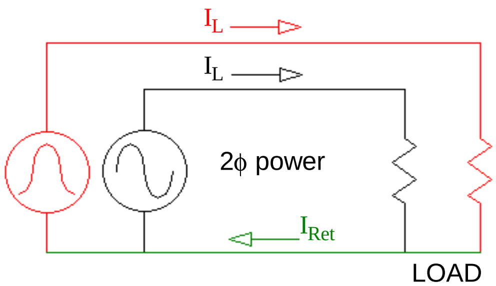
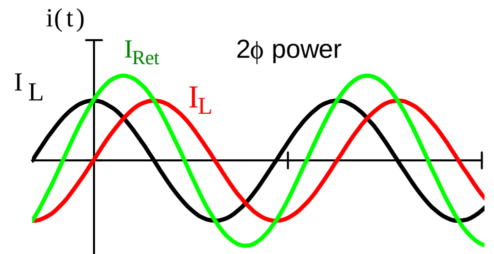
ECE 3600 3-Phase Power notes

Single phase power pulses at 120 Hz. This is not suitable for motors or generators over about 5 hp.

Two-phase power is constant as long as the two loads are balanced.

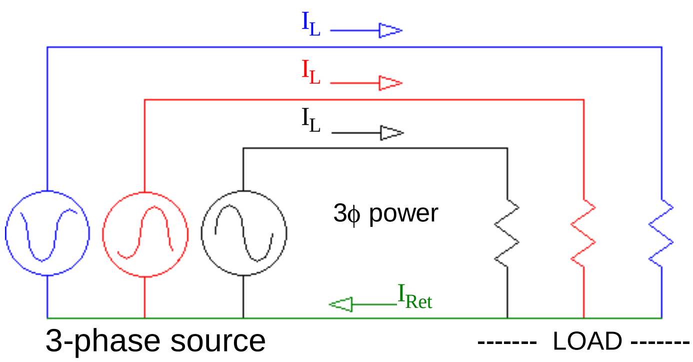


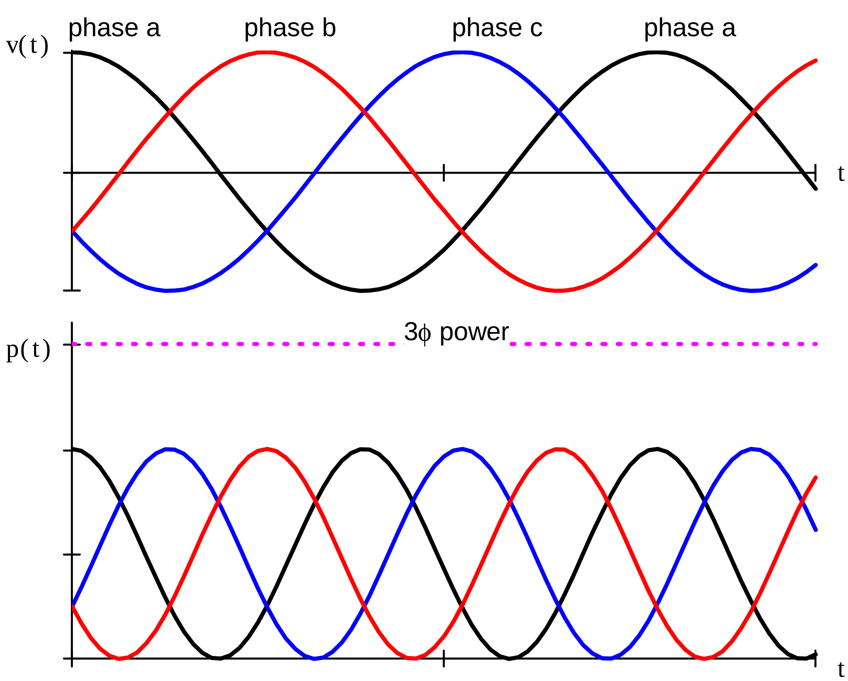
But, the return current is larger than either load current.

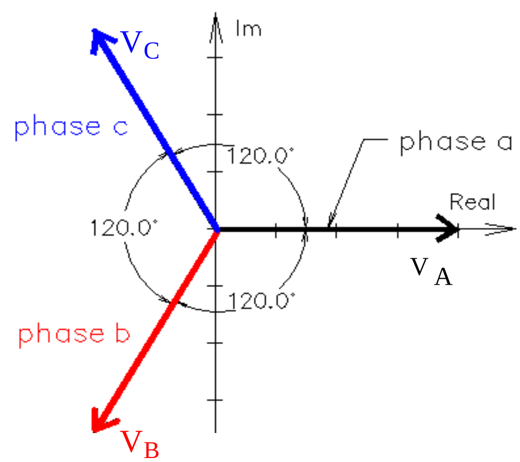


The return or "neutral" wire must be thicker than either "hot" line.

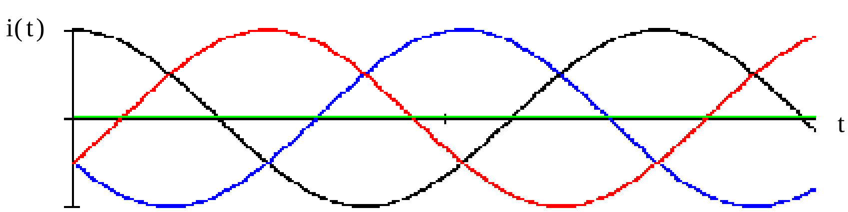
3-Phase Power

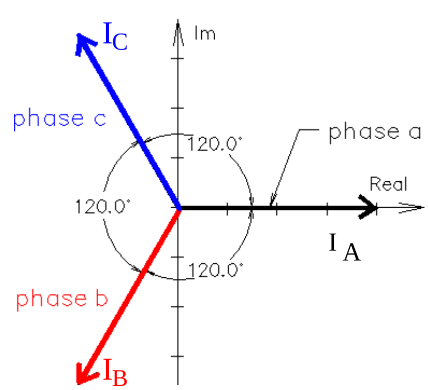






Three phase power is constant as long as the three loads are balanced.





If loads are balanced, ground return current will be zero.

If the loads are close to balanced the relatively small return current can be carried by the earth ground.

ECE 3600 3-Phase Power notes

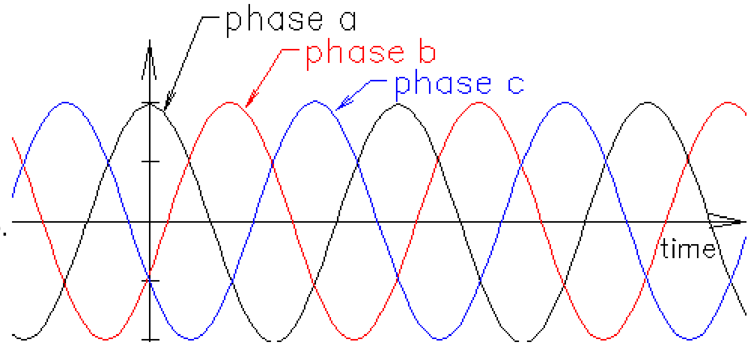
Basics

Single phase power pulses at 120 Hz. This is not good for motors or generators over about 5 hp.

Three phase power is constant as long as the three loads are balanced.

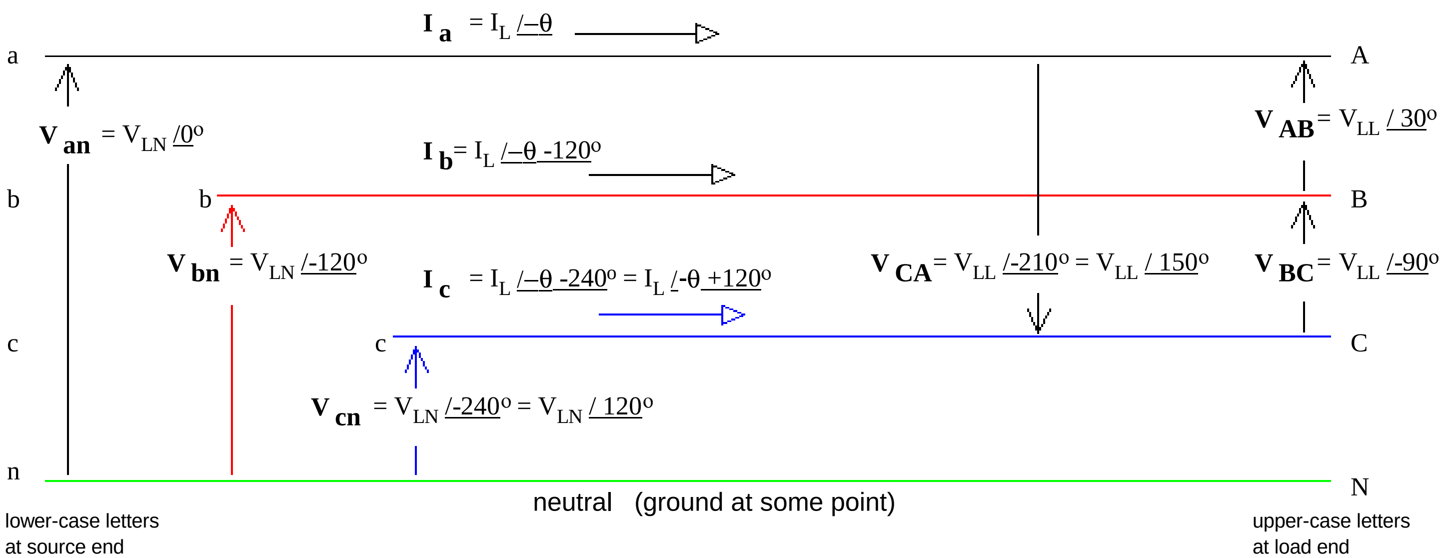
Three lines are needed to transmit 3-phase power.
If loads are balanced, ground return current will be zero.

