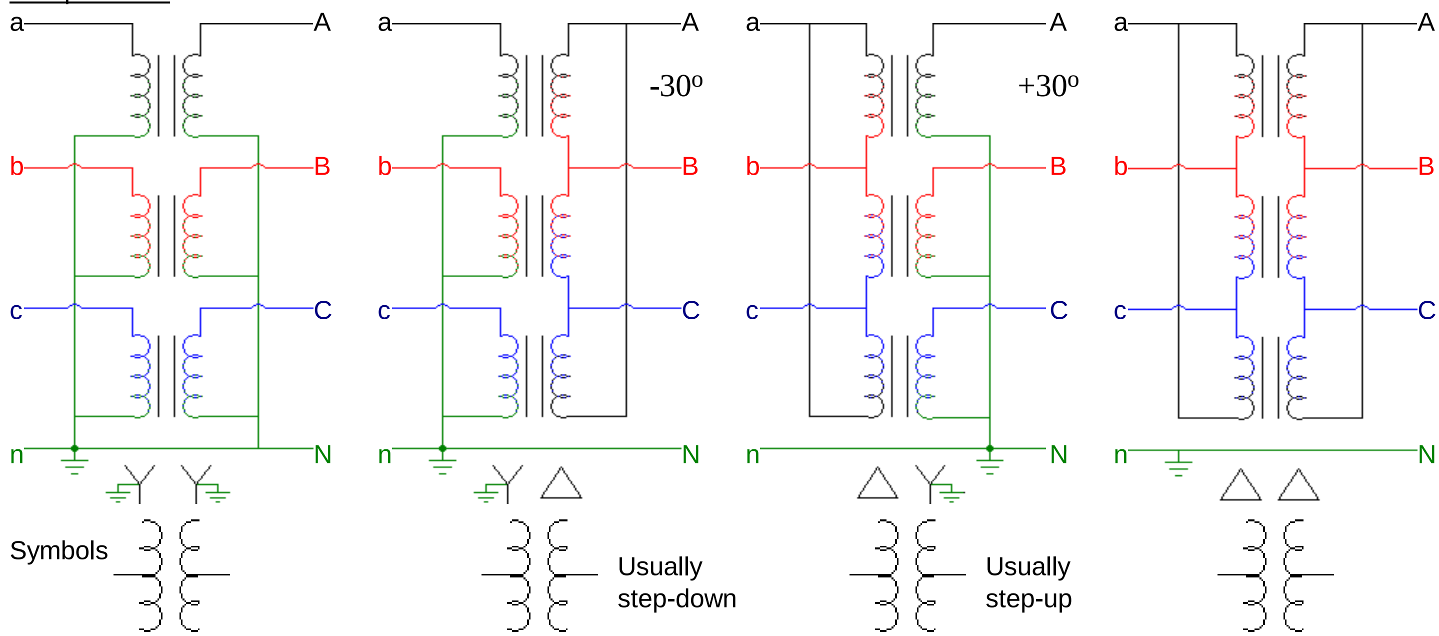


(Exporters India)

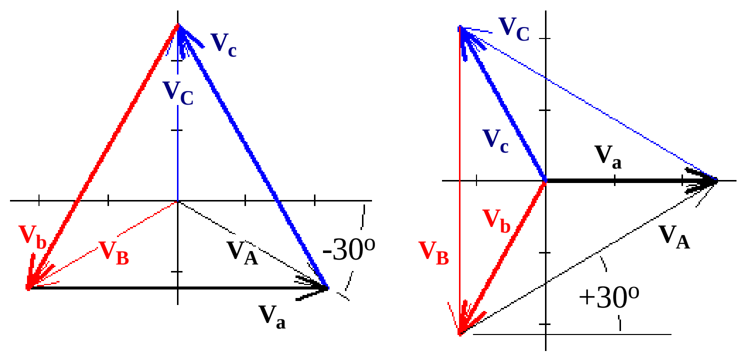
Mineral Oil is Flammable (or is that inflammable?)

(cbn.co.za)

Multiple cores



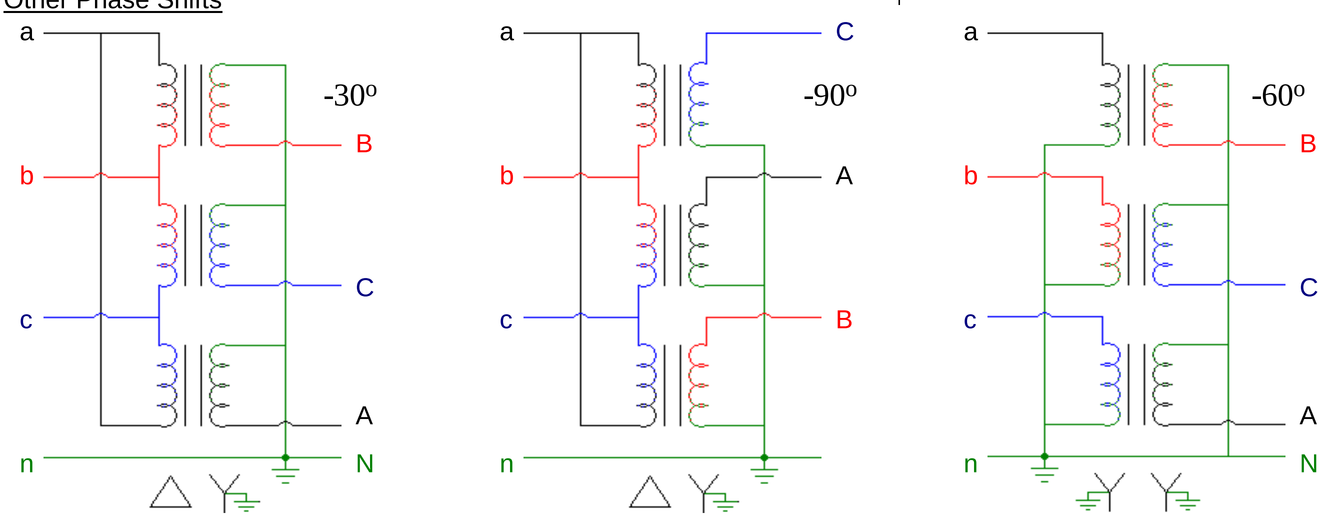
Y-Y transformers usually have a tertiary winding for the 3rd harmonic current, see below.

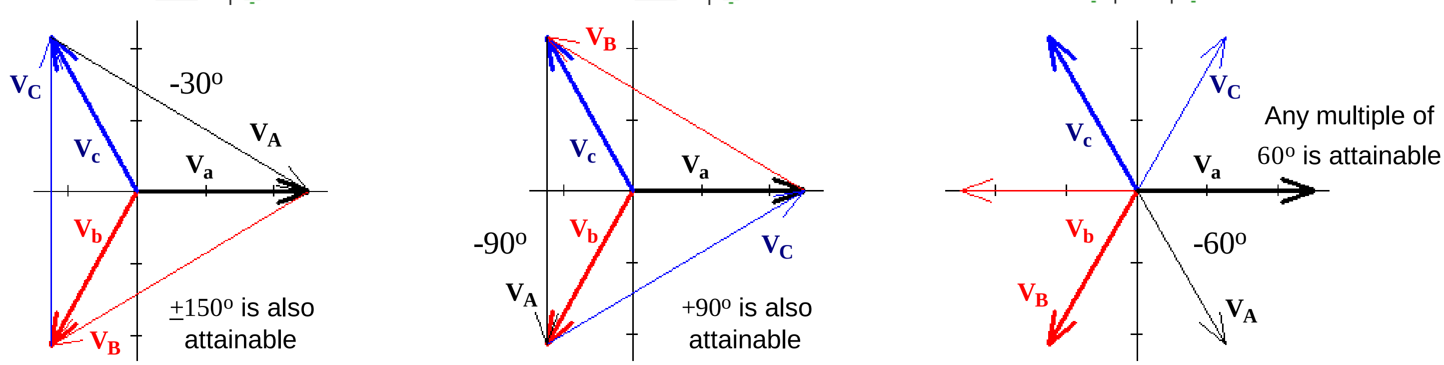


The Δ-Δ configuration can get by with just 2 transformers, at 58% VA.

