

Name: _____

ECE 3600 homework # 1

Due: Fri, 1/13/23

e

Base your answers on class lecture & discussion, books and/or internet research. Some possible sources:

<http://www.nerc.com/>

http://en.wikipedia.org/wiki/Electricity_generation

<http://www.energy.gov/energysources/electricpower.htm>

http://en.wikipedia.org/wiki/Relative_cost_of_electricity_generated_by_different_sources

1. What is the name of the organization which ensures the reliability of power in North America?

2. Electric Utilities have been forced to break up into two separate companies responsible for:
 - a.

 - b.

3. What does deregulation provide for independent power producers (IPPs)?

4. The current bottleneck to overall system capacity.

5. What are the advantages of a highly interconnected system? (List at least 2). Also give a disadvantage.

6. Rank the sources of electrical energy in the US (highest to lowest %)
 - 1.
 - 2.
 - 3.
 - 4.
 5. Other

7. List 3 of the "Other" sources.
 - 1.
 - 2.
 - 3.

8. Rank the sources of electrical energy in the US by environmental and social negatives (worst to best). Assume "Other" is all the 3 you listed above. Consider petroleum just a little worse than natural gas (due to the danger of spills). Also give (in your opinion) the worst environment or social negative of each. Your answers here may be subjective.
 - 1.
 - 2.
 - 3.
 - 4.

9. Rank the sources of electrical energy in the US cost per kWh.

List Nat gas twice, once for single cycle and once for combined-cycle. Choose one of the "Other" that you listed above. Initial costs are amortized over the life of the generation facility. You will have to make some guesses and may qualify your answers.

1. (cheapest)
- 2.
- 3.
- 4.
- 5.
6. (most expensive).

10. Give the approximate efficiencies of each type of power plant:

- a. Hydroelectric
- b. Rankine-cycle steam turbine plants, regardless of the source of heat.
(coal, oil, gas-steam, nuclear, solar-steam, geothermal)
- c. Single-cycle (Brayton-cycle) gas turbine
- d. Combined-cycle (Brayton-cycle flowed by Rankine-cycle)

11. In nuclear fission reactions, what is particle is crucial to the chain reaction and is used to control the reaction rate?

12. a) Why can't a wind turbine's coefficient of performance (conversion of wind energy to rotational mechanical energy) be 100%?

b) What two things can be controlled to maximize the coefficient of performance?

c) What is the biggest single problem of wind power?

13. a) Do photovoltaic cells produce AC or DC power?

b) What are the 2 biggest problems of photovoltaic cells?

14. What is cogeneration?

15. Some power sources are used to supply base loads and some are used to supply peak loads. Give some reasons to differentiate the sources in this way.

Base loads

Peak loads

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16. Requirements of the power system

1.

2.

3.

4.

5.

17. What two things are constantly monitored by the power company to assure that they meeting the demand.

1.

2.

18. Sensors placed around the network can let operators know if these requirements are being met What is the name of this system:

Review of Phasors

ECE 3600

A. Stolp
9/3/08
rev.

For steady-state sinusoidal response ONLY

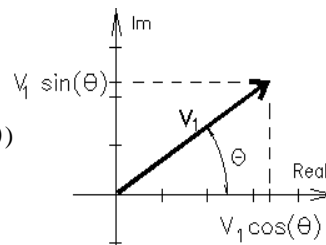
Phasors

Time domain

$$v(t) = \sqrt{2} \cdot V_1 \cdot \cos(377 \cdot t + \theta)$$

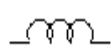
Phasor, frequency domain (RMS)

$$\mathbf{V}_1 = V_1 \cdot e^{j\theta} = V_1 \angle \theta = V_1 \cdot \cos(\theta) + j \cdot V_1 \cdot \sin(\theta)$$



Impedances,

Inductor



$$v_L = L \cdot \frac{d}{dt} i_L = L \cdot \frac{d}{dt} I_p \cdot e^{j(\omega t + \theta)} = j \cdot \omega \cdot L \cdot [I_p \cdot e^{j(\omega t + \theta)}]$$

$$\mathbf{V}_L(\omega) = j \cdot \omega \cdot L \cdot \mathbf{I}(\omega)$$

AC impedance

$$\mathbf{Z}_L = j \cdot \omega \cdot L$$

Capacitor



$$i_C = C \cdot \frac{d}{dt} v_C = C \cdot \frac{d}{dt} V_p \cdot e^{j(\omega t + \theta)} = j \cdot \omega \cdot C \cdot [V_p \cdot e^{j(\omega t + \theta)}]$$

$$\mathbf{I}_C(\omega) = j \cdot \omega \cdot C \cdot \mathbf{V}(\omega)$$

$$\mathbf{V}_C(\omega) = \frac{1}{j \cdot \omega \cdot C} \cdot \mathbf{I}(\omega)$$

$$\mathbf{Z}_C = \frac{1}{j \cdot \omega \cdot C} = \frac{-j}{\omega \cdot C}$$

Resistor



$$v_R = i_R \cdot R$$

$$\mathbf{V}_R(\omega) = R \cdot \mathbf{I}(\omega)$$

$$\mathbf{Z}_R = R$$

You can use impedances just like resistances as long as you deal with the complex arithmetic.
ALL the DC circuit analysis techniques will work with AC.

series:



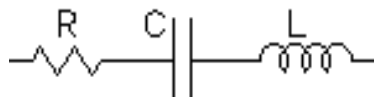
$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \dots$$

$$f := 60 \text{ Hz}$$

$$\omega := 2 \cdot \pi \cdot f$$

$$\omega = 377 \frac{\text{rad}}{\text{sec}}$$

Example:



$$R := 20 \cdot \Omega$$

$$L := 80 \text{ mH}$$

$$C := 60 \cdot \mu\text{F}$$

$$j \cdot \omega \cdot L = 30.159j \cdot \Omega$$

$$\frac{1}{j \cdot \omega \cdot C} = -44.21j \cdot \Omega$$

$$\mathbf{Z}_{eq} := R + \frac{1}{j \cdot \omega \cdot C} + j \cdot \omega \cdot L = 20 \cdot \Omega - 44.21j \cdot \Omega + 30.16j \cdot \Omega = 20 - 14.05j \cdot \Omega$$

$$\sqrt{(20 \cdot \Omega)^2 + (14.05 \cdot \Omega)^2} = 24.44 \cdot \Omega$$

$$\text{atan}\left(\frac{-14.05 \cdot \Omega}{20 \cdot \Omega}\right) = -35.09 \cdot \text{deg}$$

$$\mathbf{Z}_{eq} = 24.44 \Omega \angle -35.1^\circ$$

$$\text{If: } \mathbf{V} := 120 \cdot \text{V} \cdot e^{j \cdot 0 \cdot \text{deg}}$$

$$\mathbf{I} := \frac{\mathbf{V}}{\mathbf{Z}_{eq}} = \frac{120 \cdot \text{V}}{24.44 \cdot \Omega} = 4.91 \cdot \text{A} \quad \angle 0 - -35.1 = 35.1 \text{ deg}$$

$$4.91 \cdot \cos(35.1 \cdot \text{deg}) = 4.017$$

$$4.91 \cdot \sin(35.1 \cdot \text{deg}) = 2.823$$

$$\mathbf{I} = 4.017 + 2.822j \cdot \text{A}$$

slight roundoff error

Voltage divider:

$$V_{Z_n} = V_{total} \cdot \frac{Z_n}{Z_1 + Z_2 + Z_3 + \dots}$$

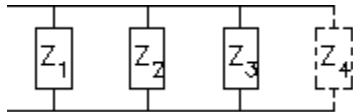
Eg: $V_C := V \cdot \frac{j\omega C}{Z_{eq}} = 120 \cdot V \cdot e^{j0 \cdot \text{deg}} \cdot \frac{44.21 \cdot e^{-j90 \cdot \text{deg}} \cdot \Omega}{24.44 \cdot e^{-j35.1 \cdot \text{deg}} \cdot \Omega}$

$$120 \cdot V \cdot \frac{44.21 \cdot \Omega}{24.44 \cdot \Omega} = 217.07 \cdot V \quad \angle 0 + -90 - -35.1 = -54.9 \text{ deg}$$

$$V_C = 217.1V \angle -54.9^\circ \quad V_C = 124.771 - 177.604j \cdot V$$

$$217.1 \cdot \cos(-54.9 \cdot \text{deg}) = 124.8 \quad 217.1 \cdot \sin(-54.9 \cdot \text{deg}) = -177.6$$

parallel:



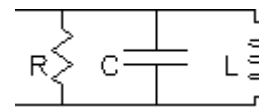
$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \dots}$$

Example:

$$f := 60 \cdot \text{Hz}$$

$$\omega := 2 \cdot \pi \cdot f$$

$$\omega = 377 \cdot \frac{\text{rad}}{\text{sec}}$$



$$L := 80 \cdot \text{mH}$$

$$R := 20 \cdot \Omega$$

$$C := 60 \cdot \mu\text{F}$$

$$j \cdot \omega \cdot L = 30.159j \cdot \Omega$$

$$\frac{1}{\omega \cdot L} = 3.316 \cdot 10^{-2} \cdot \frac{1}{\Omega}$$

$$\frac{1}{j \cdot \omega \cdot C} = -44.21j \cdot \Omega$$

$$\omega \cdot C = 2.262 \cdot 10^{-2} \cdot \frac{1}{\Omega}$$

$$Z_{eq} := \frac{1}{\frac{1}{R} + \frac{1}{\left(\frac{1}{j \cdot \omega \cdot C}\right)} + \frac{1}{j \cdot \omega \cdot L}} = \frac{1}{\frac{1}{R} + j \cdot \omega \cdot C - \frac{j}{\omega \cdot L}} = \frac{1}{\frac{1}{20 \cdot \Omega} + 2.262 \cdot 10^{-2} \cdot j \cdot \frac{1}{\Omega} - 3.316 \cdot 10^{-2} \cdot j \cdot \frac{1}{\Omega}} = \frac{1}{\left(5 \cdot 10^{-2} - 1.054 \cdot 10^{-2} \cdot j\right) \cdot \frac{1}{\Omega}}$$

$$= \frac{1}{\left(5 \cdot 10^{-2} - 1.054 \cdot 10^{-2} \cdot j\right) \cdot \frac{1}{\Omega}} \cdot \frac{5 \cdot 10^{-2} + 1.054 \cdot 10^{-2} \cdot j}{5 \cdot 10^{-2} + 1.054 \cdot 10^{-2} \cdot j} = 19.149 + 4.037j \cdot \Omega$$

$$\sqrt{\left(5 \cdot 10^{-2} \cdot \frac{1}{\Omega}\right)^2 + \left(1.054 \cdot 10^{-2} \cdot \frac{1}{\Omega}\right)^2} = 5.11 \cdot 10^{-2} \cdot \frac{1}{\Omega} \quad \text{atan}\left(\frac{-1.054 \cdot 10^{-2} \cdot \Omega}{5 \cdot 10^{-2} \cdot \Omega}\right) = -11.9 \cdot \text{deg}$$

$$\frac{1}{5.11 \cdot 10^{-2} \cdot \frac{1}{\Omega}} = 19.569 \cdot \Omega \quad \angle 0 - -11.9 = 11.9 \text{ deg} \quad Z_{eq} = 19.57\Omega \angle 11.9^\circ$$

ff: $V := 120 \cdot V \cdot e^{j0 \cdot \text{deg}} \quad I := \frac{V}{Z_{eq}} = \frac{120 \cdot V}{19.57 \cdot \Omega} = 6.132 \cdot A \quad \angle 0 - 11.9 = -11.9 \text{ deg}$

$$6.132 \cdot \cos(-11.9 \cdot \text{deg}) = 6$$

$$6.132 \cdot \sin(-11.9 \cdot \text{deg}) = -1.264$$

$$I = 6 - 1.265j \cdot A$$

slight roundoff error

Current divider:

$$I_{Z_n} = I_{total} \cdot \frac{\frac{1}{Z_n}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \dots}$$

Eg: $I_L := I \cdot \frac{\frac{1}{j \cdot \omega \cdot L}}{\frac{1}{R} + j \cdot \omega \cdot C + \frac{1}{j \cdot \omega \cdot L}} = I \cdot \frac{\left(\frac{1}{j \cdot \omega \cdot L}\right)}{\left(\frac{1}{Z_{eq}}\right)} = I \cdot \frac{Z_{eq}}{j \cdot \omega \cdot L}$

$$= 6.132 \cdot A \cdot e^{j-11.9 \cdot \text{deg}} \cdot \frac{19.57 \cdot e^{j11.9 \cdot \text{deg}} \cdot \Omega}{30.159 \cdot e^{j90 \cdot \text{deg}} \cdot \Omega}$$

$$I_L = 6.132 \cdot A \cdot \frac{19.57 \cdot \Omega}{30.159 \cdot \Omega} = 3.979 \cdot A$$

$$\angle -11.9 + 11.9 - 90 = -90 \text{ deg}$$

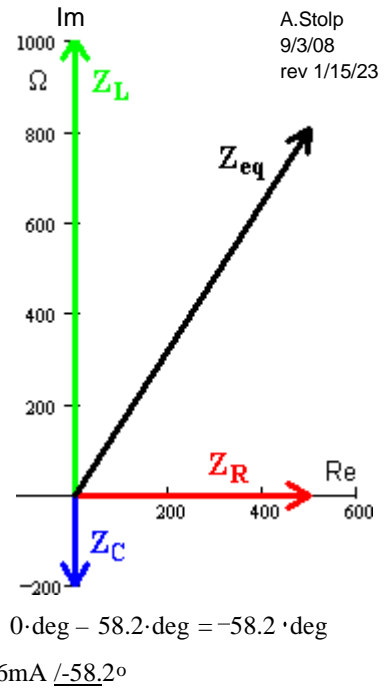
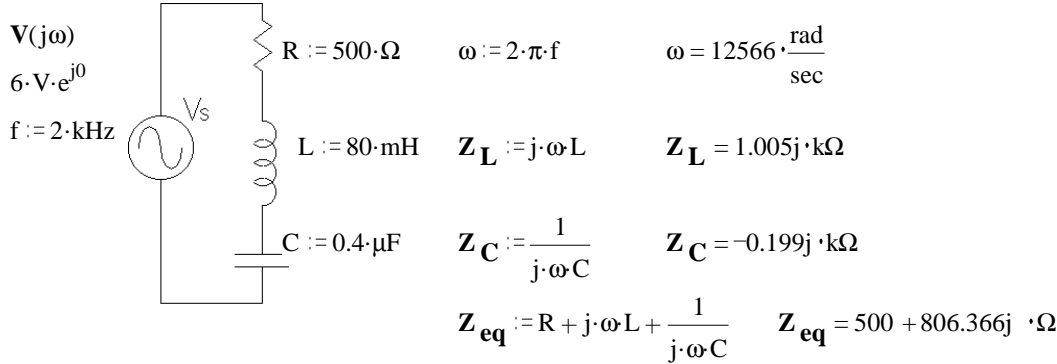
$$I_L = -3.979 \cdot 10^3 j \cdot \text{mA}$$

Duh... $\frac{V}{j \cdot \omega \cdot L} = -3.979 \cdot 10^3 j \cdot \text{mA}$

ECE 3600 Phasor Examples

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9/3/08
rev 1/15/23

Ex. 1 Find V_R , V_L , and V_C in polar phasor form. $f := 2 \cdot \text{kHz}$



$\sqrt{500^2 + 806^2} = 948.491$ $\text{atan}\left(\frac{806}{500}\right) = 58.187 \cdot \text{deg}$ $Z_{eq} = 948.5 \Omega / 58.2^\circ$
 find the current: $I := \frac{6 \cdot V \cdot e^{j0}}{Z_{eq}}$ magnitude: $\frac{6 \cdot V}{948.5 \Omega} = 6.326 \cdot \text{mA}$ angle: $0 \cdot \text{deg} - 58.2 \cdot \text{deg} = -58.2 \cdot \text{deg}$
 $I = 6.326 \text{mA} / -58.2^\circ$

find the magnitude

find the angle

$V_R := I \cdot R$ $6.326 \cdot \text{mA} \cdot 500 \cdot \Omega = 3.163 \cdot \text{V}$ $-58.2 \cdot \text{deg} + 0 \cdot \text{deg} = -58.2 \cdot \text{deg}$ $V_R = 3.163 \text{V} / -58.2^\circ$
 $V_L := I \cdot Z_L$ $6.326 \cdot \text{mA} \cdot 1005 \cdot \Omega = 6.358 \cdot \text{V}$ $-58.2 \cdot \text{deg} + 90 \cdot \text{deg} = 31.8 \cdot \text{deg}$ $V_L = 6.358 \text{V} / 31.8^\circ$
 $V_C := I \cdot Z_C$ $6.326 \cdot \text{mA} \cdot (-199) \cdot \Omega = -1.259 \cdot \text{V}$ $-58.2 \cdot \text{deg} + (90) \cdot \text{deg} = 31.8 \cdot \text{deg}$ $V_C = -1.259 \text{V} / 31.8^\circ$
 OR: $6.326 \cdot \text{mA} \cdot (199) \cdot \Omega = 1.259 \cdot \text{V}$ $-58.2 \cdot \text{deg} + (-90) \cdot \text{deg} = -148.2 \cdot \text{deg}$ $V_C = 1.259 \text{V} / -148.2^\circ$