The 3 lines coming into your house are **NOT** 3-phase.
They are +120 V, Gnd, -120 V



(The two 120s are 180º out-of-phase, allowing for 240 V connections)

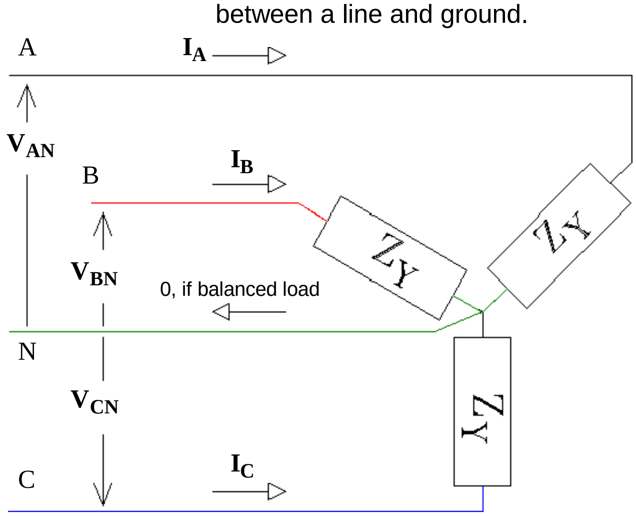
3-phase outlets have 4 connections

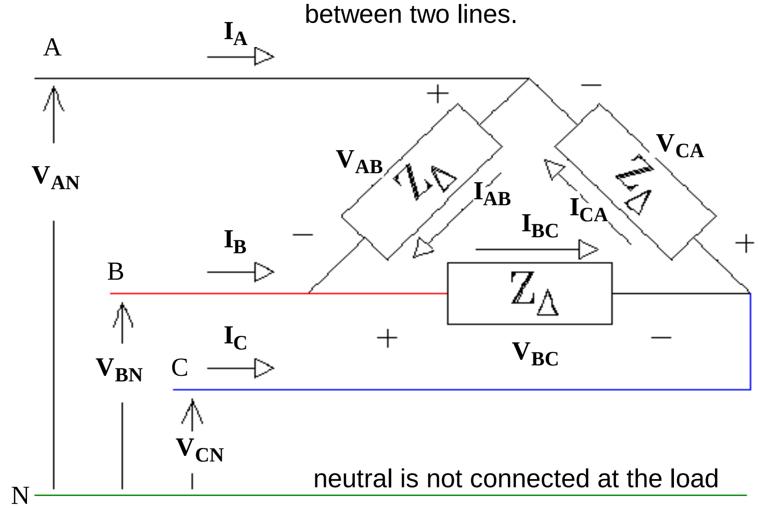


Connections to the 3 Lines

Wye connection:
Connect each load or generator phase between a line and ground.

Delta connection:
Connect each load or generator phase between two lines.





$$|V_{AN}| = |V_{BN}| = |V_{CN}| = V_{LN} = \frac{V_{LL}}{\sqrt{3}} = \frac{V_L}{\sqrt{3}}$$

$$|I_A| = |I_B| = |I_C| = I_L = \sqrt{3} \cdot I_{LL}$$

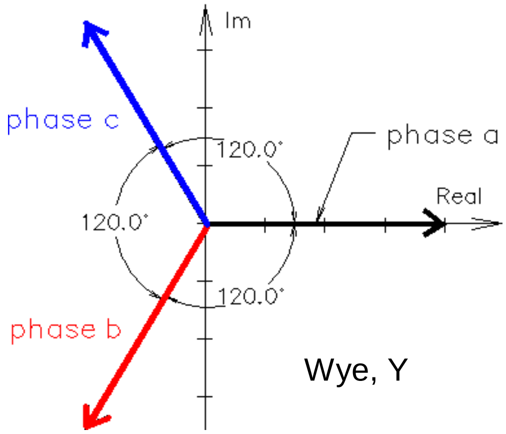
$$|V_{AB}| = |V_{BC}| = |V_{CA}| = V_{LL} = \sqrt{3} \cdot V_{LN} = V_L$$

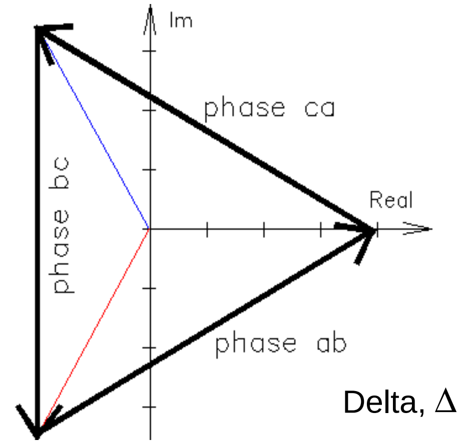
$$|I_{AB}| = |I_{BC}| = |I_{CA}| = I_{LL} = \frac{I_L}{\sqrt{3}}$$

To get equivalent line currents with equivalent voltages: $Z_Y = \frac{Z_\Delta}{3}$ $\quad Z_\Delta = 3 \cdot Z_y$

Wye, Y, connection:
Connect each load or generator phase between a line and ground.

Delta, Δ, connection:
Connect each load or generator phase between two lines.





$$V_{LN} = \frac{V_{LL}}{\sqrt{3}} \qquad I_L = \sqrt{3} \cdot I_{LL}$$

(Δ-connection)

$$V_{LL} = \sqrt{3} \cdot V_{LN} \qquad I_{LL} = \frac{I_L}{\sqrt{3}}$$

Apparent Power: $\left|\mathbf{S}_{3\phi}\right| = 3 \cdot \left|\mathbf{S}_{1\phi}\right| = 3 \cdot V_{LN} \cdot I_L = 3 \cdot V_{LL} \cdot I_{LL} = \sqrt{3} \cdot V_{LL} \cdot I_L$

Power: $P_{3\phi} = 3 \cdot P_{1\phi} = 3 \cdot V_{LN} \cdot I_L \cdot pf = 3 \cdot V_{LL} \cdot I_{LL} \cdot pf = \sqrt{3} \cdot V_{LL} \cdot I_L \cdot pf = S_{3\phi} \cdot pf$

$pf = \cos(\theta)$

Reactive power: $Q_{3\phi} = 3 \cdot Q_{1\phi} = 3 \cdot V_{LN} \cdot I_L \cdot \sin(\theta)$ etc... $= \sqrt{\left(\left|\mathbf{S}_{3\phi}\right|\right)^2 - P_{3\phi}^{\ 2}}$

Cautions about "L" subscripts:

I_L is always the line current, same as would flow in a Y-connected device.

V_L is always the line-to-line voltage, same as across a Δ-connected device.

When a single phase is taken from a 3-phase panel, then the line voltage (V_L) of that single phase is the line-to-neutral voltage of the 3-phase input to that panel, so the value of V_L changes in the panel (isn't that nice?).

Z_L could be the load impedance, either Y-connected or Δ-connected, or it could be the line impedance-- the impedance in the line itself, between the source and the load.

Cautions about "ϕ" or "ph" subscripts:

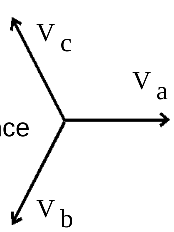
In our book: V_ϕ = the voltage across a single phase of a source or load and depends on the connection of that load, V_{LN} for Y-connected devices and V_{LL} for Δ-connected devices.