Thick lines are the inputs
Thin lines are the outputs

Other Phase Shifts

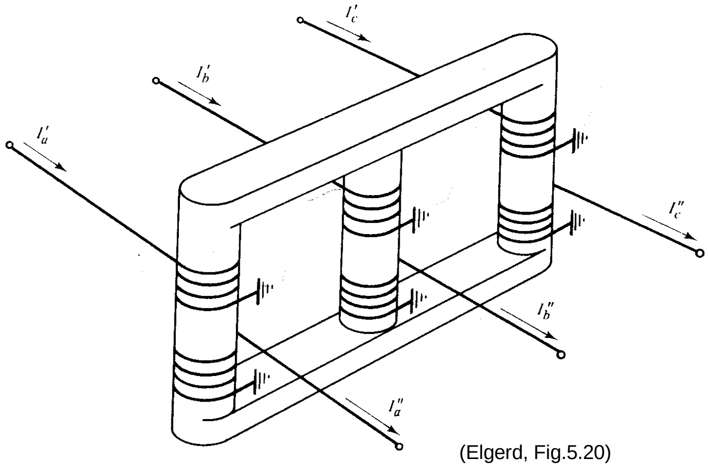




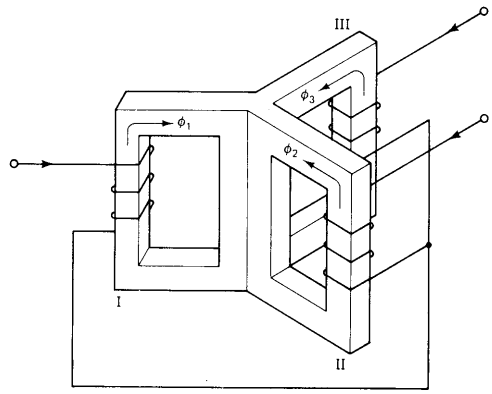
These can also be wired Y- Δ

Single-Core 3-phase Transformers

Cheaper and have less core loss than using individual cores or transformers.



(Elgerd, Fig.5.20)



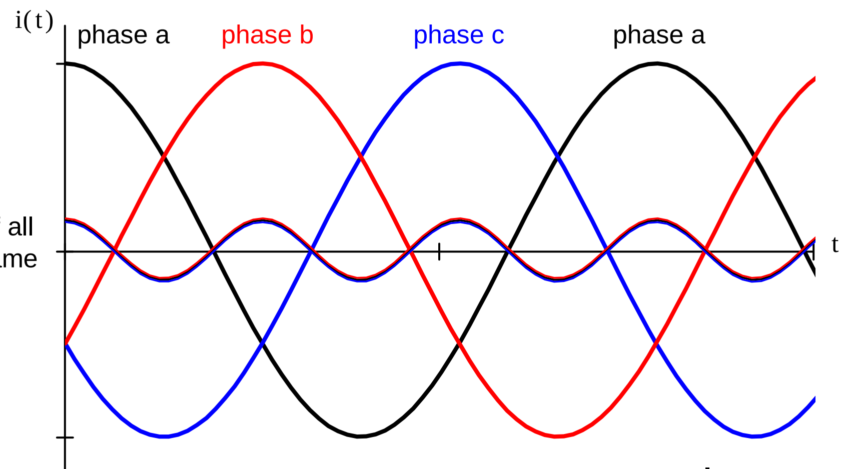
(Ryff, Fig.7.18)

Single-core transformers can also create all phase shifts shown on the previous page.

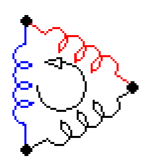
Third-Harmonic Currents

Third-harmonic currents (due to B-H non-linearity) add up to a significant neutral current.

The 3rd harmonic of all 3 phases are the same



Any Δ-connected winding will allow the third-harmonic current to flow in a loop.

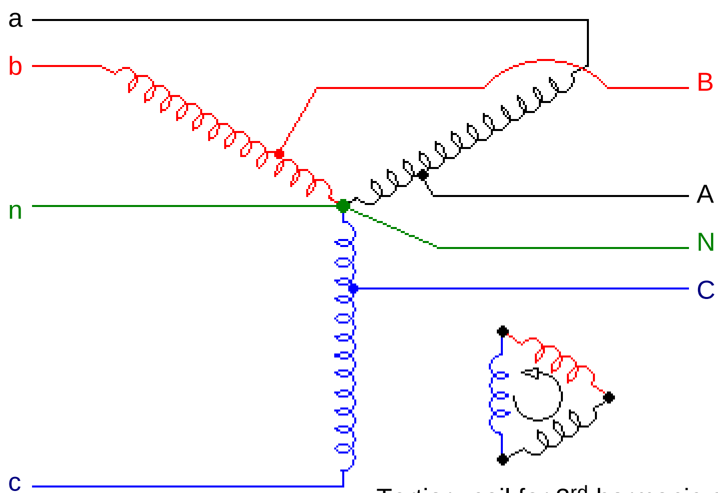


(iStock)

(ITG LLC)

3-phase autotransformers

Becoming more popular because they're cheaper for a given VA.

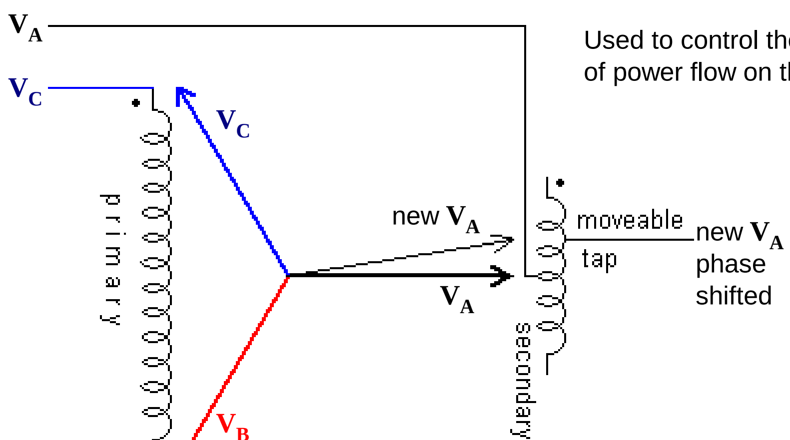


Tertiary coil for 3rd harmonic current. Sometimes this is used for "station power", that is, used to power the substation.

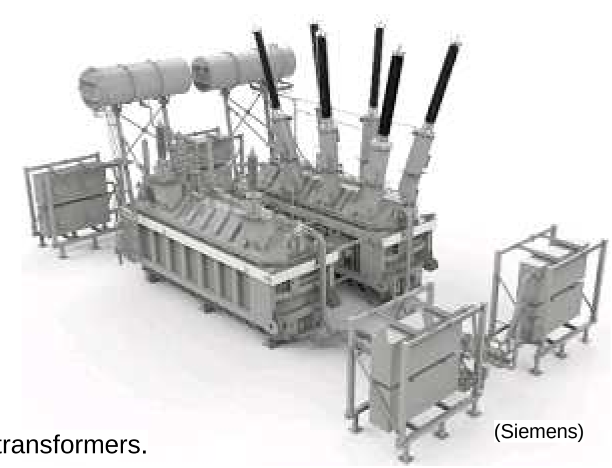
(Arn)

345kV/138kV Autotransformer at Terminal Substation in Salt Lake City. Note oil tank and cooling fins.

Phase-Shifting Transformers



Used to control the direction of power flow on the network

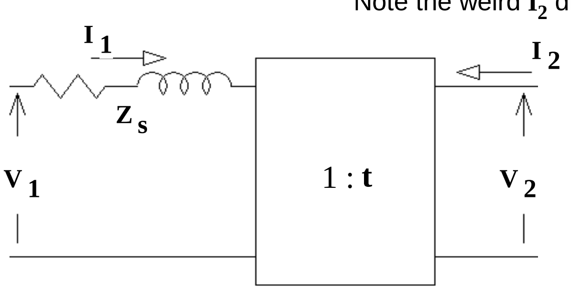


(Siemens)

Phases B and C are shifted in exactly the same way, with two other transformers.

Off-Nominal Turns Ratio

Note the weird I_2 direction



If there is a phase shift, t will be complex

$$Z_s = \frac{1}{Y_s} \qquad \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{bmatrix} Y_s & -\frac{Y_s}{t} \\ -\frac{Y_s}{\bar{t}} & \frac{Y_s}{(|t|)^2} \end{bmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$\bar{t}$ = complex conjugate of t

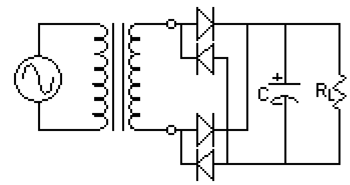
Transformer Examples

b

Ex.1 a) An ideal transformer has 360 turns on the primary winding and 36 turns on the secondary. If the primary is connected across a 120 V (rms) generator, what is the rms output voltage?

$$120\cdot\text{volt}\cdot\frac{36}{360} = 12\cdot\text{volt}$$

b) If you used a full-wave rectifier and a capacitor to make a DC power supply with this transformer, what DC voltage should you get?