OR, you can also find these voltages directly, using a voltage divider. I.E. to find V_C directly:

$$V_C := \frac{\frac{1}{j \cdot \omega C}}{R + j \cdot \omega L + \frac{1}{j \cdot \omega C}} \cdot 6 \cdot V = \frac{1}{R \cdot (j \cdot \omega C) + j \cdot \omega L \cdot (j \cdot \omega C) + 1} \cdot 6 \cdot V = \frac{1}{R \cdot (j \cdot \omega C) - \omega^2 \cdot L \cdot C + 1} \cdot 6 \cdot V$$

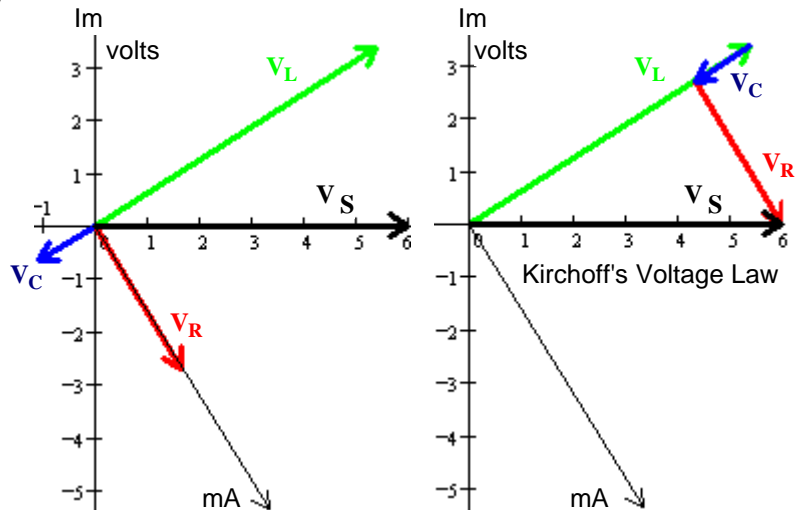
$$= \frac{1}{(1 - \omega^2 \cdot L \cdot C) + j \cdot \omega R \cdot C} \cdot 6 \cdot V \quad (1 - \omega^2 \cdot L \cdot C) = -4.053 \quad j \cdot \omega R \cdot C = 2.513j$$

$$= \frac{6 \cdot V}{-4.053 + 2.513j} \cdot \frac{(-4.053 - 2.513j)}{(-4.053 - 2.513j)} = \frac{6 \cdot V \cdot (-4.053 - 2.513j)}{(-4.053)^2 + 2.513^2}$$

$6 \cdot V \cdot (-4.053 - 2.513j) = -24.318 - 15.078j \cdot \text{V}$
 $(-4.053)^2 + 2.513^2 = 22.742$
 $= \left(\frac{-24.318}{22.742} - \frac{15.078j}{22.742} \right) \cdot \text{V} = -1.069 - 0.663j \cdot \text{V}$
 magnitude: $\sqrt{1.069^2 + 0.663^2} = 1.258$
 angle: $\text{atan}\left(\frac{-0.663}{-1.069}\right) = 31.81 \cdot \text{deg}$

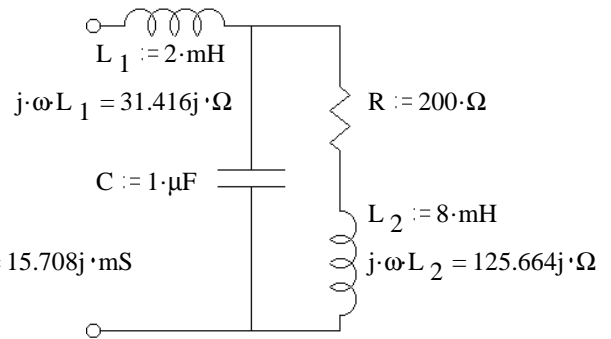
but this is actually in the third quadrant, so modify your calculator's results:

$31.81 \cdot \text{deg} - 180 \cdot \text{deg} = -148.19 \cdot \text{deg}$
 $= 1.258 \text{V} / -148.2^\circ$



ECE 3600 Phasor Examples p2

Ex. 2 a) Find Z_{eq} . $f := 2.5 \cdot \text{kHz}$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 15708 \cdot \frac{\text{rad}}{\text{sec}}$



$$Z_{eq} = j \cdot \omega L_1 + \frac{1}{\frac{1}{R + j \cdot \omega L_2} + \frac{1}{j \cdot \omega C}} = j \cdot \omega L_1 + \frac{1}{\frac{1}{R + j \cdot \omega L_2} + j \cdot \omega C}$$

$$j \cdot \omega L_1 = 31.416j \cdot \Omega$$

$$C := 1 \cdot \mu\text{F}$$

$$j \cdot \omega C = 15.708j \cdot \text{mS}$$

$$R := 200 \cdot \Omega$$

$$L_2 := 8 \cdot \text{mH}$$

$$j \cdot \omega L_2 = 125.664j \cdot \Omega$$

$$Z_{eq} := j \cdot \omega L_1 + \frac{1}{\frac{1}{R + j \cdot \omega L_2} + j \cdot \omega C} = 31.416j \cdot \Omega + \frac{1}{\frac{1}{(200 + 125.664j) \cdot \Omega} + 15.708j \cdot \text{mS}}$$

$$= 31.416j \cdot \Omega + \frac{1}{(3.585 - 2.252j + 15.708j) \cdot \text{mS}} = 31.416j \cdot \Omega + (18.487 - 69.391j) \cdot \Omega = 18.487 - 37.975j \cdot \Omega$$

$$|Z_{eq}| = 42.238 \cdot \Omega \quad \arg(Z_{eq}) = -64.043 \cdot \text{deg}$$

b) $V_{in} := 12 \cdot \text{V} \cdot e^{j \cdot 20 \cdot \text{deg}}$ Find I_{L1} , V_C $I_{L1} := \frac{V_{in}}{Z_{eq}} = \frac{12 \cdot \text{V}}{42.24 \cdot \Omega} = 284.1 \cdot \text{mA}$ $20 \cdot \text{deg} - (-64.04) \cdot \text{deg} = 84.04 \cdot \text{deg}$

$$I_{L1} = 284.1 \text{mA} / 84.04^\circ = 284.1 \cdot \text{mA} \cdot e^{j \cdot 84.04 \cdot \text{deg}} \quad I_{L1} = 29.485 + 282.569j \cdot \text{mA}$$

$$V_C := I_{L1} \cdot (18.486 - 69.384j) \cdot \Omega = 284.1 \cdot \text{mA} \cdot \sqrt{18.486^2 + 69.384^2} \cdot \Omega = 20.4 \cdot \text{V} \quad 84.04 \cdot \text{deg} + \text{atan}\left(\frac{-69.384}{18.486}\right) = 8.959 \cdot \text{deg}$$

To find V_C directly:

$$V_C := \frac{\frac{1}{R + j \cdot \omega L_2}}{j \cdot \omega L_1 + \frac{1}{\frac{1}{R + j \cdot \omega L_2} + j \cdot \omega C}} \cdot V_{in} = \frac{1}{j \cdot \omega L_1 \cdot \left(\frac{1}{R + j \cdot \omega L_2} + j \cdot \omega C\right) + 1} \cdot V_{in} \quad V_C = 20.153 + 3.178j \cdot \text{V}$$