I_ϕ Also depends on connection.

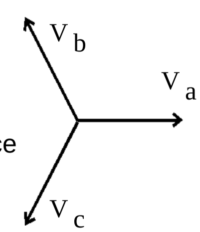
In **some** books: $V_\phi = V_{ph} = V_{LN}$ $\quad I_\phi = I_{ph}$ = current in a Y-connection <-- DON'T USE in this class

Phase sequences:

abc, "positive" sequence



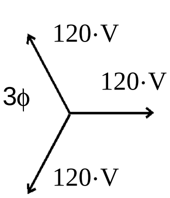
acb, "negative" or "reverse" sequence



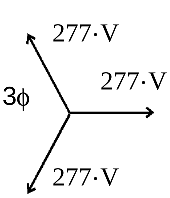
Common usage: $V_L = V_{LL}$ "line voltage" = line-to-line voltage

An unspecified voltage or a "line" voltage must always be assumed to be line-to-line,

208 3ϕ

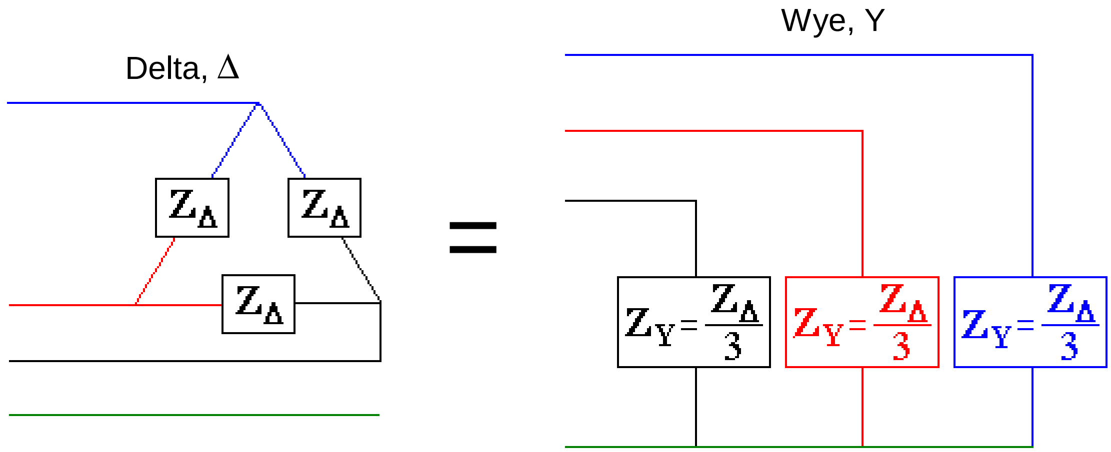


480 3ϕ



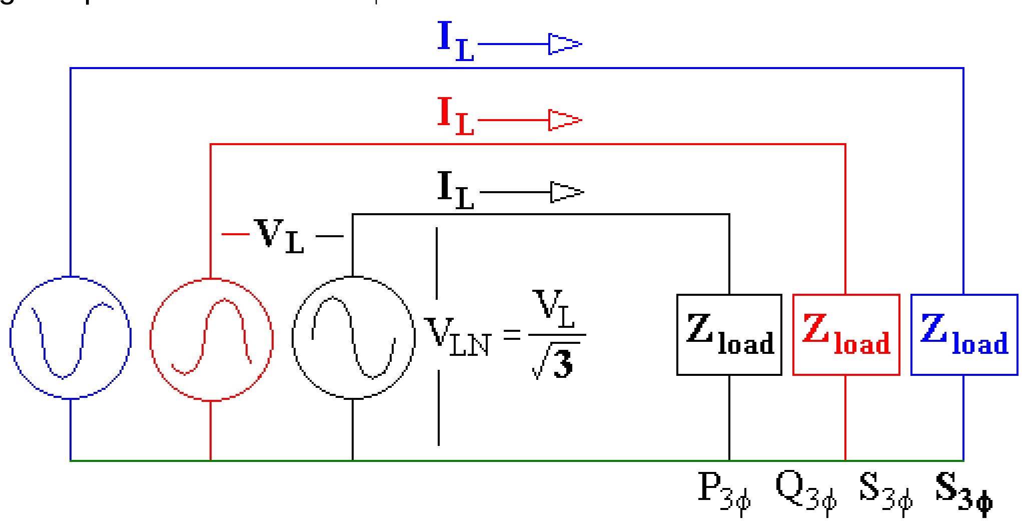
Our Approach Only works if system is **Balanced** (Always so in our class)

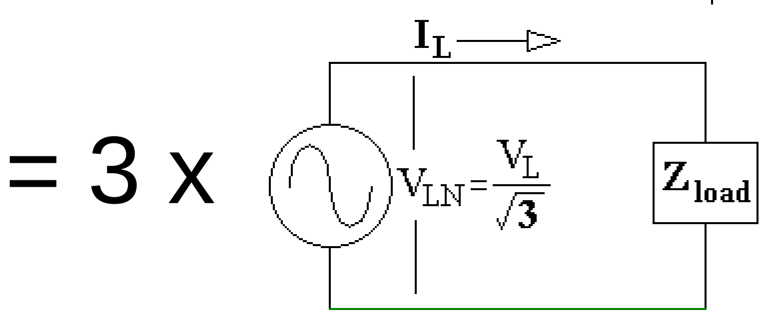
1) Change all Δ-connected loads to equivalent Y-connected loads $Z_Y = \frac{Z_\Delta}{3}$



2) Find all voltages as V_{LN}, especially $V_{LN} = \frac{V_L}{\sqrt{3}}$

3) Change all power numbers to 1ϕ.





$$P_{1\phi} = \frac{P_{3\phi}}{3} \qquad Q_{1\phi} = \frac{Q_{3\phi}}{3}$$

$$\mathbf{S}_{1\phi} = \frac{\mathbf{S}_{3\phi}}{3}$$

$$S_{1\phi} = \left|\mathbf{S}_{1\phi}\right| = \frac{S_{3\phi}}{3}$$

4) Solve the remaining single-phase problem.

5) Return to "line" voltages and 3ϕ powers, as necessary.

$$V_L = \sqrt{3} \cdot V_{LN}$$

$$P_{3\phi} = 3 \cdot P_{1\phi}$$
$$Q_{3\phi} = 3 \cdot Q_{1\phi}$$
$$\left|\mathbf{S}_{3\phi}\right| = 3 \cdot \left|\mathbf{S}_{1\phi}\right|$$
$$\mathbf{S}_{3\phi} = 3 \cdot \mathbf{S}_{1\phi}$$

In rare cases, you may also need: $I_\Delta = I_{LL} = \frac{I_L}{\sqrt{3}}$

and: $Z_\Delta = 3 \cdot Z_Y$

A.Stolp
9/9/09
rev 9/5/20

ECE 3600 3-Phase Examples

Ex. 1 A Y-connected load is connected to 208-V, 3-phase.
It draws 1.2kW of power at a power factor of 75%, leading. $P_{3\phi} := 1.2 \cdot kW$ $\quad pf := 0.75$

a) Find the apparent power and the reactive power.

$S_{3\phi} := \dfrac{P_{3\phi}}{pf}$ $\quad S_{3\phi} = 1.6 \cdot kVA$ $\quad Q_{3\phi} := -\sqrt{S_{3\phi}^{\,2} - P_{3\phi}^{\,2}}$ $\quad Q_{3\phi} = -1.058 \cdot kVAR$

Negative because the power factor is leading.

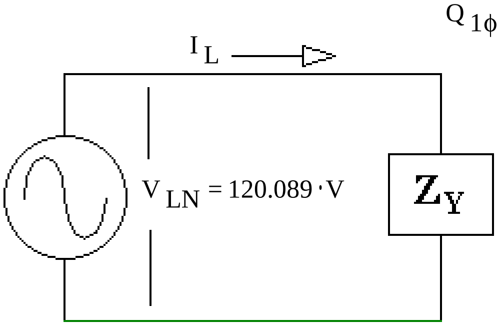
b) Find the line current.