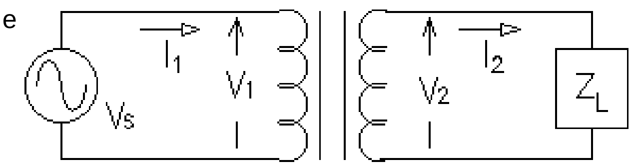
$$12\cdot V\cdot\sqrt{2} - 2\cdot 0.7\cdot V = 15.6\cdot V$$ less under load

peak 2 diodes

Ex.2 A transformer has $N_1 = 320$ turns and $N_2 = 1000$ turns. If the input voltage is $v(t) = (255\ V)\cos(\omega t)$, what rms voltage is developed across the secondary coil?

$$\frac{255\cdot\text{volt}}{\sqrt{2}}\cdot\frac{1000}{320} = 563\cdot\text{volt}$$

Ex.3 A transformer is rated at 480V / 120V, 1.2kVA. Assume the transformer is ideal and all voltages and currents are RMS.



$|\mathbf{Z}_L| = 20\cdot\Omega$

$pf := 75\cdot\%$ lagging

$\mathbf{V}_L := 110\cdot V$

a) What is the current rating of the primary?

$$\frac{1.2\cdot kVA}{480\cdot V} = 2.5\cdot A$$

b) What is the current rating of the secondary?

$$\frac{1.2\cdot kVA}{120\cdot V} = 10\cdot A$$

c) The secondary has 100 turns of wire. How many turns does the primary have?

$N_2 := 100$ $\quad N_1 := \frac{480\cdot V}{120\cdot V}\cdot N_2$ $\quad N_1 = 400$ turns

d) $\mathbf{V}_L := 110\cdot V$ How big is the source voltage ($|\mathbf{V}_S|$)?

$\mathbf{V}_S := \frac{N_1}{N_2}\cdot\mathbf{V}_L$ $\quad \mathbf{V}_S = 440\cdot V$

e) The secondary load ($\mathbf{Z}_L$) has a magnitude of 20 Ω at a power factor of 75%. Find the secondary current, $\mathbf{I}_2$ (magnitude **and <u>angle</u>**).

$pf := 75\cdot\%$

$\mathbf{I}_2 = \frac{\mathbf{V}_L}{20\cdot\Omega} = 5.5\cdot A$ $\quad pf = 0.75$ $\quad \text{acos}(pf) = 41.4\cdot\text{deg}$ $\quad \mathbf{I}_2 = 5.5A\ \underline{/-41.4^o}$

f) Find the primary current, $\mathbf{I}_1$ (magnitude **and <u>angle</u>**).

$\mathbf{I}_1 = \frac{100}{400}\cdot 5.5\cdot A = 1.375\cdot A$ $\quad \text{acos}(pf) = 41.4\cdot\text{deg}$ $\quad \mathbf{I}_1 = 1.375A\ \underline{/-41.4^o}$

Transformer is ideal, so angle is exactly the same as the load.

g) How much average power does the load dissipate?

$$P_L = |\mathbf{V}_2|\cdot|\mathbf{I}_2|\cdot pf = 110\cdot V\cdot 5.5\cdot A\cdot 75\cdot\% = 453.8\cdot\text{watt}$$

h) How much average power does the power source ($\mathbf{V}_S$) supply?

$$P_S = P_L = 454\cdot\text{watt}$$

i) What is the load as seen by $\mathbf{V}_S$? (magnitude **and <u>angle</u>**)

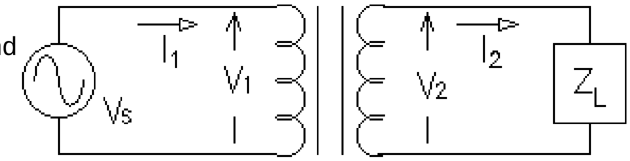
$\left(\frac{400}{100}\right)^2\cdot 20\cdot\Omega = 320\cdot\Omega$ $\quad \text{acos}(pf) = 41.4\cdot\text{deg}$ $\quad \mathbf{Z}_{eq} = 320\Omega\ \underline{/41.4^o}$

OR: $\frac{440\cdot V}{1.375\cdot A} = 320\cdot\Omega\ \ \underline{/\ 0 - -41.4^o}$

Transformer Examples p2

Ex.4 A transformer is rated at 480V/240V, 1.2kVA.
Assume the transformer is ideal and all voltages and currents are RMS.

How much power does the load consume?



load

$|\mathbf{V_S}| = 440 \cdot V$ $\qquad |\mathbf{Z_L}| = 16 \cdot \Omega$

$pf := 80 \cdot \%$ lagging

$\mathbf{V_L} := \mathbf{V_S} \cdot \left(\frac{240}{480}\right)$ $\qquad |\mathbf{V_L}| = 220 \cdot V$

$I_2 := \frac{|\mathbf{V_L}|}{|\mathbf{Z_L}|}$ $\qquad P_L := |\mathbf{V_L}| \cdot I_2 \cdot pf$ $\qquad P_L = 2.42 \cdot kW$

Ex.5 The transformer shown in the circuit below is ideal. It is rated at 220/110 V, 200 VA, 60 Hz

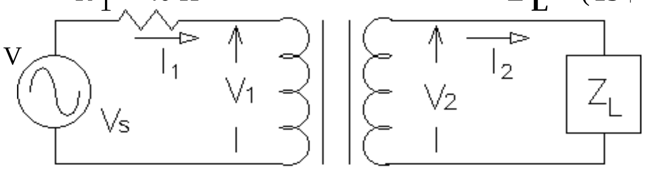
Find the following:

a) The primary current (magnitude).

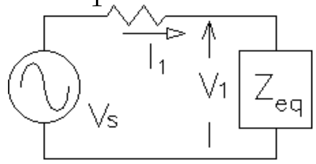
$|\mathbf{I_1}| = ?$

$R_1 := 40 \cdot \Omega$ $\qquad \mathbf{Z_L} := (15 + 20 \cdot j) \cdot \Omega$

$V_S := 120 \cdot V$



$R_1 = 40 \cdot \Omega$



$\mathbf{Z_{eq}} = \left(\frac{N_1}{N_2}\right)^2 \cdot \mathbf{Z_L}$

$\mathbf{Z_{eq}} := \left(\frac{220}{110}\right)^2 \cdot \mathbf{Z_L}$

$\mathbf{Z_{eq}} = 60 + 80j \cdot \Omega$

$R_1 + \mathbf{Z_{eq}} = 100 + 80j \cdot \Omega$

$I_1 := \frac{V_S}{\sqrt{100^2 + 80^2} \cdot \Omega}$ $\qquad I_1 = 0.937 \cdot A$

b) The primary voltage (magnitude).

$|\mathbf{V_1}| = ?$

$V_1 := I_1 \cdot \sqrt{60^2 + 80^2} \cdot \Omega$ $\qquad V_1 = 93.7 \cdot V$

c) The secondary voltage (magnitude).

$|\mathbf{V_2}| = ?$

$V_2 = \frac{110}{220} \cdot V_1 = 46.85 \cdot V$

d) The power supplied by the source.

$P_S = ?$

$P_S = I_1^2 \cdot 100 \cdot \Omega = 87.8 \cdot W$

e) Is this transformer operating within its ratings? Show your evidence.

$I_{1max} = \frac{200 \cdot VA}{220 \cdot V} = 0.909 \cdot A \quad < \quad I_1 = 0.937 \cdot A$

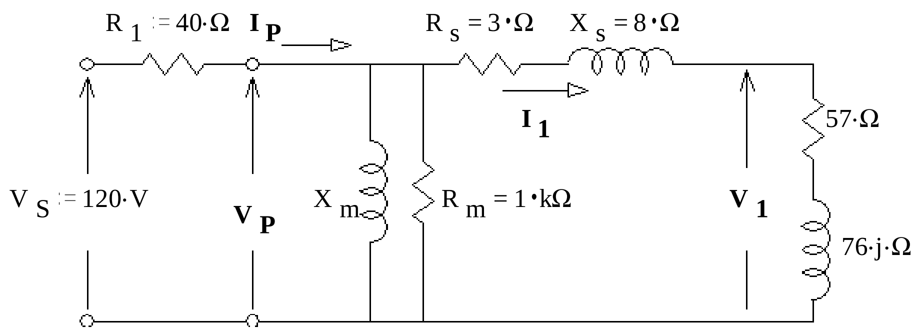
ALWAYS CHECK CURRENT

NO

Transformer Examples p3

Ex.6 Repeat Ex.5 with a non-ideal transformer whose characteristics are shown below.

$R_m := 1 \cdot k\Omega$ $X_m := 400 \cdot \Omega$ $R_s := 3 \cdot \Omega$ $X_s := 8 \cdot \Omega$ $N := 1.95$



$\mathbf{Z_{eq}} := (1.95)^2 \cdot (15 + 20 \cdot j) \cdot \Omega$

$\mathbf{Z_{eq}} = 57.037 + 76.05j \cdot \Omega$

Find the following:

a) The primary current (magnitude).