You could then use another voltage divider to find V_R or V_{L2} .

c) Find I_{L2} $I_{L2} := \frac{V_C}{R + j \cdot \omega L_2} = \frac{20.4 \cdot \text{V} \cdot e^{j \cdot 8.96 \cdot \text{deg}}}{236.202 \cdot \Omega \cdot e^{j \cdot 32.142 \cdot \text{deg}}} = \frac{20.4 \cdot \text{V}}{236.202 \cdot \Omega} / 8.96 - 32.142^\circ = 86.4 \text{mA} / -23.18$

Or, directly by Current divider: $I_{L2} := \frac{\frac{1}{R + j \cdot \omega L_2}}{j \cdot \omega C + \frac{1}{R + j \cdot \omega L_2}} \cdot I_{L1} = \frac{1}{j \cdot \omega C \cdot (R + j \cdot \omega L_2) + 1} \cdot I_{L1} = 79.404 - 34.001j \cdot \text{mA}$

d) How about I_C ? $I_C := \frac{V_C}{\left(\frac{1}{j \cdot \omega C}\right)} = V_C \cdot j \cdot \omega C = 20.4 \text{V} / 8.96^\circ \cdot 15.708 \text{mS} / 90^\circ = 320 \text{mA} / 98.96^\circ$

Or, directly by Current divider: $I_C := \frac{j \cdot \omega C}{j \cdot \omega C + \frac{1}{R + j \cdot \omega L_2}} \cdot I_{L1}$

This current is greater than the input current. What's going on?

The angle between I_C & I_{L2} is big enough that they somewhat cancel each other out, partial resonance.

Check Kirchoff's Current Law: $I_C + I_{L2} = 29.485 + 282.569j \cdot \text{mA} = I_{L1} = 29.485 + 282.569j \cdot \text{mA}$

ECE 3600 Phasor Examples p3

Ex. 3 a) Find Z_2 .

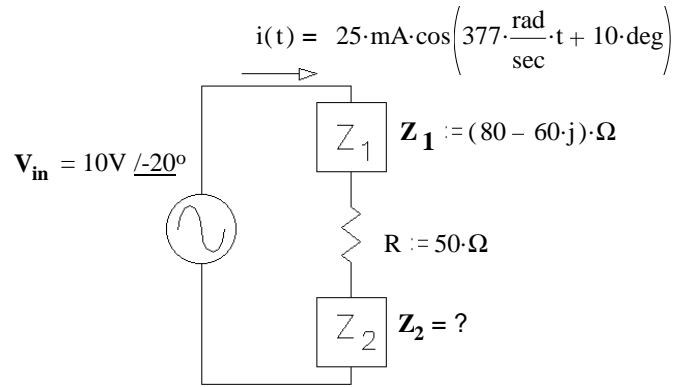
$$I := 25 \cdot \text{mA} \cdot e^{j \cdot 10 \cdot \text{deg}}$$

$$V_{in} := 10 \cdot \text{V} \cdot e^{-j \cdot 20 \cdot \text{deg}}$$

$$Z_T := \frac{V_{in}}{I} = \frac{10 \cdot \text{V}}{25 \cdot \text{mA}} \angle -20 - 10^\circ = 400 \Omega \angle -30^\circ$$

$$Z_T = 346.41 - 200j \cdot \Omega$$

$$Z_2 := Z_T - R - Z_1 = (346.41 - 200j) \cdot \Omega - 50 \cdot \Omega - (80 - 60j) \cdot \Omega = 216.41 - 140j \cdot \Omega$$



- b) Circle 1: i) The source current leads the source voltage <--- answer, because $10^\circ > -20^\circ$.
 ii) The source voltage leads the source current

Ex. 4 a) The impedance Z_1 (above) is made of two components in series. What are they and what are their values?

$$Z_1 = 80 - 60j \cdot \Omega \quad \omega := 377 \cdot \frac{\text{rad}}{\text{sec}}$$

Must have a resistor because there is a real part.

$$R := \text{Re}(Z_1)$$

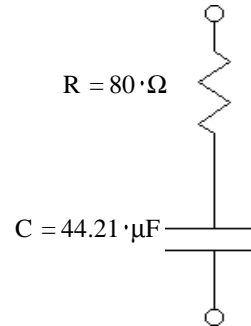
$$R = 80 \cdot \Omega$$

Must have a capacitor because the imaginary part is negative.

$$\text{Im}(Z_1) = -60 \cdot \Omega = \frac{-1}{\omega C}$$

$$C := \frac{-1}{\omega \text{Im}(Z_1)}$$

$$C = 44.21 \cdot \mu\text{F}$$



b) The impedance Z_1 is made of two components in parallel. What are they and what are their values?

$$Z_1 = 80 - 60j \cdot \Omega$$

Must have a resistor because there is a real part.

Must have a capacitor because the imaginary part is negative.

$$Z_1 = \frac{1}{\frac{1}{R} + j \cdot \omega C}$$

$$\frac{1}{Z_1} = \frac{1}{(80 - 60j) \cdot \Omega} \cdot \frac{(80 + 60j)}{(80 + 60j)} = \frac{80 + 60j}{80^2 + 60^2} = \frac{80 + 60j}{10,000} \cdot \frac{1}{\Omega}$$

$80^2 + 60^2 = 10000$

$$\frac{1}{Z_1} = 8 + 6j \cdot \text{mS} = 0.008 + 0.006j \cdot \frac{1}{\Omega} = \frac{1}{R} + j \cdot \omega C$$

$$\frac{1}{R} = 0.008 \cdot \frac{1}{\Omega}$$

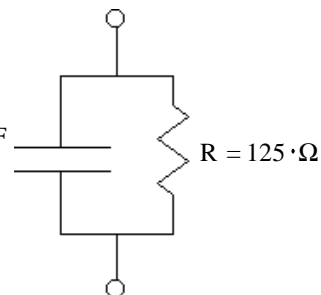
$$R := \frac{1}{0.008 \cdot \frac{1}{\Omega}}$$

$$R = 125 \cdot \Omega$$

$$\omega C = 0.006 \cdot \frac{1}{\Omega}$$

$$C := \frac{0.006 \cdot \frac{1}{\Omega}}{\omega}$$

$$C = 15.915 \cdot \mu\text{F}$$



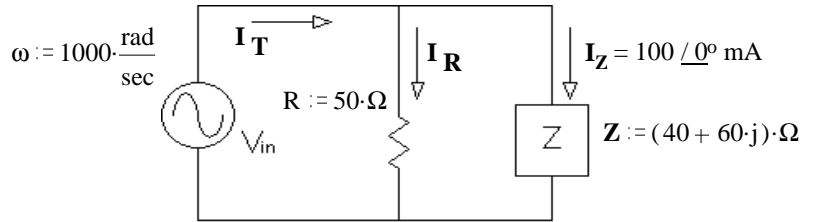
ECE 3600 Phasor Examples p4

Ex. 5 a) Find V_{in} in polar form.

$$I_Z := 100 \cdot \text{mA} \quad Z := (40 + 60j) \cdot \Omega$$

$$V_{in} := I_Z \cdot Z \quad V_{in} = 4 + 6j \cdot \text{V}$$

$$\sqrt{4^2 + 6^2} = 7.211 \quad \text{atan}\left(\frac{6}{4}\right) = 56.31 \cdot \text{deg} \quad V_{in} = 7.21 \text{V} / \underline{-56.3^\circ}$$



b) Find I_T in polar form. $I_R := \frac{V_{in}}{R} = \frac{(4 + 6j) \cdot \text{V}}{50 \cdot \Omega} = \frac{4 \cdot \text{V}}{50 \cdot \Omega} + \frac{6j \cdot \text{V}}{50 \cdot \Omega} = 80 + 120j \cdot \text{mA}$

$$I_T := I_R + I_Z = (80 + 120j) \cdot \text{mA} + 100 \cdot \text{mA} = 180 + 120j \cdot \text{mA}$$

$$|I_T| = 216.3 \cdot \text{mA} \quad \arg(I_T) = 33.69 \cdot \text{deg} \quad I_T = 216.3 \text{mA} / \underline{33.7^\circ}$$