Our Approach 1) Change all Δ-connected loads to equivalent Y-connected loads $\mathbf{Z_Y} = \dfrac{\mathbf{Z_\Delta}}{3}$ NOT NEEDED

2) Find all voltages as $V_{LN} := \dfrac{208 \cdot V}{\sqrt{3}}$ $\quad V_{LN} = 120.089 \cdot V$

3) Change all power numbers to 1ϕ. $P_{1\phi} := \dfrac{P_{3\phi}}{3}$ $\quad P_{1\phi} = 400 \cdot W$ $\quad S_{1\phi} := \dfrac{S_{3\phi}}{3}$ $\quad S_{1\phi} = 533.333 \cdot VA$

$Q_{1\phi} := \dfrac{Q_{3\phi}}{3}$ $\quad Q_{1\phi} = -352.767 \cdot VAR$



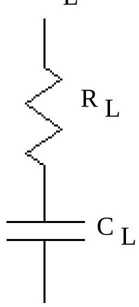
$S_{1\phi} = 533.333 \cdot VA$

$I_L := \dfrac{S_{1\phi}}{V_{LN}}$ $\quad I_L = 4.441 \cdot A$

c) Find the values of the load components, assumming they are connected in series.

The components must be a resistor and a capacitor because there is some real power and the power factor is leading.

$I_L = 4.441 \cdot A$ $\qquad$ assume $\omega = 377 \cdot \dfrac{rad}{sec}$



$R_L := \dfrac{P_{1\phi}}{I_L^{\,2}} \quad = \quad \dfrac{P_{1\phi}}{I_L^{\,2}} = 20.28 \cdot \Omega$

$X_C := \dfrac{Q_{1\phi}}{I_L^{\,2}}$ $\quad X_C = -17.885 \cdot \Omega \quad = \quad -\dfrac{1}{\omega \cdot C_L}$ $\quad C_L := -\dfrac{1}{\omega \cdot X_C}$ $\quad C_L = 148.3 \cdot \mu F$

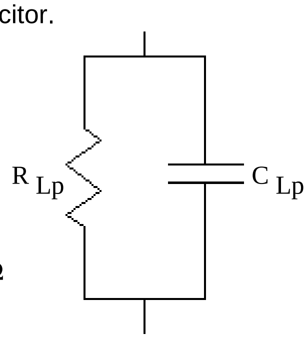
d) Find the values of the load components, assumming they are connected in parallel.

Still a resistor and a capacitor.

$V_\phi = 208 \cdot V$

$R_{Lp} := \dfrac{V_{LN}^{\,2}}{P_{1\phi}}$

$R_{Lp} = 36.053 \cdot \Omega$



$X_C := \dfrac{V_{LN}^{\,2}}{Q_{1\phi}}$

$X_C = -40.881 \cdot \Omega \quad = \quad -\dfrac{1}{\omega \cdot C_{Lp}}$

$C_{Lp} := -\dfrac{1}{\omega \cdot X_C}$ $\quad C_{Lp} = 64.9 \cdot \mu F$

e) Correct the power factor with Y-connected components.
Need inductors

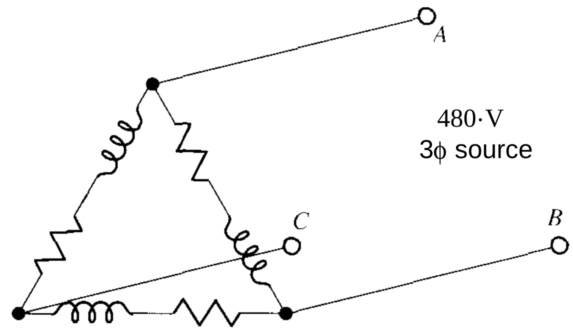
$$Q_{1\phi Ind} := -Q_{1\phi} = \frac{V_\phi^2}{\omega \cdot L_Y} \qquad L_Y := \frac{V_\phi^2}{\omega \cdot -Q_{1\phi}} \qquad L_Y = 325.3 \cdot mH$$

f) Correct the power factor with Δ-connected components.

$$L_\Delta := \frac{\left(\sqrt{3} \cdot V_\phi\right)^2}{\omega \cdot -Q_{1\phi}} \qquad L_\Delta = 975.9 \cdot mH$$

OR $\omega \cdot L_\Delta = \mathbf{Z}_\Delta = 3 \cdot \mathbf{Z}_y = 3 \cdot \omega \cdot L_Y \qquad 3 \cdot L_Y = 975.9 \cdot mH$

Ex. 2 From F08, exam 1, Find the following:



480·V
3φ source

All $\mathbf{Z} := (30 + 12 \cdot j) \cdot \Omega$

a) The line current that would be measured by an ammeter.

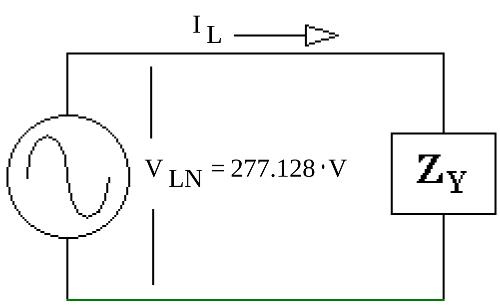
$V_{LL} := 480 \cdot V \qquad \mathbf{Z}_\Delta := (30 + 12 \cdot j) \cdot \Omega$

Our Approach
1) Change all Δ-connected loads to equivalent Y-connected loads

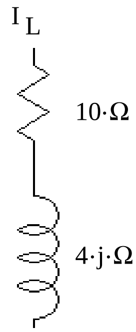
$$\mathbf{Z}_Y := \frac{\mathbf{Z}_\Delta}{3} \qquad \mathbf{Z}_Y = (10 + 4j) \cdot \Omega$$

2) Find all voltages as V_{LN} $\quad V_{LL} = 480 \cdot V \quad V_{LN} := \frac{V_{LL}}{\sqrt{3}} \quad V_{LN} = 277.128 \cdot V$

3) Change all power numbers to 1φ. NOT NEEDED

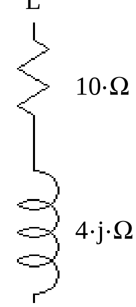


$\mathbf{Z}_Y = (10 + 4j) \cdot \Omega$ =



$$I_L := \frac{V_{LN}}{\left|\mathbf{Z}_Y\right|} = \frac{277.128 \cdot V}{\sqrt{10^2 + 4^2} \cdot \Omega} = 25.731 \cdot A$$

b) The power consumed by the three-phase load.



$P_{1\phi} = I_L^2 \cdot 10 \cdot \Omega = 6.62 \cdot kW$

$P_{3\phi} = 3 \cdot \left(I_L^2 \cdot 10 \cdot \Omega\right) = 19.86 \cdot kW$

c) The value of Y-connected impedances that would result in exactly the same line currents and same pf.

$$\mathbf{Z}_Y = (10 + 4j) \cdot \Omega$$

d) The value of Y-connected capacitors that would correct the pf.

$$Q_{1\phi} := \sqrt{S_{1\phi}^2 - P_{1\phi}^2} \qquad Q_{1\phi} := \sqrt{\left(V_{LN} \cdot I_L\right)^2 - (6.62 \cdot kW)^2} \qquad Q_{1\phi} = 2.65 \cdot kVAR$$

so we need:

$$Q_C := -Q_{1\phi} \qquad Q_C = -2.65 \cdot kVAR = -\frac{V_{LN}^2}{\left(\frac{1}{\omega \cdot C}\right)} = -V_{LN}^2 \cdot \omega \cdot C \qquad C := \frac{Q_C}{-V_{LN}^2 \cdot \omega} \qquad C = 91.5 \cdot \mu F$$

Ex. 3 For the three-phase delta-connected load in fig P1 .7,
The line-to-line voltage and line current are:

$\mathbf{V}_{\mathbf{AB}} := 480\cdot V \;\; \underline{/0^o}$ $\qquad \mathbf{I}_{\mathbf{A}} = 10A\underline{/-40^o}$

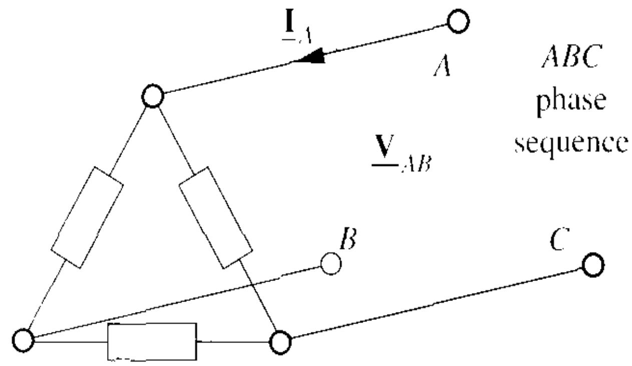
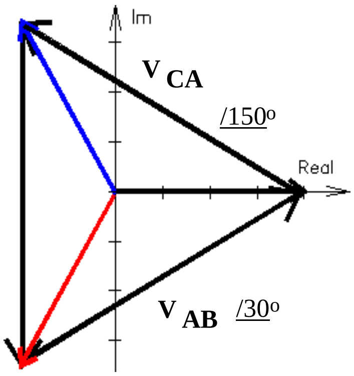


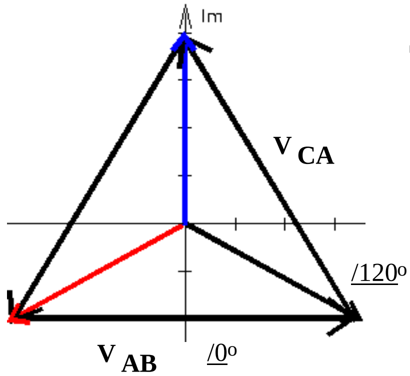
Figure P1.7

a) What is $\mathbf{V}_{\mathbf{CA}}$?