$|\mathbf{I_P}| = ?$

$R_s + X_s \cdot j + \mathbf{Z_{eq}} = 60.037 + 84.05j \cdot \Omega$

$$\frac{1}{\frac{1}{X_m \cdot j} + \frac{1}{R_m} + \frac{1}{(60 + 84 \cdot j) \cdot \Omega}} = 43.689 + 68.412j \cdot \Omega$$

$R_1 + (43.689 + 68.412 \cdot j) \cdot \Omega = 83.689 + 68.412j \cdot \Omega$

$$\mathbf{I_P} := \frac{120 \cdot V}{(83.689 + 68.412 \cdot j) \cdot \Omega} \qquad \mathbf{I_P} = 0.86 - 0.703j \cdot A$$

$|\mathbf{I_P}| = 1.11 \cdot A$

b) The primary voltage (magnitude).

$|\mathbf{V_P}| = ?$

$\mathbf{V_P} := \mathbf{I_P} \cdot (43.689 + 68.412 \cdot j) \cdot \Omega \qquad \mathbf{V_P} = 85.619 + 28.105j \cdot V$

$|\mathbf{V_P}| = 90.114 \cdot V$

c) The secondary voltage (magnitude).

$|\mathbf{V_2}| = ?$

$$\mathbf{I_1} := \frac{\mathbf{V_P}}{(60 + 84 \cdot j) \cdot \Omega} \qquad \mathbf{I_1} = 0.704 - 0.517j \cdot A$$

$\mathbf{V_1} := \mathbf{I_1} \cdot (57 + 76 \cdot j) \cdot \Omega \qquad \mathbf{V_1} = 79.375 + 24.026j \cdot V$

$|\mathbf{V_1}| = 82.931 \cdot V \qquad |\mathbf{V_2}| = \frac{1}{N} \cdot 82.931 \cdot V = 42.529 \cdot V$

OR, simply:

$$I_1 := \frac{90.114 \cdot V}{\sqrt{60^2 + 84^2} \cdot \Omega} \qquad I_1 = 0.873 \cdot A \qquad V_2 = \frac{I_1 \cdot \sqrt{57^2 + 76^2} \cdot \Omega}{1.95} = 42.529 \cdot V$$

d) The power supplied by the source.

$P_S = ?$

$P_S = 120 \cdot V \cdot \mathrm{Re}(\mathbf{I_P}) = 103.14 \cdot W$

e) Is this transformer operating within its ratings? Show your evidence.

$$I_{2max} = \frac{200 \cdot VA}{110 \cdot V} = 1.818 \cdot A \quad > \quad |\mathbf{I_1}| \cdot N = 1.702 \cdot A$$

YES

f) Find the efficiency, assuming that the only useful output is from $\mathbf{Z_L}$.

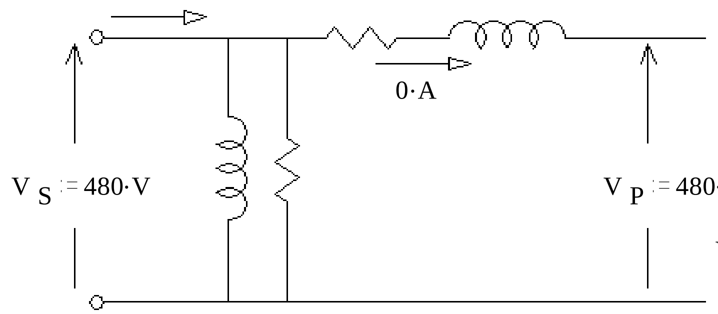
$$\eta = \frac{(|\mathbf{I_1}|)^2 \cdot 57 \cdot \Omega}{103.14 \cdot W} \cdot 100 \cdot \% = 42.12 \cdot \%$$

Ex.7 Find the voltage regulation and full-load efficiency of the transformer with the following ratings and characteristics.

Rated at 480/120 V, 2 kVA, 60 Hz $\quad R_m := 8.4 \cdot k\Omega \quad X_m := 2 \cdot k\Omega \quad R_s := 5 \cdot \Omega \quad X_s := 15 \cdot \Omega$

$V_{Prated} := 480 \cdot V \quad V_{Srated} := 120 \cdot V \quad S_{rated} := 2000 \cdot VA$

No load:



$V_2 := 120 \cdot V$

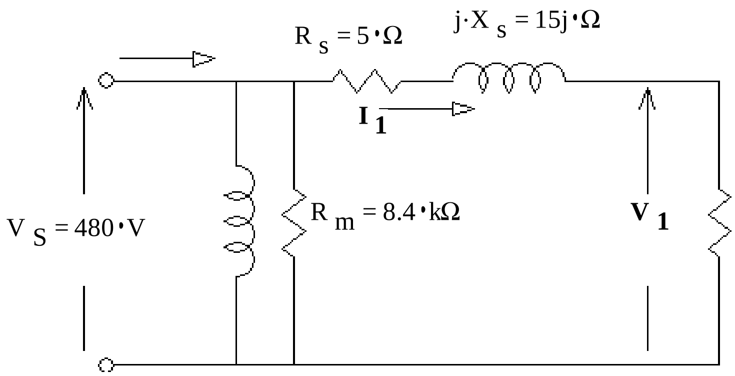
$V_{NL1} := 480$

Full load: $\quad R_L := \frac{(120 \cdot V)^2}{2000 \cdot W} \quad R_L = 7.2 \cdot \Omega$

Find the actual turns ratio: (See page 3 of notes)

$$R_x := \frac{{V_{Prated}}^2}{S_{rated}} - 2 \cdot R_s \qquad R_{eq} := \frac{R_x + \sqrt{{R_x}^2 - 4 \cdot \left({R_s}^2 + {X_s}^2\right)}}{2}$$

Turns ratio $N := \sqrt{\frac{R_{eq}}{R_L}} \quad N = 3.778$



$N^2 \cdot R_L = 102.767 \cdot \Omega$

NOT: $\left(\frac{480}{120}\right)^2 \cdot R_L = 115.2 \cdot \Omega$

$$\mathbf{I_1} := \frac{V_S}{R_{eq} + R_s + 15j \cdot \Omega} \qquad \mathbf{I_1} = 4.369 - 0.608j \cdot A$$

$$\mathbf{V_1} := \mathbf{I_1} \cdot R_{eq} \qquad \mathbf{V_1} = 449.031 - 62.5j \cdot V$$

$$V_{FL1} := \left|\mathbf{V_1}\right| \qquad V_{FL1} = 453.359 \cdot V$$

OR, simply:

$$I_1 := \frac{V_S}{\sqrt{\left(R_{eq} + R_s\right)^2 + {X_s}^2}} \qquad I_1 = 4.412 \cdot A$$

$$V_{FL1} = I_1 \cdot R_{eq} = 453.359 \cdot V$$

Voltage regulation: $\quad \%VR = \frac{V_{no\_load} - V_{full\_load}}{V_{full\_load}} \cdot 100 \cdot \% = \frac{V_S - V_{FL1}}{V_{FL1}} \cdot 100 \cdot \% = 5.876 \cdot \%$

Efficiency $\quad \eta = \frac{P_{out}}{P_{in}} \cdot 100 \cdot \% = \frac{P_{out}}{P_{out} + P_{losses}} \cdot 100 \cdot \%$

$$P_{out} := {I_1}^2 \cdot R_{eq} \qquad P_{out} = 2 \cdot kW$$

$$P_{losses} := {I_1}^2 \cdot 5 \cdot \Omega + \frac{(480 \cdot V)^2}{8400 \cdot \Omega} \qquad P_{losses} = 0.125 \cdot kW$$

$$\eta = \frac{P_{out}}{P_{out} + P_{losses}} \cdot 100 \cdot \% = 94.129 \cdot \%$$

Transformer Examples p5

Ex.8 A 500/100-V, 2.5-kVA transformer is subjected to an OC test and a SC test with the results below.

a) Draw a model of this transformer and find the values of all the elements of the model, including the turns ratio.

During the open-circuit test: $I_{OC} := 0.5 \cdot A$ $\quad P_{OC} := 150 \cdot W$ $\quad V_{sec} := 108 \cdot V$

During the short-circuit test: $V_{SC} := 22 \cdot V$ $\quad P_{SC} := 70 \cdot W$

$S_{rated} := 2.5 \cdot kVA$ $\quad V_{Prated} := 500 \cdot V$ $\quad I_{Prated} := \frac{S_{rated}}{V_{Prated}}$ $\quad I_{Prated} = 5 \cdot A$

Open-circuit test used to find R_m and X_m and turns ratio.