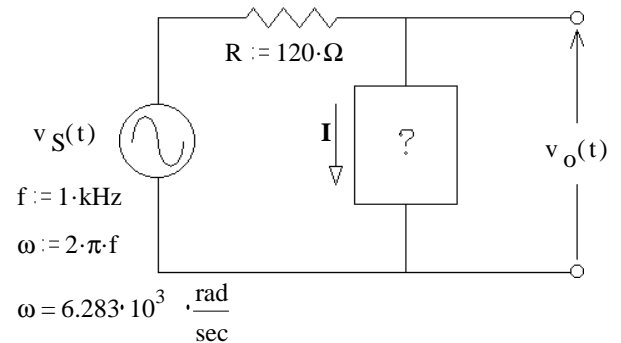
c) Circle 1: i) I_T leads V_{in} ii) V_{in} leads I_T answer ii), $56.3^\circ > 33.7^\circ$

Ex. 6 You need to design a circuit in which the the "output" voltage leads the input voltage ($v_S(t)$) by 30° of phase.

a) What should go in the box: R, L, C?

$$V_o = \frac{Z_{\text{box}}}{R + Z_{\text{box}}} \cdot V_S$$

angle of $\frac{Z_{\text{box}}}{R + Z_{\text{box}}}$ is 30° .



This can only happen if the angle of Z_{box} is positive, so Z_{box} is an inductor

b) Find its value. $V_o = \frac{j \cdot \omega L}{R + j \cdot \omega L} \cdot V_S$ angle: $\frac{j \cdot \omega L}{R + j \cdot \omega L}$ is $90 - \text{atan}\left(\frac{\omega L}{R}\right) = 30^\circ$ so $\text{atan}\left(\frac{\omega L}{R}\right) = 60^\circ$.

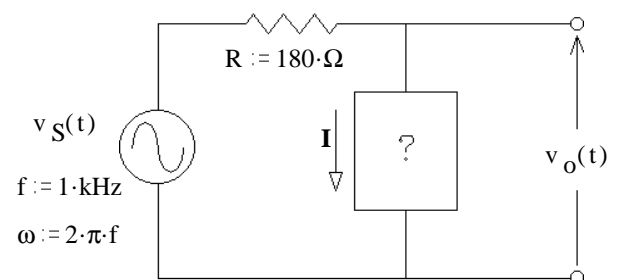
$$\frac{\omega L}{R} = \tan(60 \cdot \text{deg}) = 1.732 \quad L := \frac{R \cdot 1.732}{\omega} \quad L = 33.1 \cdot \text{mH}$$

Ex. 7 You need to design a circuit in which the the "output" voltage lags the input voltage ($v_S(t)$) by 40° of phase.

a) What should go in the box: R, L, C?

$$V_o = \frac{Z_{\text{box}}}{R + Z_{\text{box}}} \cdot V_S$$

angle of $\frac{Z_{\text{box}}}{R + Z_{\text{box}}}$ is -40° .



This can only happen if the angle of Z_{box} is negative, so Z_{box} is a capacitor

b) Find its value. $V_o = \frac{1}{R + \frac{1}{j \cdot \omega C}} \cdot V_S$ angle: $\frac{1}{R + \frac{1}{j \cdot \omega C}}$ is $-90 - \text{atan}\left(\frac{1}{\omega C \cdot R}\right) = -90 - \text{atan}\left(-\frac{1}{\omega C \cdot R}\right)$ so $\text{atan}\left(-\frac{1}{\omega C \cdot R}\right) = -50^\circ$

$$-\frac{1}{\omega C \cdot R} = \tan(-50 \cdot \text{deg}) = -1.192 \quad C := \frac{1}{\omega R \cdot 1.192} \quad C = 0.742 \cdot \mu\text{F}$$

ECE 3600 Phasor Examples p5

Ex. 8 The magnitudes of I_1 and I_2 are 3A and 2A. They lag the supply voltage by 20° and 30° , respectively.

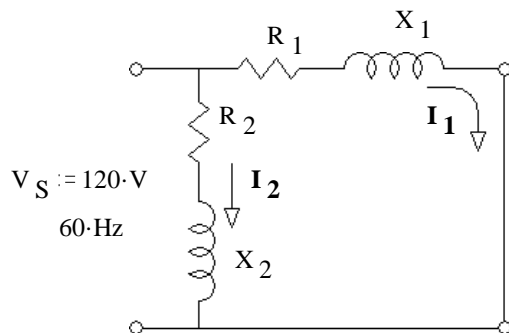
a) Find the values of R_1 , R_2 , X_1 and X_2 .

$$\mathbf{Z}_1 := \frac{120\text{-V}}{3\text{-A} \cdot e^{-j20\text{-deg}}} \quad \mathbf{Z}_1 = 37.588 + 13.681j \cdot \Omega$$

$$R_1 := \text{Re}(\mathbf{Z}_1) \quad R_1 = 37.588 \cdot \Omega$$

$$X_1 := \text{Im}(\mathbf{Z}_1) \quad X_1 = 13.681 \cdot \Omega$$

$$\mathbf{Z}_2 := \frac{120\text{-V}}{2\text{-A} \cdot e^{-j30\text{-deg}}} \quad \mathbf{Z}_2 = 51.962 + 30j \cdot \Omega \quad R_2 := \text{Re}(\mathbf{Z}_2) \quad R_2 = 51.962 \cdot \Omega \quad X_2 := \text{Im}(\mathbf{Z}_2) \quad X_2 = 30 \cdot \Omega$$



b) Add C to the circuit such that I_{1C} leads I_2 by 90° . Find the value of C.

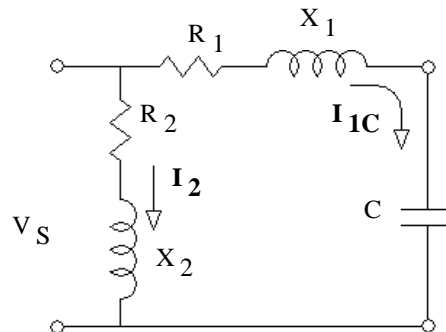
$$\omega := 2 \cdot \pi \cdot 60\text{-Hz}$$

$$\mathbf{I}_{1C} = \frac{120\text{-V}}{R_1 + j \cdot X_1 + j \cdot X_C} \quad \text{needs to be at an angle of } +50^\circ$$

$$\text{So: } \text{atan}\left(\frac{X_1 + X_C}{R_1}\right) = -50\text{-deg}$$

$$\frac{X_1 + X_C}{R_1} = \tan(-50\text{-deg})$$

$$X_C := R_1 \cdot \tan(-50\text{-deg}) - X_1 \quad X_C = -58.476 \cdot \Omega = \frac{-1}{\omega C} \quad C := \frac{-1}{\omega X_C} \quad C = 45.4 \cdot \mu\text{F}$$



c) Change C so that the magnitudes of I_{1C} and I_2 are the same. Find the new C.

$$|\mathbf{I}_{1C}| = \left| \frac{120\text{-V}}{R_1 + j \cdot X_1 + j \cdot X_C} \right| \quad \text{needs to be } 2\text{A} \quad \text{So: } |R_1 + j \cdot X_1 + j \cdot X_C| = 60 \cdot \Omega$$

$$\sqrt{R_1^2 + (X_1 + X_C)^2} = 60 \cdot \Omega$$

$$(X_1 + X_C) = \sqrt{(60 \cdot \Omega)^2 - R_1^2} = 46.767 \cdot \Omega$$

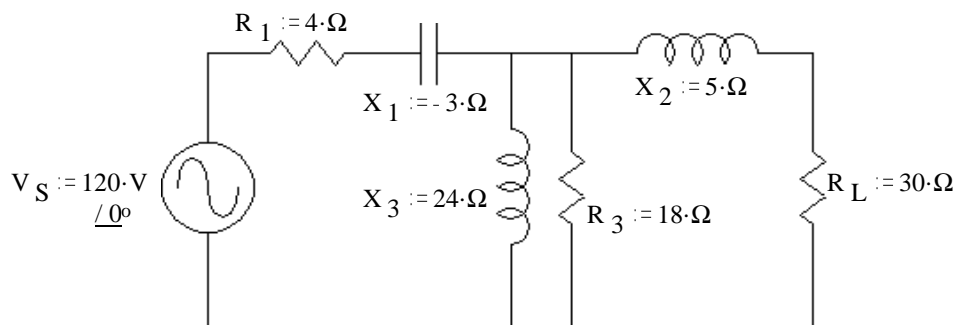
$$X_C := \sqrt{(60 \cdot \Omega)^2 - R_1^2} - X_1 \quad X_C = 33.086 \cdot \Omega = \frac{-1}{\omega C} \quad \text{NOT POSSIBLE}$$