Normal phase angles



Rotate CW 30 deg



$\mathbf{V}_{\mathbf{CA}} := 480\cdot V \;\; \underline{/120^o}$

$= 480\cdot V \;\; \underline{/-240^o}$

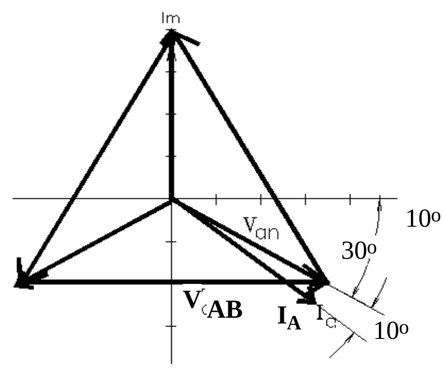
b) What is the phase current in the load?

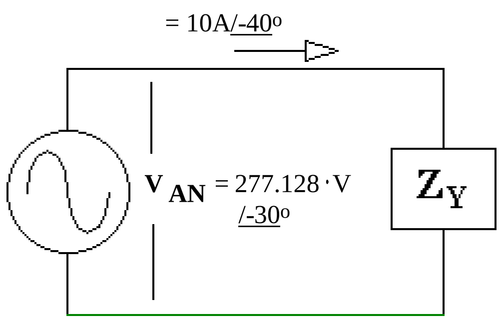
$$I_{LL} = \frac{I_L}{\sqrt{3}} \qquad \frac{10\cdot A}{\sqrt{3}} = 5.774\cdot A$$

c) What is the time-average power into the load?

$\mathbf{V}_{\mathbf{AN}} := \dfrac{480\cdot V}{\sqrt{3}} \;\; \underline{/-30^o}$ $\qquad$ Since $\mathbf{I}_{\mathbf{A}} = 10A\underline{/-40^o}$ $\qquad$ **I** lags **V** by 10^o

$\theta := 10\cdot deg$





$$P_{1\phi} = (277.128\cdot V\cdot 10\cdot A)\cdot\cos(\theta) = 2.729\cdot kW$$

$$P_{3\phi} = 3\cdot(277.128\cdot V\cdot 10\cdot A)\cdot\cos(\theta) = 8.188\cdot kW$$

d) What is the phase impedance?

$\mathbf{Z}_{\mathbf{Y}} := \dfrac{277.128\cdot V}{10\cdot A} \;\; \underline{/-30-(-40)^o}$ $\qquad \mathbf{Z}_{\mathbf{Y}} = 27.71\cdot\Omega \;\; \underline{/10^o}$

$\mathbf{Z}_{\Delta} = 3\cdot\mathbf{Z}_{\mathbf{Y}} = 83.14\cdot\Omega \;\; \underline{/10^o}$

Ex. 4 In the three-phase circuit shown in Fig. P1.9. find the following:

a) The line current that would be measured by an ammeter.

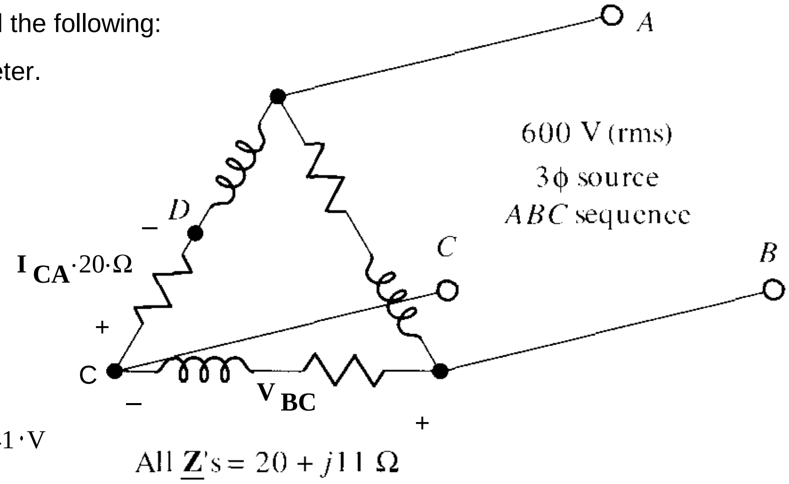


Figure P1.9

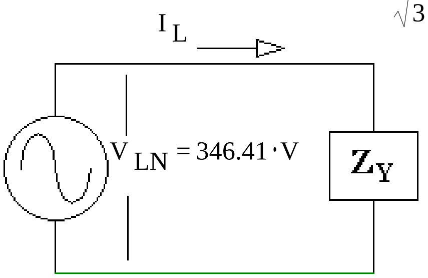
Direct way

$V_{LL} := 600\cdot V$ $\quad Z_\Delta := (20 + 11\cdot j)\cdot\Omega$

$I_{AB} := \left|\frac{V_{LL}}{Z_\Delta}\right|$ $\quad I_{AB} = 26.286\cdot A$

$I_A := \sqrt{3}\cdot I_{AB}$ $\quad I_A = 45.53\cdot A$

Our Approach $\quad V_{LN} := \frac{600\cdot V}{\sqrt{3}}$ $\quad V_{LN} = 346.41\cdot V$

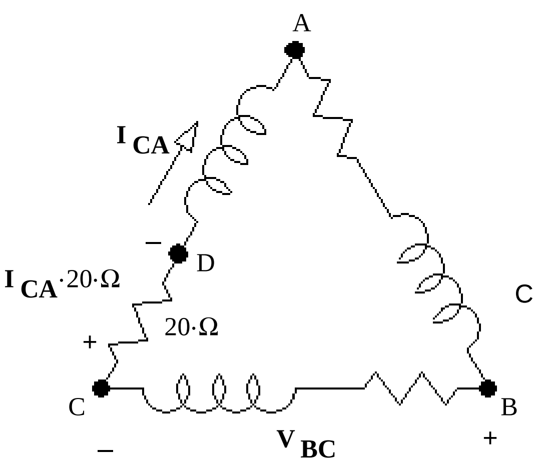


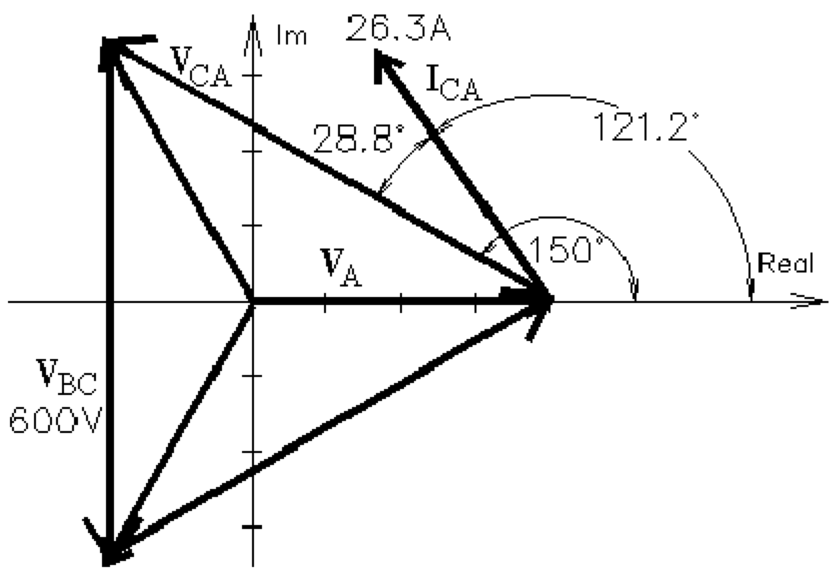
$Z_Y := \frac{Z_\Delta}{3}$ $\quad Z_Y = 6.667 + 3.667j \;\cdot\Omega$

$I_L := \frac{V_{LN}}{|Z_Y|} = \frac{346.41\cdot V}{\sqrt{6.667^2 + 3.667^2}}$ $\quad I_L = 45.53\cdot A$

b) The power factor of the three-phase load. $\quad \theta := \operatorname{atan}\left(\frac{11}{20}\right)$ $\quad \theta = 28.811\cdot deg$ $\quad$ pf $\quad \cos(\theta) = 0.876$

c) The voltage that would be measured between B and D by a voltmeter.