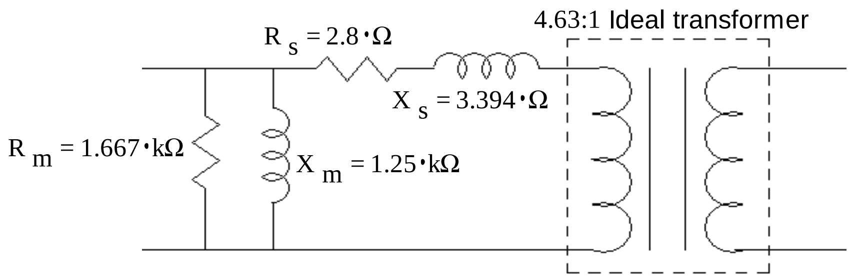
$V_{OC} := V_{Prated}$ $\quad R_m := \frac{V_{OC}^2}{P_{OC}}$ $\quad R_m = 1.667 \cdot k\Omega$

$Q_{OC} := \sqrt{(V_{OC} \cdot I_{OC})^2 - P_{OC}^2}$ $\quad Q_{OC} = 200 \cdot VAR$ $\quad X_m := \frac{V_{OC}^2}{Q_{OC}}$ $\quad X_m = 1.25 \cdot k\Omega$

Turns ratio: $N := \frac{V_{Prated}}{V_{sec}}$ $\quad N = 4.63$

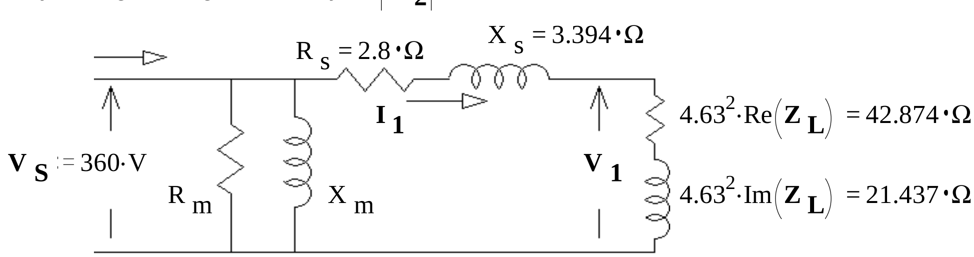
Short-circuit test used to find R_s and X_s $\quad I_{SC} := I_{Prated}$ $\quad R_s := \frac{P_{SC}}{I_{SC}^2}$ $\quad R_s = 2.8 \cdot \Omega$

$Q_{SC} := \sqrt{(V_{SC} \cdot I_{SC})^2 - P_{SC}^2}$ $\quad Q_{SC} = 0.085 \cdot kVAR$ $\quad X_s := \frac{Q_{SC}}{I_{SC}^2}$ $\quad X_s = 3.394 \cdot \Omega$



b) The transformer is connected to a primary source voltage of 360V and loaded with $\mathbf{Z_L} := (2 + 1 \cdot j) \cdot \Omega$

Find the secondary voltage. Magnitude only. $|\mathbf{V_2}| = ?$



$$|\mathbf{V_1}| = V_1 := \mathbf{V_S} \cdot \frac{\sqrt{(42.874 \cdot \Omega)^2 + (21.437 \cdot \Omega)^2}}{\sqrt{(R_s + 42.874 \cdot \Omega)^2 + (X_s + 21.437 \cdot \Omega)^2}} \qquad V_1 = 331.935 \cdot V$$

$$|\mathbf{V_2}| = V_2 = \frac{V_1}{4.63} = 71.69 \cdot V$$

c) Is this transformer operating within its ratings? Show all evidence and calculate needed to to determine this.

$$|\mathbf{I_2}| = I_2 := \frac{\mathbf{V_S}}{\sqrt{(R_s + 42.874 \cdot \Omega)^2 + (X_s + 21.437 \cdot \Omega)^2}} \cdot N \qquad I_2 = 32.059 \cdot A > I_{Srated} = \frac{S_{rated}}{100 \cdot V} = 25 \cdot A$$

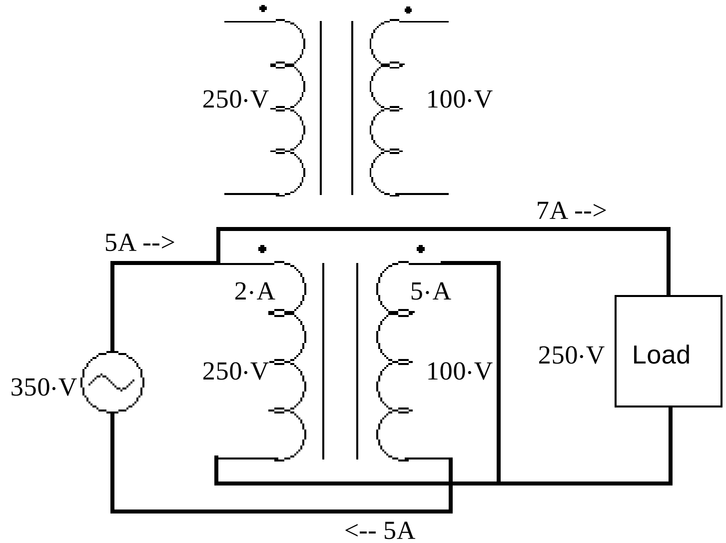
NO !

Transformer Examples p6

Ex.9 You have a 250/100-V, 500-VA transformer.

a) Show the necessary connections to use this transformer to transform 350 V to 250 V. Also show the 350-V source and the load.

b) Connected this way, determine the maximum power that could be converted from 350 V to 250 V without overloading the transformer.



ratings: $\frac{500\cdot VA}{250\cdot V} = 2\cdot A$ $\quad \frac{500\cdot VA}{100\cdot V} = 5\cdot A$

new VA rating and maximum power: $5\cdot A\cdot 350\cdot V = 1.75\cdot kVA$

OR: $7\cdot A\cdot 250\cdot V = 1.75\cdot kVA$

$1.75\cdot kW$

c) Besides the right impedance magnitude, what other characteristic must the load posses in order to actually use this much power?

Load must be purely resistive (power factor is 1).

d) Could this transformer also be used to transform 280 V to 200 V ? If yes, what is the maximum power that could be transformed?

Same connections as above Maximum power: $5\cdot A\cdot 280\cdot V = 1.4\cdot kW$

Ex.10 A 745kV/138kV, 750-MVA transformer is shown.

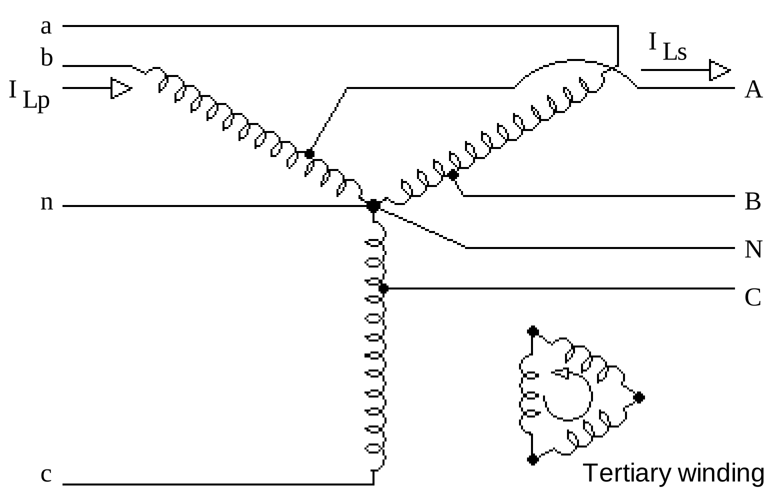
a) What is the purpose of the tertiary winding?

To allow 3rd harmonic currents to flow without affecting currents outside the transformer.

b) Find the maximum I_{Lp} ans I_{Ls}.

$$I_{Lp.rated} := \frac{\left(\frac{750\cdot MVA}{3}\right)}{\left(\frac{345\cdot kV}{\sqrt{3}}\right)} \qquad I_{Lp.rated} = 1255\cdot A$$

$$I_{Ls.rated} := \frac{345}{138}\cdot I_{Lp.rated} \qquad I_{Ls.rated} = 3138\cdot A$$



c) Find the currents flowing in the transformer when operated at rated capacity.