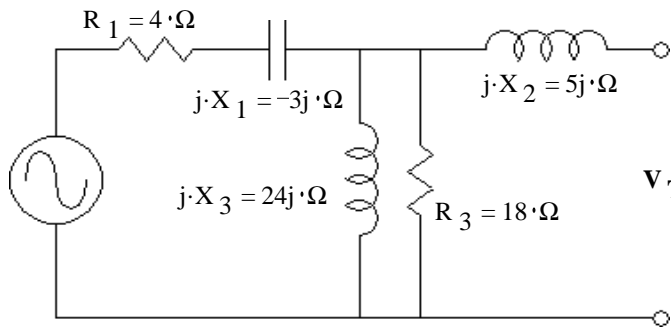
$$\text{So: } (X_1 + X_C) = -46.767 \cdot \Omega$$

$$\text{And: } X_C := -\sqrt{(60 \cdot \Omega)^2 - R_1^2} - X_1 \quad X_C = -60.448 \cdot \Omega = \frac{-1}{\omega C} \quad C := \frac{-1}{\omega X_C} \quad C = 43.9 \cdot \mu\text{F}$$

You'll use a very similar method to find start- and run- capacitors for single-phase induction motors.

Ex. 9 a) In the circuit below R_L is the load resistor. Find and draw the Thevenin equivalent of the rest of the circuit.



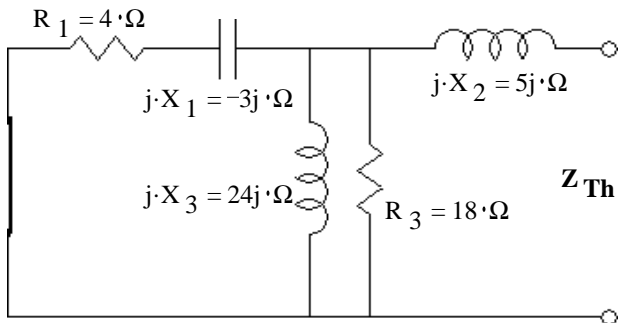


$$\mathbf{V}_{Th} := \mathbf{V}_S \frac{\left(\frac{1}{j \cdot X_3} + \frac{1}{R_3} \right)}{R_1 + j \cdot X_1 + \left(\frac{1}{j \cdot X_3} + \frac{1}{R_3} \right)}$$

$$|\mathbf{V}_{Th}| = 104.645 \cdot \text{V}$$

$$\arg(\mathbf{V}_{Th}) = 16.899 \cdot \text{deg}$$

$$\mathbf{V}_{Th} = 100.126 + 30.418j \cdot \text{V}$$



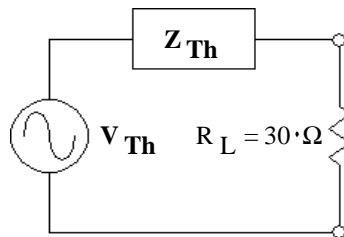
$$\mathbf{Z}_{Th} := \frac{1}{\frac{1}{R_1 + j \cdot X_1} + \frac{1}{j \cdot X_3} + \frac{1}{R_3}} + j \cdot X_2$$

$$|\mathbf{Z}_{Th}| = 5.3962 \cdot \Omega$$

$$\arg(\mathbf{Z}_{Th}) = 40.587 \cdot \text{deg}$$

$$\mathbf{Z}_{Th} = 4.098 + 3.511j \cdot \Omega$$

b) Use the Thevenin equivalent to find the current through the load resistor and the voltage across the load resistor.



$$\mathbf{I}_{RL} := \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + R_L}$$

$$|\mathbf{I}_{RL}| = 3.053 \cdot \text{A}$$

$$\arg(\mathbf{I}_{RL}) = 11.02 \cdot \text{deg}$$

$$\mathbf{I}_{RL} = 2.997 + 0.584j \cdot \text{A}$$

$$\mathbf{V}_{RL} := \mathbf{I}_{RL} \cdot R_L$$

$$|\mathbf{V}_{RL}| = 91.584 \cdot \text{V}$$

$$\arg(\mathbf{V}_{RL}) = 11.02 \cdot \text{deg}$$

$$\mathbf{V}_{RL} = 89.895 + 17.507j \cdot \text{V}$$

c) Find a replacement for R_L in order to maximize the power delivered to R_L .

$$R_L := |\mathbf{Z}_{Th}|$$

$$R_L = 5.396 \cdot \Omega$$

d) Find the new current and voltage for the load resistor.

$$\mathbf{I}_{RL} := \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + R_L}$$

$$\mathbf{I}_{RL} = 10.32 - 0.612j \cdot \text{A}$$

$$|\mathbf{I}_{RL}| = 10.338 \cdot \text{A}$$

$$\arg(\mathbf{I}_{RL}) = -3.395 \cdot \text{deg}$$

$$\mathbf{V}_{RL} := \mathbf{I}_{RL} \cdot R_L$$

$$\mathbf{V}_{RL} = 55.687 - 3.303j \cdot \text{V}$$

$$|\mathbf{V}_{RL}| = 55.785 \cdot \text{V}$$

$$\arg(\mathbf{V}_{RL}) = -3.395 \cdot \text{deg}$$

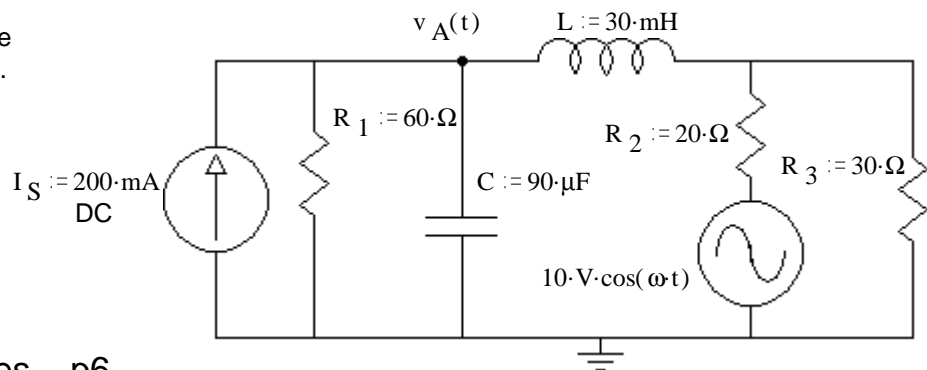
You'll use a Thevenin equivalent circuit to analyze induction motors.

Ex. 10 The circuit shown has two sources. The current source is DC and the voltage source is 60Hz.

Using superposition, find the nodal voltage $v_A(t)$. Be sure to redraw the circuit twice as part of your solution.