Using $\mathbf{V_A}$ as reference (0º):

$V_{BC} := 600\cdot V\cdot e^{-j\cdot 90\cdot deg}$

$I_{CA} := 26.286\cdot A\cdot e^{j\cdot(150 - 28.811)\cdot deg}$

$V_{CD} := I_{CA}\cdot 20\cdot\Omega$

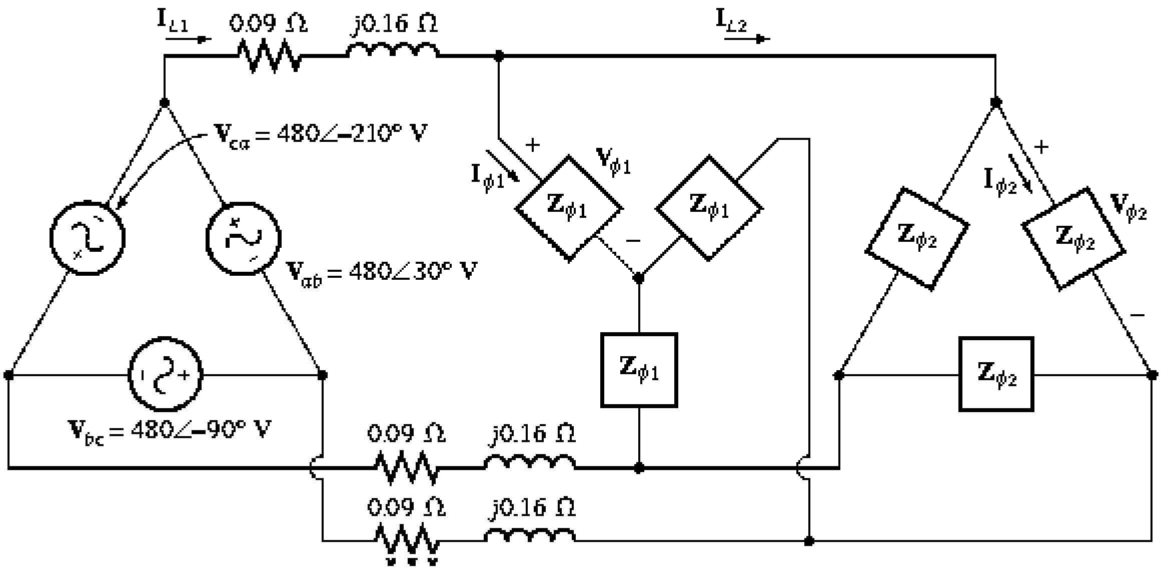
$V_{CD} = -272.251 + 449.734j \;\cdot V$

$V_{BD} := V_{BC} + V_{CD}$ $\quad V_{BD} = -272.251 - 150.266j \;\cdot V$ $\quad |V_{BD}| = 311\cdot V$

(must be the sum, NOT the difference, see the + and - signs on the drwawing.)

Ex. 5 When all you have is impedances and an input voltage, it gets messy & luckly, it's not a common problem.

Textbook problem 2-2. Figure P2-1 shows a three-phase power system with two loads. The Δ-connected generator is producing a line voltage of 480 V, and the line impedance is 0.09 + j0.16 Ω. Load 1 is Y-connected, with a phase impedance of 2.5Ω /36.87° and load 2 is Δ-connected, with a phase impedance of 5Ω /-20° .



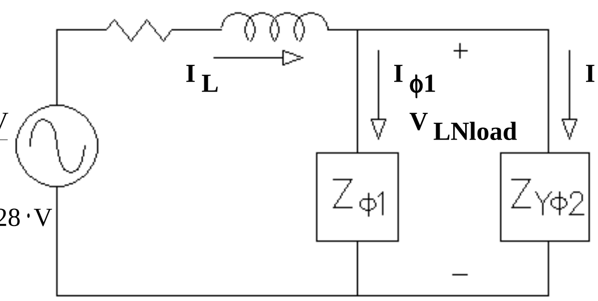
$Z_{\phi1} := 2.5 \cdot e^{j \cdot 36.87 \cdot deg} \cdot \Omega$ $\quad Z_{\phi2} := 5 \cdot e^{-j \cdot 20 \cdot deg} \cdot \Omega$

a) What is the line voltage at the two loads?

Find an equivalent Y-only circuit: $Z_{line} := (0.09 + 0.16 \cdot j) \cdot \Omega$

$V_Y := \dfrac{480 \cdot V}{\sqrt{3}}$

$V_Y = 277.128 \cdot V$



$Z_{Y\phi2} := \dfrac{Z_{\phi2}}{3}$

$Z_{Y\phi2} = 1.566 - 0.57j \ \cdot \Omega$

$Z_{Yloads} := \dfrac{1}{\dfrac{1}{Z_{\phi1}} + \dfrac{1}{Z_{Y\phi2}}}$ $\qquad Z_{Yloads} = 1.13 + 0.044j \ \cdot \Omega$ $\qquad \left|Z_{Yloads}\right| = 1.131 \cdot \Omega$

$\arg\left(Z_{Yloads}\right) = 2.254 \cdot deg$

$Z_{Ytot} := Z_{line} + Z_{Yloads}$ $\qquad Z_{Ytot} = 1.22 + 0.204j \ \cdot \Omega$ $\qquad \left|Z_{Ytot}\right| = 1.237 \cdot \Omega$

$\arg\left(Z_{Ytot}\right) = 9.516 \cdot deg$

$I_L := \dfrac{V_Y}{Z_{Ytot}}$ $\qquad I_L = 220.998 - 37.047j \ \cdot A$ $\qquad \left|I_L\right| = 224.082 \cdot A$

$\arg\left(I_L\right) = -9.516 \cdot deg$

$V_{LNload} := I_L \cdot Z_{Yloads}$ $\qquad V_{LNload} = 251.311 - 32.025j \ \cdot V$ $\qquad \left|V_{LNload}\right| = 253.343 \cdot V$

$\arg\left(V_{LNload}\right) = -7.262 \cdot deg$

$V_{Lload} := V_{LNload} \cdot \sqrt{3}$ $\qquad V_{Lload} = 435.283 - 55.47j \ \cdot V$ $\qquad \left|V_{Lload}\right| = 438.803 \cdot V$

b) What is the voltage drop on the transmission lines?

$\mathbf{V}_{\mathbf{linedrop}} := \mathbf{I}_{\mathbf{L}} \cdot \mathbf{Z}_{\mathbf{line}}$ $\quad \mathbf{V}_{\mathbf{linedrop}} = 25.817 + 32.025j \cdot V$ $\quad \left|\mathbf{V}_{\mathbf{linedrop}}\right| = 41.136 \cdot V$

$\arg\left(\mathbf{V}_{\mathbf{linedrop}}\right) = 51.126 \cdot deg$

Check: $\mathbf{V}_{\mathbf{Y}} - \mathbf{V}_{\mathbf{LNload}} = 25.817 + 32.025j \cdot V$

c) Find the real and reactive powers supplied to each load.

$I_{\phi 1} := \dfrac{\left|\mathbf{V}_{\mathbf{LNload}}\right|}{\left|\mathbf{Z}_{\boldsymbol{\phi}\mathbf{1}}\right|}$ $\quad I_{\phi 1} = 101.337 \cdot A$ $\quad I_{\phi 2} := \dfrac{\left|\mathbf{V}_{\mathbf{LNload}}\right|}{\left|\mathbf{Z}_{\mathbf{Y}\boldsymbol{\phi}\mathbf{2}}\right|}$ $\quad I_{\phi 2} = 152.006 \cdot A$

$P_{3\phi 1} := 3 \cdot I_{\phi 1}^{2} \cdot \mathrm{Re}\left(\mathbf{Z}_{\boldsymbol{\phi}\mathbf{1}}\right)$ $\quad P_{3\phi 1} = 61.615 \cdot kW$ $\quad P_{3\phi 2} := 3 \cdot I_{\phi 2}^{2} \cdot \mathrm{Re}\left(\mathbf{Z}_{\mathbf{Y}\boldsymbol{\phi}\mathbf{2}}\right)$ $\quad P_{3\phi 2} = 108.562 \cdot kW$

$Q_{3\phi 1} := 3 \cdot I_{\phi 1}^{2} \cdot \mathrm{Im}\left(\mathbf{Z}_{\boldsymbol{\phi}\mathbf{1}}\right)$ $\quad Q_{3\phi 1} = 46.212 \cdot kVAR$ $\quad Q_{3\phi 2} := 3 \cdot I_{\phi 2}^{2} \cdot \mathrm{Im}\left(\mathbf{Z}_{\mathbf{Y}\boldsymbol{\phi}\mathbf{2}}\right)$ $\quad Q_{3\phi 2} = -39.513 \cdot kVAR$

d) Find the real and reactive power losses in the transmission line.