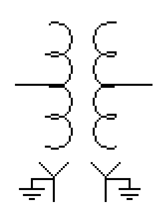
Current from primary terminal to the tap: $I_p = I_{Lp.rated} = 1255\cdot A$

Current from neutral to the tap: $I_p = I_{Ls.rated} - I_{Lp.rated} = 1883\cdot A$

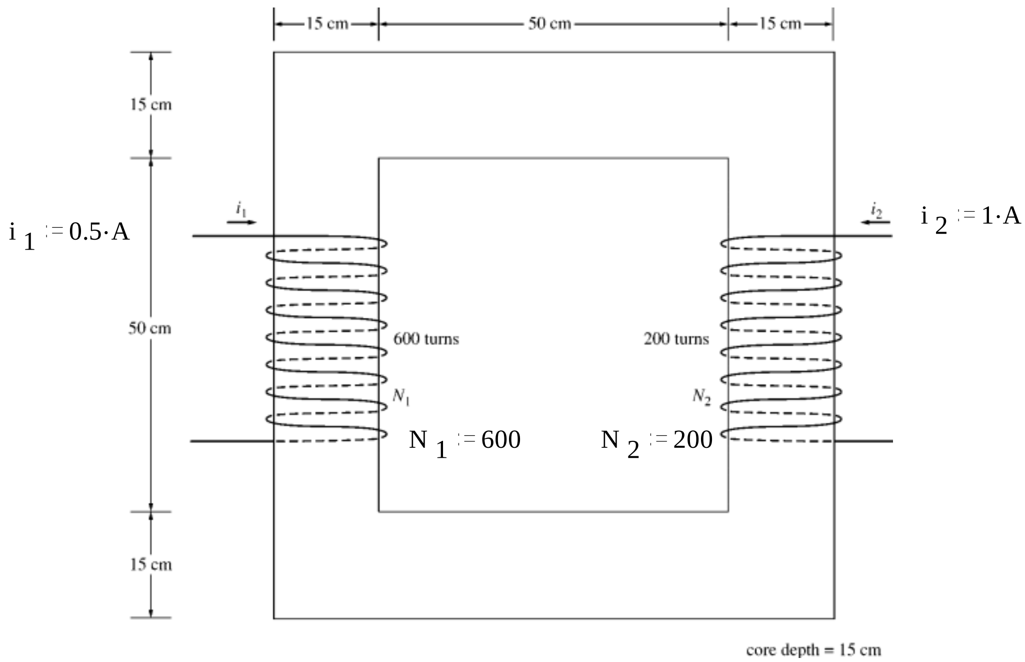
Current from tap to secondary terminal and out of the transformer: $I_s = I_{Ls.rated} = 3138\cdot A$

d) At what fraction of the total turns is the tap located? $\frac{138}{345} = 0.4 = \frac{4}{10}$ OR at 40%

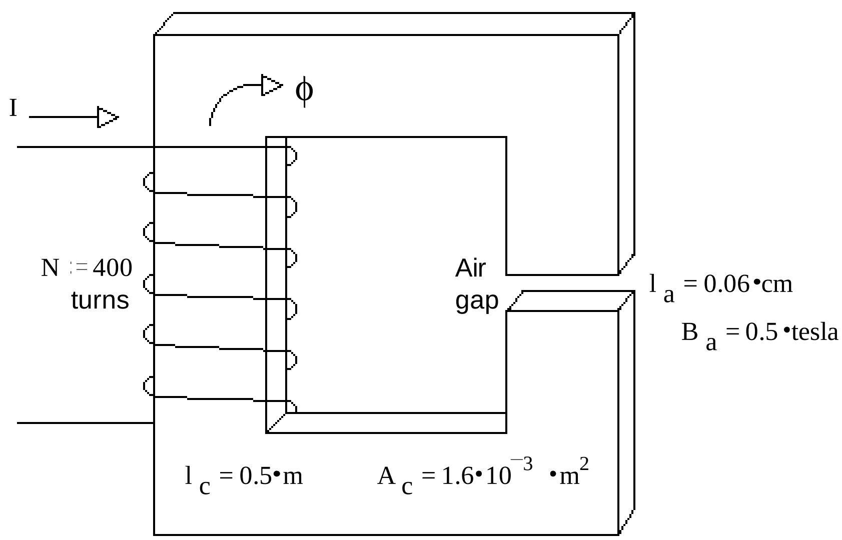
e) What one-line symbol would be used for this transformer?



1. Textbook problem 1-7 (p49)
   A two-legged core is shown in below. The winding on the left leg of the core (N_1) has 600 turns, and the winding on the right (N_2) has 200 turns. The coils are wound in the directions shown in the figure. If the dimensions are as shown, then what flux would be produced by currents $i_1 = 0.5$ A and $i_2 = 1.00$ A? Assume $\mu_r = 1000$ and constant.



2. Textbook Example 1-2 (p20) with:
   Mean magnetic length: $l_c := 50 \cdot cm$ Air gap length: $l_a := 0.06 \cdot cm$
   Core cross-sectional area: $A_c := 16 \cdot cm^2$ Relative permeability of core: $\mu_r := 4000$
   Effective air-gap cross-sectional area is 5% more than the core.
   a) Find the total reluctance of the core with the air gap. $\mathcal{R}_{eq} = ?$
   b) Find the required current so that the flux density of in the air gap is: $B_a := 0.5 \cdot tesla$ I = ?



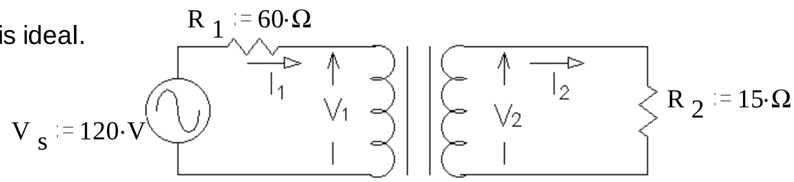
Answers

1. $0.0054 \cdot Wb$ 2. a) $3.464 \cdot 10^5 \cdot \frac{A \cdot turns}{Wb}$ b) $727 \cdot mA$

1. A step-up transformer is designed to have an output voltage of 2200 V (rms) when the primary is connected across a 240 V (rms) source.
   a) If there are 150 turns on the primary winding, how many turns are required on the secondary?
   b) If a load resistor across the secondary draws a current of 1.2 A, what is the current in the primary, assuming ideal conditions?

2. An ideal transformer has a turns ratio ($N = N_1/N_2$) of 1.5 . It is desired to operate a 200 Ω resistive load at 150 V (rms).
   a) Find the secondary and primary currents.
   b) Find the source voltage ($\mathbf{V_1}$).
   c) Find the power dissipated in the load resistor and the power delivered to the primary from the source.
   d) Find the impedance the source sees looking into the primary winding by calculating $\mathbf{Z_{eq}} = N^2\,\mathbf{Z_L}$ and again by calculating $\mathbf{V_1} / \mathbf{I_1}$.

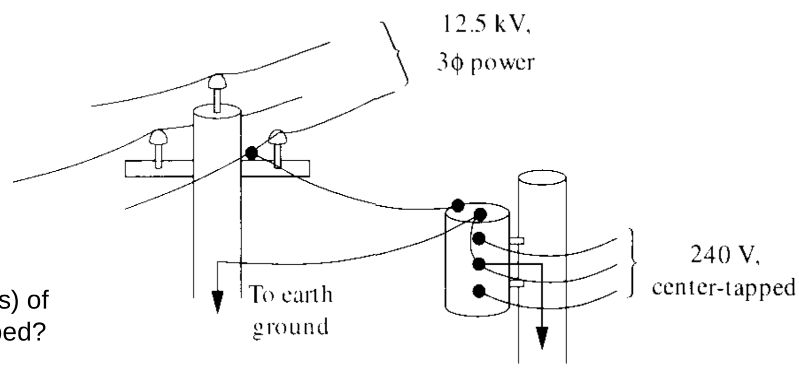
3. The transformer shown in the circuit below is ideal. It is rated at 120/30 V, 80 VA, 60 Hz
   Find the following:
   a) I_1 = ?   b) V_2 = ?



4. An ideal transformer has a rating of 500/125 V, 10 kVA, 60 Hz. It is loaded with an impedance of 5Ω at 80% pf (0.80). The source voltage applied to the primary winding is 440 V (rms). Find:
   a) the load voltage
   b) the load current
   c) the kVA delivered to load
   d) the power delivered to load
   e) the primary current
   f) the power factor of primary
   g) the impedance the source sees looking into primary.

5. An ideal transformer is rated to deliver 400 kVA at 460 V to a customer.
   a) How much current can the transformer supply to the customer?
   b) If the customer's load is purely resistive (i.e. if the pf = 1), what is the maximum power the customer can receive?
   c) If the customer's power factor is 0.8 (lagging), what is the maximum usable power the customer can receive?
   d) What is the maximum power if the power factor is 0.7 (lagging)?
   e) If the customer requires 300 kW to operate, what is the minimum allowable power factor given the rating of this transformer?

6. The city of Murray, Utah. distributes power to neighborhoods with a 12.47-kV three-phase system. (12.47 kV is the line voltage.) Each group of houses is served from one phase and ground, and transformed to 240/120 V by a pole transformer, as shown.



   a) What is the turns ratio (primary/secondary turns) of the pole transformer to give 240 V, center-tapped?
   b) When a 1500-W toaster-oven is turned on, how much does the current increase in the high-voltage wire? Assume the power factor is unity and the transformer is 100% efficient.
   c) Repeat b) for a clothes drier that draws 15 A.