$$v_A(t) = ?$$

$$\omega := 2 \cdot \pi \cdot 60 \cdot \text{Hz}$$



Eliminate voltage source

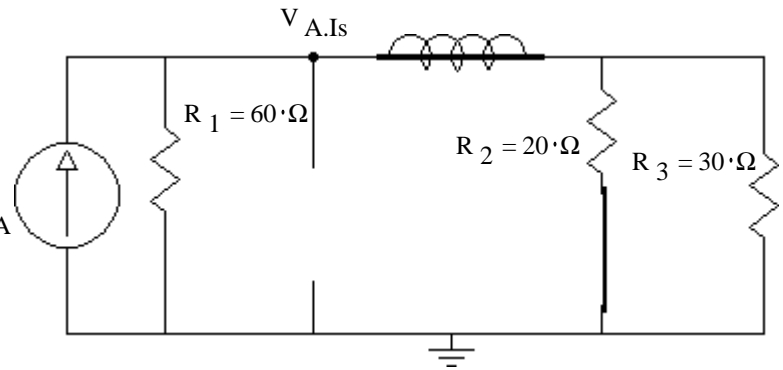
$$R_{eq} := \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$R_{eq} = 10 \cdot \Omega$$

$$V_{A,Is} := I_S \cdot R_{eq}$$

$$V_{A,Is} = 2 \cdot V$$

$$I_S = 200 \cdot \text{mA}$$



Eliminate current source

Let's use nodal analysis

node A

$$I_L = I_1 + I_C$$

$$\frac{V_B - V_A}{j \cdot \omega L} = \frac{V_A}{R_1} + V_A \cdot j \cdot \omega C$$

$$V_B - V_A = \left(\frac{V_A}{R_1} + V_A \cdot j \cdot \omega C \right) \cdot (j \cdot \omega L)$$

$$V_B = \left(\frac{V_A}{R_1} + V_A \cdot j \cdot \omega C \right) \cdot j \cdot \omega L + V_A$$

node B

$$I_2 = I_L + I_3$$

$$\frac{V_S - V_B}{R_2} = \frac{V_B - V_A}{j \cdot \omega L} + \frac{V_B}{R_3}$$

$$\frac{V_S}{R_2} + \frac{V_A}{j \cdot \omega L} = V_B \cdot \left(\frac{1}{j \cdot \omega L} + \frac{1}{R_3} + \frac{1}{R_2} \right) = V_B \cdot (83.333 - 88.419 \cdot j) \cdot \text{mS} = V_B \cdot 121.5 \cdot \text{mS} \cdot e^{-46.696 \cdot \frac{\pi}{180} \cdot j}$$

$$V_B = \frac{V_S}{R_2 \cdot \left(\frac{1}{j \cdot \omega L} + \frac{1}{R_3} + \frac{1}{R_2} \right)} + \frac{V_A}{j \cdot \omega L \cdot \left(\frac{1}{j \cdot \omega L} + \frac{1}{R_3} + \frac{1}{R_2} \right)} = \left(\frac{V_A}{R_1} + V_A \cdot j \cdot \omega C \right) \cdot j \cdot \omega L + V_A$$

Equate to node A equation:

$$\frac{V_S}{R_2 \cdot \left(\frac{1}{j \cdot \omega L} + \frac{1}{R_3} + \frac{1}{R_2} \right)} = \left(\frac{V_A}{R_1} + V_A \cdot j \cdot \omega C \right) \cdot j \cdot \omega L + V_A - \frac{V_A}{1 + j \cdot \omega L \cdot \left(\frac{1}{R_3} + \frac{1}{R_2} \right)}$$

$$1 + j \cdot \omega L \cdot \left(\frac{1}{R_3} + \frac{1}{R_2} \right) = 1 + 0.942j$$

$$= V_A \cdot \left[\left(\frac{1}{R_1} + j \cdot \omega C \right) \cdot j \cdot \omega L + 1 - \frac{1}{1 + j \cdot \omega L \cdot \left(\frac{1}{R_3} + \frac{1}{R_2} \right)} \right]$$

$$\left(\frac{1}{R_1} + j \cdot \omega C \right) \cdot j \cdot \omega L = -0.384 + 0.188j$$

$$V_A := \frac{V_S}{R_2 \cdot \left(\frac{1}{j \cdot \omega L} + \frac{1}{R_3} + \frac{1}{R_2} \right)} \cdot \frac{1}{\left[\left(\frac{1}{R_1} + j \cdot \omega C \right) \cdot j \cdot \omega L + 1 - \frac{1}{1 + j \cdot \omega L \cdot \left(\frac{1}{R_3} + \frac{1}{R_2} \right)} \right]}$$

$$V_A = 4.796 - 3.5j \cdot V$$

$$|V_A| = 5.938 \cdot V \quad \arg(V_A) = -36.12 \cdot \text{deg}$$

$$V_{A,Vs} = 5.938 \cdot V \cdot \cos(377 \cdot t - 36.1 \cdot \text{deg})$$

$$= \frac{V_S}{R_2 \cdot 121.5 \cdot \text{mS} \cdot e^{-j \cdot 46.696 \cdot \text{deg}} \cdot \left[(-0.384 + 0.188 \cdot j) + 1 - \frac{1}{1 + 0.942 \cdot j} \right]}$$

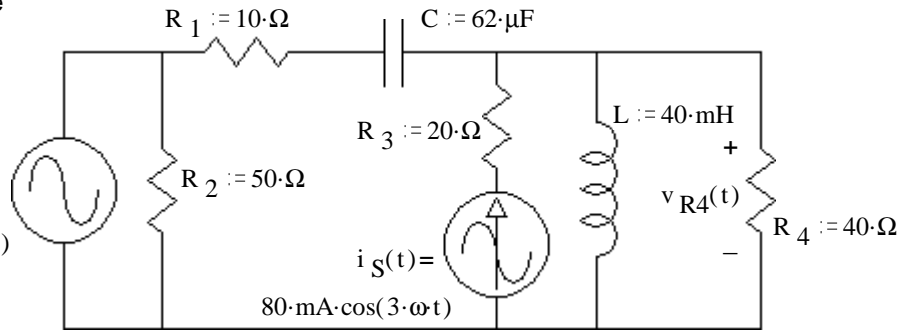
Add the results $v_A(t) = 2 \cdot V + 5.938 \cdot V \cdot \cos(377 \cdot t - 36.1 \cdot \text{deg})$

Ex. 11 The circuit shown has two sources. The frequency of the current source is the third harmonic of the voltage source.

Using superposition, find the voltage across R_4 . Be sure to redraw the circuit twice as part of your solution.

$v_{R4}(t) = ?$

$v_S(t) := 10 \cdot V \cdot \cos(\omega t)$
 $f := 60 \cdot \text{Hz}$
 $\omega := 2 \cdot \pi \cdot f$



Eliminate current source

$Z_C = \frac{1}{j \cdot \omega C} = -42.784j \cdot \Omega$

$V_{R4.Vs} := V_S \cdot \frac{\frac{1}{\left(\frac{1}{j \cdot \omega L} + \frac{1}{R_4}\right)}}{R_1 + \frac{1}{j \cdot \omega C} + \frac{1}{\left(\frac{1}{j \cdot \omega L} + \frac{1}{R_4}\right)}}$

$V_{R4.Vs} = -2.875 + 3.138j \cdot V$
 $|V_{R4.Vs}| = 4.256 \cdot V \quad \arg(V_{R4.Vs}) = 132.5 \cdot \text{deg}$
 $v_{R4.Vs}(t) := 4.256 \cdot V \cdot \cos(\omega t + 132.5 \cdot \text{deg})$

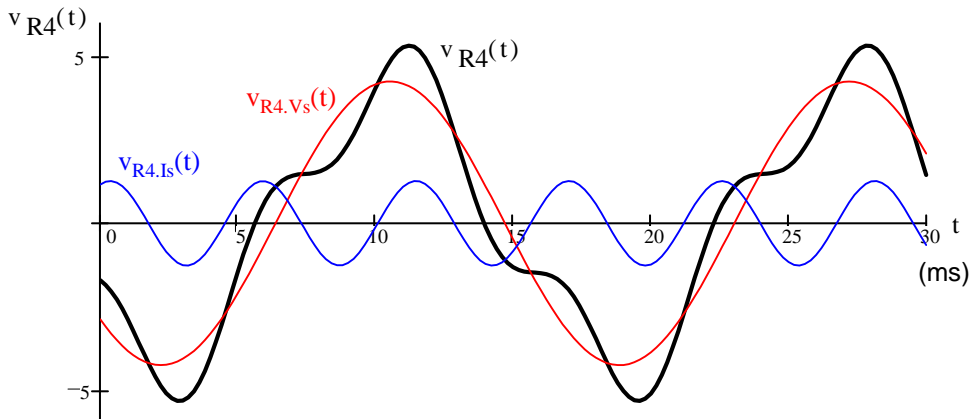
Eliminate voltage source

$Z_C = \frac{1}{j \cdot 3 \cdot \omega C} = -14.261j \cdot \Omega$

$V_{R4.Is} := I_S \cdot \frac{1}{\left(\frac{1}{R_1 + \frac{1}{j \cdot 3 \cdot \omega C}} + \frac{1}{j \cdot 3 \cdot \omega L} + \frac{1}{R_4}\right)}$