$P_{3\phi L} := 3 \cdot \left(\left|\mathbf{I}_{\mathbf{L}}\right|\right)^{2} \cdot \mathrm{Re}\left(\mathbf{Z}_{\mathbf{line}}\right)$ $\quad P_{3\phi L} = 13.557 \cdot kW$

$Q_{3\phi L} := 3 \cdot \left(\left|\mathbf{I}_{\mathbf{L}}\right|\right)^{2} \cdot \mathrm{Im}\left(\mathbf{Z}_{\mathbf{line}}\right)$ $\quad Q_{3\phi L} = 24.102 \cdot kVAR$

e) Find the real power, reactive power, and power factor supplied by the generator.

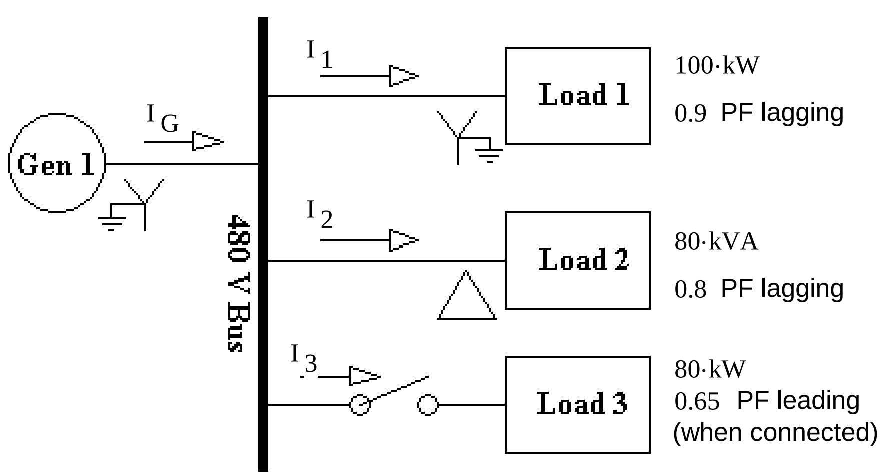
$P_{3\phi gen} := P_{3\phi L} + P_{3\phi 1} + P_{3\phi 2}$ $\quad P_{3\phi gen} = 183.734 \cdot kW$

$Q_{3\phi gen} := Q_{3\phi L} + Q_{3\phi 1} + Q_{3\phi 2}$ $\quad Q_{3\phi gen} = 30.801 \cdot kVAR$ $\quad pf = \dfrac{P_{3\phi gen}}{3 \cdot \left|\mathbf{V}_{\mathbf{Y}}\right| \cdot \left|\mathbf{I}_{\mathbf{L}}\right|} = 0.986$ lagging

f) What is the efficiency of this system? $\quad \eta = \dfrac{P_{3\phi 1} + P_{3\phi 2}}{P_{3\phi gen}} = 92.621 \cdot \%$

The next example uses a "one-line diagram" to show how a generator is connected to 3 loads. In these diagrams, one line represents all 3 phases and nuetral. Because the individual lines are not shown, there may be notes or symbols to indicate Y or Δ connections. All powers given will be 3-phase values, all voltages will be line voltages (that is line-to-line) and all currents will line currents. The term "bus" refers common connection area.

Ex. 6 The one-line diagram below shows a single, Y-connected generator and 3 loads. Assume all lines are lossless.



Find:

a) The phase voltage and currents in Load 1.

$V_{LL} := 480 \cdot V \qquad V_{LN} := \frac{V_{LL}}{\sqrt{3}} \qquad V_{LN} = 277.128 \cdot V = V_{L1\phi}$

$pf_{L1} := 0.9 \qquad S_{L1.1\phi} := \frac{100 \cdot kW}{3 \cdot pf_{L1}} \qquad I_1 := \frac{S_{L1.1\phi}}{V_{LN}} \qquad I_1 = 133.646 \cdot A = I_{L1\phi}$

b) The phase voltage and currents in Load 2.

$V_{LL} := 480 \cdot V = V_{L2\phi} \qquad pf_{L2} := 0.8 \qquad S_{L2.1\phi} := \frac{80 \cdot kVA}{3}$

$I_2 := \frac{S_{L2.1\phi}}{V_{LN}} \qquad I_2 = 96.225 \cdot A = \sqrt{3} \cdot I_{L2\phi} \qquad I_{L2\phi} = \frac{I_2}{\sqrt{3}} = 55.556 \cdot A$

c) The real, reactive and apparent power supplied by the generator with the switch to load 3 open.

$P_1 := 100 \cdot kW \qquad P_2 := 80 \cdot kVA \cdot pf_{L2} \qquad P_2 = 64 \cdot kW \qquad P_G := P_1 + P_2 \qquad P_G = 164 \cdot kW$

$Q_1 := \sqrt{\left(\frac{100 \cdot kW}{pf_{L1}}\right)^2 - (100 \cdot kW)^2} \qquad Q_1 = 48.432 \cdot kVAR \qquad Q_2 := \sqrt{(80 \cdot kVA)^2 - (64 \cdot kW)^2} \qquad Q_1 = 48.432 \cdot kVAR$

$Q_G := Q_1 + Q_2 \qquad Q_G = 96.432 \cdot kVAR$

$S_G := \sqrt{P_G^{\,2} + Q_G^{\,2}} \qquad S_G = 190.25 \cdot kVAR$

d) The total line current from the generator, I_G, with the switch to load 3 open.

$I_G = \frac{\left(\frac{S_G}{3}\right)}{V_{LN}} = 228.836 \cdot A$

e) The real, reactive and apparent power supplied by the generator with the switch to load 3 closed.

$pf_{L3} := 0.65 \qquad S_{L3.1\phi} := \frac{80 \cdot kW}{3 \cdot pf_{L3}} \qquad Q_3 := -\sqrt{\left(\frac{80 \cdot kW}{pf_{L3}}\right)^2 - (80 \cdot kW)^2} \qquad Q_3 = -93.53 \cdot kVAR$

$P_G := P_1 + P_2 + 80 \cdot kW \qquad P_G = 244 \cdot kW \qquad Q_G := Q_1 + Q_2 + Q_3 \qquad Q_G = 2.902 \cdot kVAR$

$S_G := \sqrt{P_G^{\,2} + Q_G^{\,2}} \qquad S_G = 244.017 \cdot kVAR$

f) How does the total line apparent power from the generator, S_G, compare to the sum of the three individual apparent powers, $S_1 + S_2 + S_3$? If they aren't equal, why not? (Switch closed)

$3 \cdot S_{L1.1\phi} + 80 \cdot kVAR + 3 \cdot S_{L3.1\phi} = 314.188 \cdot kVAR \neq S_G = 244.017 \cdot kVAR$

Can't Add Magnitudes

g) The total line current from the generator, I_G, with the switch to load 3 closed.

$I_G := \frac{\left(\frac{S_G}{3}\right)}{V_{LN}} \qquad I_G = 293.507 \cdot A$

h) How does the total line current from the generator, I_G, compare to the sum of the three individual currents, $I_1 + I_2 + I_3$? If they aren't equal, why not? (Switch closed)

$I_3 := \frac{S_{L3.1\phi}}{V_{LN}} \qquad I_3 = 148.039 \cdot A$

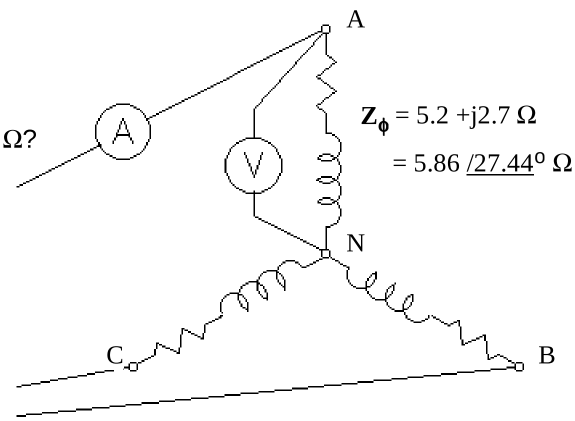
$I_1 + I_2 + I_3 = 377.909 \cdot A \neq I_G = 293.507 \cdot A$

Can't Add Magnitudes

ECE 3600 homework # 4 Due: Fri, 9/11/20 c

Note: All voltages and currents are always assumed to be RMS unless said to be otherwise.