Answers

1. a) 1375 turns b) 11 A   2. a) 0.75 A, 0.50 A b) 225 V c) 112.5 W d) 450Ω

3. a) 0.4·A b) 24V   4. a) 110 V b) 22 A c) 2.42 kVA d) 1.94 kW e) 5.5 A f) 0.80 g) 80Ω /36.9° Ω

5. a) 870·A b) 400·kW c) 320·kW d) 280·kW e) 0.75   6. a) 30 b) 208·mA

1. The parameters of a step-down transformer are shown below.
   The transformer is loaded with $\mathbf{Z}_L := (2.5 + 0.8 \cdot j) \cdot \Omega$ and the secondary voltage is $V_2 := 36 \cdot V$

   $R_m := 2 \cdot k\Omega$ $R_s := 2 \cdot \Omega$ $X_m := 800 \cdot \Omega$ $X_s := 5 \cdot \Omega$ $N := 5$

   a) Draw the model with the load connected. Label parts, voltages and currents as needed for the rest of the problem.

   b) Find the primary, source voltage. Magnitude only. $|\mathbf{V}_S| = ?$

   c) Find the total complex power supplied the primary, source voltage. $\mathbf{S}_S = P_S + j \cdot Q_S = ?$

   d) Find the magnitude of the current flowing from the primary, source voltage. $|\mathbf{I}_S| = ?$

   e) Find the efficiency of the transformer. $\eta = ?$

   f) The transformer would be fully loaded if $V_S := 208 \cdot V$ and $Z_L := 2 \cdot \Omega$ all real
   Find the voltage regulation as defined in your notes. $\%VR = ?$

2. The parameters of a step-down transformer are shown below. The primary voltage is $V_S := 120 \cdot V$
   The transformer is loaded with $\mathbf{Z_L} = R_L + jX_L$ and the secondary current is $I_2 := 3.2 \cdot A$

   $R_m := 1.5 \cdot k\Omega$ $R_s := 5 \cdot \Omega$ $X_m := 1 \cdot k\Omega$ $X_s := 7 \cdot \Omega$ $N := 4$

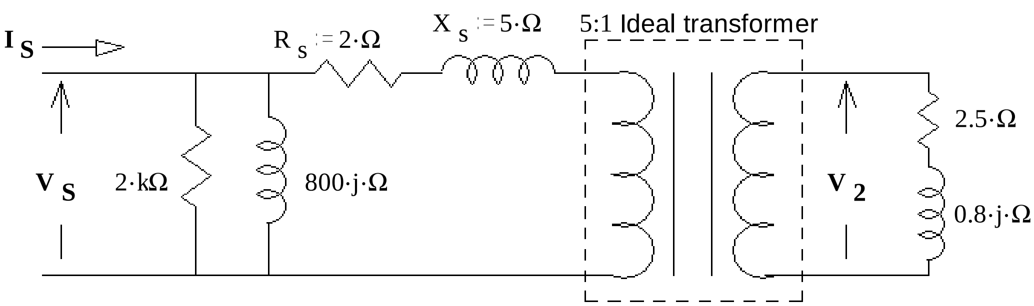
   a) The primary, source voltage provides 40 VARs $Q_S := 40 \cdot VAR$ Find X_L
   Hint: draw the model with the load.

   b) Find R_L

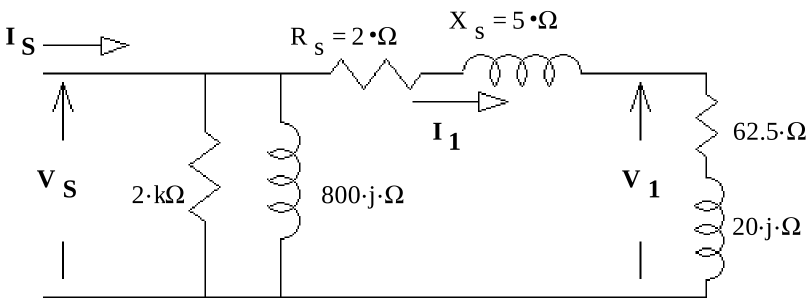
   c) Find the efficiency of this transformer. $\eta = ?$

Answers

1. a)



and/or



b) $189.7 \cdot V$ c) $503.3 + 233.1 \cdot j$ VA d) $2.923 \cdot A$ e) $93.4 \cdot \%$ f) $4.08 \cdot \%$

2. a) $2.062 \cdot \Omega$ b) $8.723 \cdot \Omega$ c) $87.5 \cdot \%$

ECE 3600 homework 8B Non-Ideal transformers, tests Due: Wed, 2/8/23 c

1. A 500V/200V, 30-kVA, 60-Hz transformer is subjected to a SC test. The secondary is short-circuited and the primary voltage is raised until rated current is flowing. This occurs when the applied voltage equals 8.2% of rated winding voltage. The transformer consumes 600 W during the test.

   a) Compute the series impedance $\mathbf{Z_s} = R_s + jX_s$ of the transformer.

   b) Compute the core flux during the SC test. (Express its magnitude in percent of normal operating flux.)

   c) Why is it permissible to assume that all of the 300 W constitute ohmic losses in R, and no part of it is core loss?

2. The same 30-kVA transformer is subjected to an OC test. The transformer consumes 500 W and draws 1.8 A.

   a) Find R_m and X_m.

   b) The open-circuit voltage measured on the secondary is 206V. Find the actual turns ratio.

3. The same 30-kVA transformer has the following impedance is hooked to the secondary: $\mathbf{Z_L} := (1.3 + 0.2 \cdot j) \cdot \Omega$

   a) Find the currents in both windings and the secondary voltage by use of the ideal (IT) model. $V_P := 500 \cdot V$

   b) Same as in part a) but now include the transformer impedances and actual turns ratio in your analysis. Take note of the change in your answers.

   c) Find the efficiency.

   d) Is the transformer current-overloaded?

4. A transformer is rated at 210V / 70V, 420VA.

   The following values were found by making the standard OC and SC tests. R_m and X_m were neglected when finding the other two components.

   $R_m := 2 \cdot k\Omega$ $X_m := 747 \cdot \Omega$

   $R_s := 4 \cdot \Omega$ $X_s := 8.2 \cdot \Omega$

   a) Draw the standard non-ideal transformer model and label the parts.

   b) What were the measurements that were taken in the standard open-circuit test? (Give me numbers)

   c) What were the measurements that were taken in the standard short-circuit test? (Give me numbers)

Answers

1. a) $0.167 + 0.663j \cdot \Omega$ b) 8.2% of normal rated value.

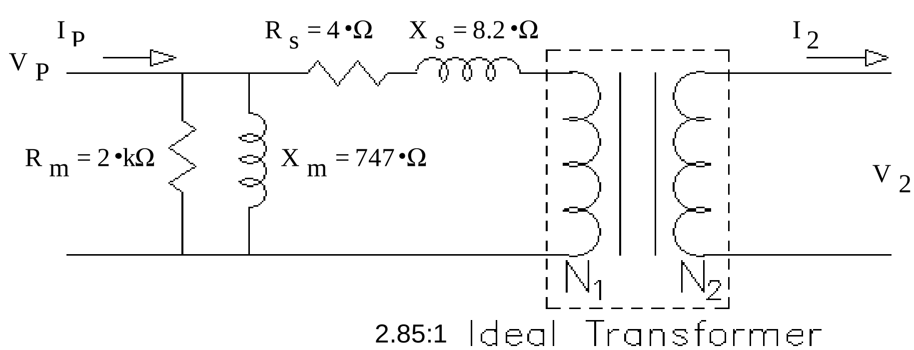
   c) Because the core flux is 8.2% of normal rated value, the core losses (which are approximately proportional to the square of the flux) are negligible.

2. a) $500 \cdot \Omega$ & $334 \cdot \Omega$ b) 2.247

3. a) $60.8 \cdot A$ $152.1 \cdot A$ $200 \cdot V$ b) $62.2 \cdot A$ a little more $151 \cdot A$ a little less $198.6 \cdot V$

   c) $95.4 \cdot \%$ d) yes, barely.

4. a)



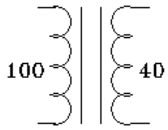
   b) $22.05 \cdot W$ 0.3A $73.68 \cdot V$ c) $16 \cdot W$ $18.25 \cdot V$

ECE 3600 homework 9A Due: Mon 2/11/23

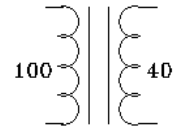
1. 5.7 A 500/200-V, 30-kVA transformer is reconnected as a 700/500-V autotransformer. Compute the new kVA rating of the device.

2. Show connections to the following 100/40-V, 200-VA transformers to get the voltage ratios desired. Compute the new VA rating of each connection.