1. The following are questions from p 78 of the textbook. These could be good closed-book exam questions.
   a) 2.1. What types of connections are possible for three-phase generators and loads?
   b) 2.2. What is meant by the term “balanced” in a balanced three-phase system?
   c) 2.3. What is the relationship between phase and line voltages and currents for a wye (Y) connection?
   d) 2.4. What is the relationship between phase and line voltages and currents for a delta (Δ) connection?
   e) 2.5. What is phase sequence?
   f) 2.7. What is a Y-Δ transform?
2. Textbook 2-1. Three impedances of $4 + j3\ \Omega$ are Δ-connected and tied to a three-phase 208-V power line. Find I_ϕ, I_L, P, Q, S (|S|), and the power factor of this load.
3. A balanced three-phase 480-V source (three line-to-neutral voltages of 277 V) supplies a balanced three-phase inductive load. The load draws a total of 9 kW at a power factor of 0.9. Calculate the phase currents and the magnitude of the per-phase load impedances, assuming a Y-connected load. Draw a phasor diagram showing all three voltages and currents, assume V_a is 0°.
   b) In order to correct the power factor, three capacitors are connected in parallel with the load impedances. Find the value of the capacitors.
4. Repeat problem 3, assuming a delta-connected load.
5. The voltmeter shown measures 120 V. Let this voltage be the phase reference (0°). The phase impedance is $\mathbf{Z_\phi} = 5.2 + j2.7 = 5.86\ \underline{/27.44^o}\ \Omega$?
   a) What is $\mathbf{V_{AB}}$ as a phasor?
   b) What would the ammeter measure?
   c) What is the apparent power?
   d) What is the real power?
   e) Correct the power factor with capacitors connected in a delta configuration, that is, find the value of the capacitors.



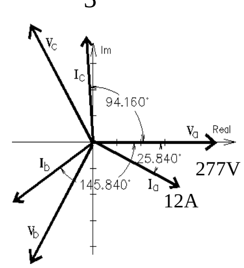
6. Three 230-V generators are connected in a wye configuration to generate three-phase power. The load consists of three balanced delta-connected impedances of $\mathbf{Z_L} = 3.8 + j1.5\ \Omega$.
   a) An ammeter is placed in one line, what would it measure?
   b) Find the total apparent power. c) Find the total real power consumed by the load.
   d) What is the phase angle between $\mathbf{I_A}$ and $\mathbf{V_{AB}}$, assuming ABC rotation?

Answers

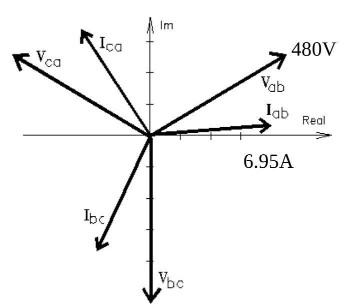
1. a) 2.1. Y & Δ b) 2.2. The 3 voltages are equal, the 3 currents are equal and the 3 loads are equal.
   c) 2.3. $V_\phi = \dfrac{V_{LL}}{\sqrt{3}} = \dfrac{V_L}{\sqrt{3}}$ $I_\phi = I_L$ d) 2.4. $V_\phi = V_{LL} = V_L$ $I_\phi = \dfrac{I_L}{\sqrt{3}}$
   e) 2.5. abc or acb f) 2.7. $Z_Y = \dfrac{Z_\Delta}{3}$

2. $41.6\cdot A\cdot A$ $72.1\cdot A$ $20.8\cdot kW$ $15.6\cdot kVAR$ $26.0\cdot kVA$

3. $12\cdot A$ lagging by 25.8^o $23\cdot\Omega$
   b) $50.2\cdot\mu F$



4. $6.95A\ \underline{/4.16^o}$ $69.1\cdot\Omega$
   b) $16.7\cdot\mu F$



5. a) $208\cdot V\cdot e^{j\cdot 30\cdot deg}$ b) $20.5\cdot A$ c) $7.37\cdot kVA$ d) $6.54\cdot kW$ e) $69.5\cdot\mu F$

6. $168\cdot A$ $117\cdot kVA$ $108\cdot kW$ -51.541^o

ECE 3600 homework # 5 & 6 Hw 5 Due: Tu, 9/15/20

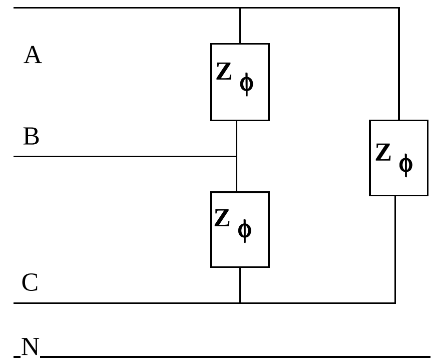
1. A 3-phase circuit is connected as shown.
   Find the following:

   3ϕ source

   $P_{3\phi} := 18 \cdot kW$

   $S_{3\phi} := 21 \cdot kVA$

   $V_S := 520 \cdot V$



   a) The load power factor, assume lagging.

   b) The line current.

   c) The phase impedance, $\mathbf{Z}_\phi$

   d) The value of Y-connected impedances that would result in exactly the same line currents and same pf.

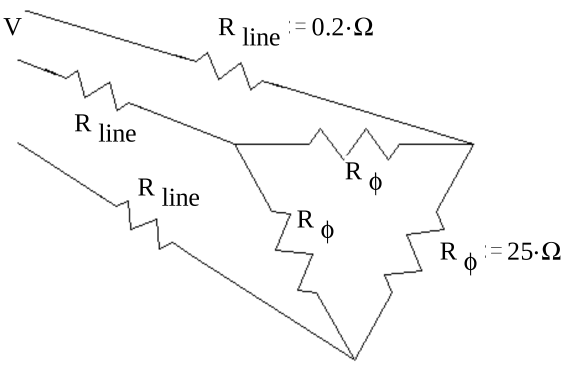
   e) The reactive power of each $\mathbf{Z}_\phi$

   f) Correct the power factor with capacitors connected in a wye configuration. $\omega := 377 \cdot \frac{rad}{sec}$

2. For the three-phase circuit shown, the R_{line} resistors represent the resistance of the distribution system. Find the following:

   $V_S := 480 \cdot V$

   3ϕ source



   a) Total power out of the source, including line and load.

   b) Line losses.

   c) Distribution system efficiency.

3. Textbook 2-6

   a) & b) For the apparent power, just the total will be sufficient.

   For the two parts below, assume the source voltage is adjusted so that the bus voltage at the plant remains $480V$ and the lines each have an impedance of $\mathbf{Z}_{line} := (0.05 + j \cdot 0.1) \cdot \Omega$

   c) With the switch open, find the magnitude of the source voltage and the efficiency of the system.

   d) With the switch closed, find the magnitude of the source voltage and the efficiency of the system.

Answers

1. a) 0.857
   b) $23.3 \cdot A$
   c) $38.6 \cdot \Omega \; \angle 31 \cdot deg$
   d) $12.9 \cdot \Omega \; \angle 31 \cdot deg$
   e) $3.61 \cdot kVAR$
   f) $106 \cdot \mu F$

2. a) $27 \cdot kW$
   b) $632.8 \cdot W$
   c) 97.7%

3. a) $59.86 \cdot kW$
   $34.56 \cdot kVAR$
   $46.04 \cdot kW$
   $34.53 \cdot kVAR$
   input:
   $105.9 \cdot kW$
   $69.09 \cdot kVAR$
   $126.4 \cdot kVA$
   $152 \cdot A$

   b) Loads 1 & 2 are the same
   Caps
   $0 \cdot W$
   $-46.06 \cdot kVAR$
   input:
   $105.9 \cdot kW$
   $23.03 \cdot kVAR$
   $108.4 \cdot kVA$
   $130.4 \cdot A$

   c) $505.4 \cdot V$
   $96.8 \cdot \%$

   d) $496.0 \cdot V$
   $97.6 \cdot \%$

ECE 3600 homework # 6 Tue: Tue, 9/18/20

1. Textbook 1-7
2. Textbook Example 1-2 with
   Mean magnetic length: $l_c := 50 \cdot cm$
   Air gap length: $l_a := 0.06 \cdot cm$
   Core cross-sectional area: $A_c := 16 \cdot cm^2$

Answers 1. $0.0054 \cdot Wb$ 2. $727 \cdot mA$