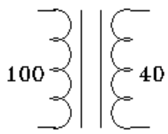
   a) 140/40 V



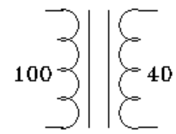
   b) 140/100 V



   c) 60/40 V



   d) 60/100 V



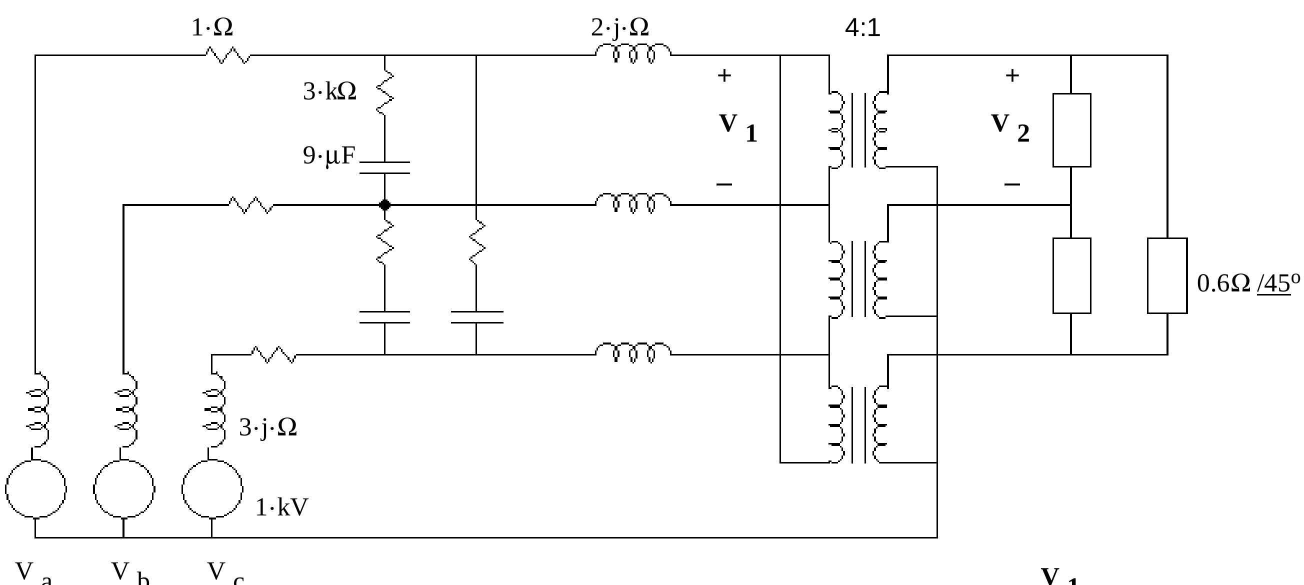
3. 5.8 The terminals of a 500/200-V transformer can be interconnected in four different ways, two of which will result in a 700/500-V autotransformer. Assume that you have interconnected the windings in the wrong way, but that you believe that you did it the right way. In other words, you think that you have a 700/500-V autotransformer when in fact you have something else. As you now connect the “700-V terminals” of your device to a 700-V source, you expect to obtain 500-V between what you presume to the “500-V terminals.” To your surprise you get an entirely different voltage.

   a) What voltage do you get?

   b) What will happen to your transformer with this kind of treatment?

ECE 3600 homework 9B Due: Wed 2/15/23

4. a) Draw a per-phase drawing of for the balanced 3-phase, 60-Hz system shown. You may neglect phase issues introduced by Y-Δ and Δ-Y connections. You may need to modify the turns ratio of the transformer to reflect Y-Δ and Δ-Y connections. Be sure to show values of the source, passive components and turns ratio on your drawing.



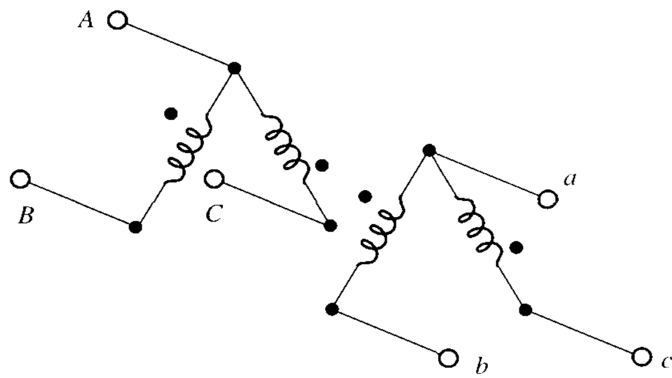
b) Find $\frac{V_1}{V_2}$ incuding phase angle

It is easy to see how to transform three-phase power with the use of three single-phase transformers, but there are two ways to transform three-phase power using only two single-phase transformers. The next two problems investigate these methods. In them, we will transform 480 V three phase to 240 V three phase; hence, the transformers have a turns ratio of 2:1. Hint: In both figures, the geometric orientation hints of the phasor relationships.

5. The configuration shown is called the “open-delta” or "V" connection, for obvious reasons. Identical 2:1 transformers are used.

   a) Show that if ABC is 480-V balanced three phase, abc is 240-V balanced three-phase. Consider the ABC voltages to be a three-phase set and prove the abc set is three-phase.

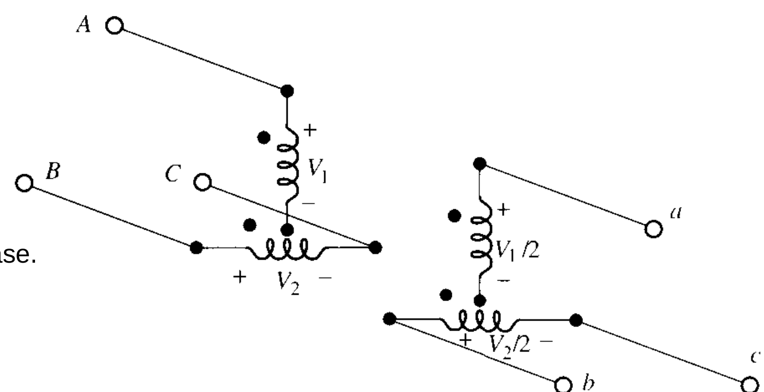
   b) If the load is 30 kVA, find the required kVA rating of the transformers to avoid overload. [You can solve this independent of part a)]



6. 1.22. The configuration shown is called the "T" connection. For this connection, the 2:1 transformers are not identical but have different voltage and kVA ratings. The bottom transformer is center-tapped so as to have equal, in-phase voltages for each half.

   a) Find the voltages V_1 and V_2 to make this transform 480-V to 240-V balanced three-phase.

   b) If the load is 30 kVA, find the required kVA rating of each transformer to avoid overload.



7. A phase-shifting transformer has a complex turns ratio of $\mathbf{t} := 4 \cdot e^{j \cdot 20 \cdot deg}$ = 4 /20°

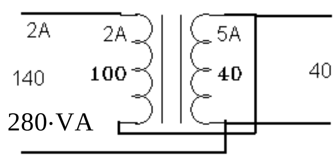
   It has a series impedance of $\mathbf{Z_S} := (0.05 + j \cdot 0.6) \cdot \Omega$

   Find the admittance matrix of this tranformer (see the last page of the transformer notes).

Answers

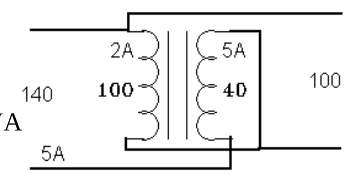
1. $105 \cdot kVA$

2. a) 140/40 V

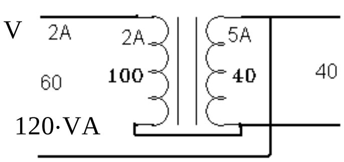


b) 140/100 V

700·VA



c) 60/40 V

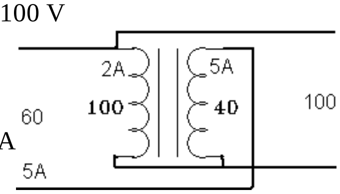


3. a) $1167 \cdot V$

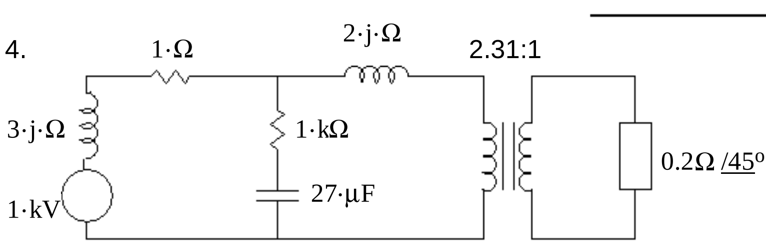
   b) The smoke gets out

d) 60/100 V

300·VA



4.



b) 2.309 / -30°

5. a) Calculate $\mathbf{V_{bc}}$ from the other two voltages and show that it has the correct magnitude and correct phase angle.

   b) $17.3 \cdot kVA$ per transformer, $34.6 \cdot kVA$ for both

6. a) $415 \cdot 7 \cdot V$ $480 \cdot V$ b) $15 \cdot kVA$ $17.3 \cdot kVA$ $32.3 \cdot kVA$ for both

7. $$\begin{pmatrix} 0.138 - 1.655 \cdot j & 0.109 + 0.401 \cdot j \\ -0.174 + 0.377 \cdot j & 8.621 \cdot 10^{-3} - 0.103 \cdot j \end{pmatrix} \cdot \frac{1}{\Omega}$$