$V_{R4.Is} = 1.165 - 0.501j \cdot V$
 $|V_{R4.Is}| = 1.268 \cdot V \quad \arg(V_{R4.Is}) = -23.25 \cdot \text{deg}$
 $v_{R4.Is}(t) := 1.268 \cdot V \cdot \cos(3 \cdot \omega t - 23.25 \cdot \text{deg})$

Add the results $v_{R4}(t) := 4.256 \cdot V \cdot \cos(\omega t + 132.5 \cdot \text{deg}) + 1.268 \cdot V \cdot \cos(3 \cdot \omega t - 23.25 \cdot \text{deg})$ $t := 0, .2.. 30$



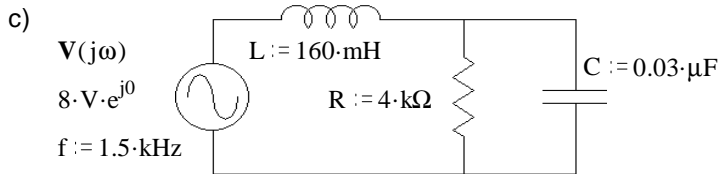
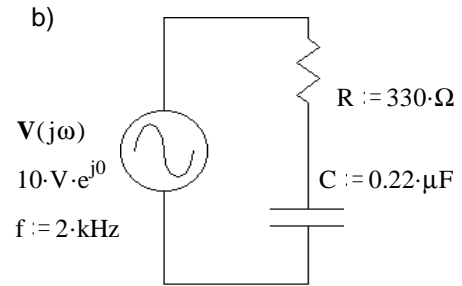
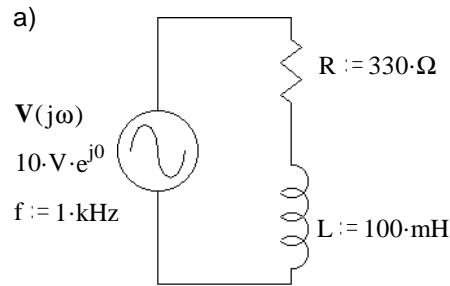
3rd harmonics like this are caused by iron cores used in transformers and motors.

Nodal analysis is used in power flow calculations

A variation of superposition is used to analyze faults on transmission lines.

- Express the impedance of a 5.2mH inductor at 60 Hz in polar form.
- A capacitor impedance has a magnitude of 240Ω at a frequency of 1.8kHz. What is the value of capacitor?

3. Find Z_{eq} in each case.

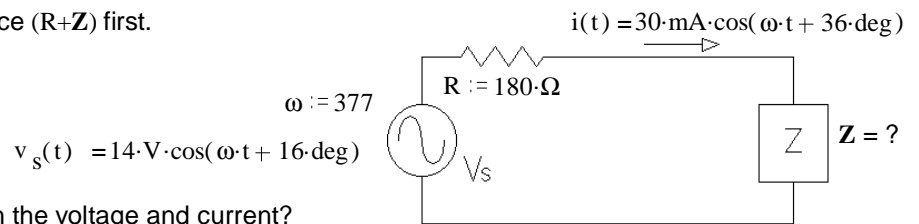


- Find the current $I(j\omega)$ in each case above.
- a) Find Z . Hint: Find the total impedance ($R+Z$) first.

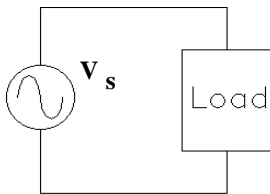
b) Which leads, current or voltage?

c) By how much?

I.E. what is the phase angle between the voltage and current?

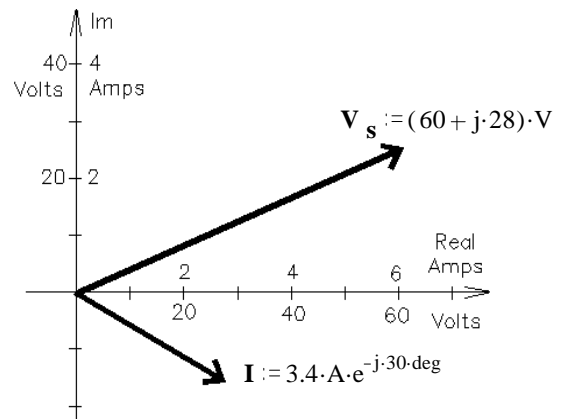


6. The phasor diagram at right shows the voltage and current in the circuit below



Assume the load consists of a resistor in series with a reactive component and the frequency is 60 Hz.

- What is the magnitude of the impedance?
- What is the value of the resistor?
- What is the reactive component (type and value)?

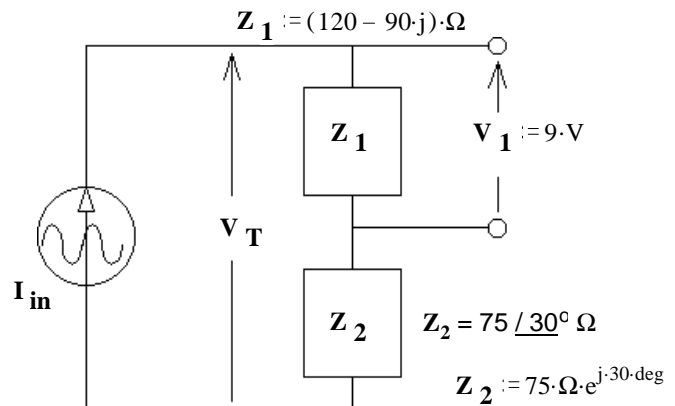


7. a) Find the AC current source, I_{in} in polar form.

b) Find V_T .

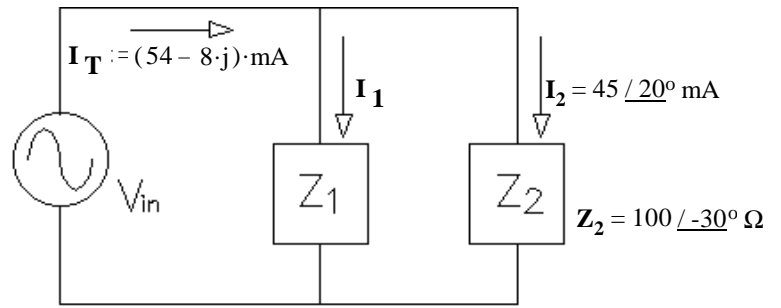
c) Choose one:

- The source current leads the source voltage.
- The source current lags the source voltage.



ECE 3600 homework 2A p2

8. a) Find Z_1 .



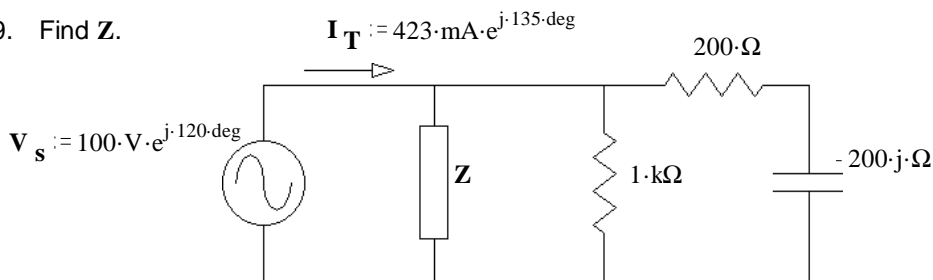
b) To make Z_1 in the simplest way, what part(s) would you need? Just determine the needed part(s) from the list below and state why you made that choice, don't find the values.

- | | | | | |
|-------------------|-------------------|-----------|--------------|----------------|
| resistor | capacitor | inductor | power supply | current source |
| Thevenin resistor | Ideal transformer | voltmeter | ammeter | scope |

c) Choose one: i) I_2 leads the source voltage (V_{in}) ii) I_2 lags the source voltage (V_{in})

d) Choose one: i) I_1 leads I_2 ii) I_1 lags I_2

9. Find Z .



Answers

1. $1.96 \Omega / 90^\circ$

2. $0.368 \mu\text{F}$

3. a) $(330 + 628.3j) \cdot \Omega = 709.7 \Omega / 62.29^\circ$ b) $(330 - 361.7j) \cdot \Omega = 489.6 \Omega / -47.63^\circ$ c) $1.82 \text{k}\Omega / -15.2^\circ$

4. a) $(6.6 - 12.5j) \cdot \text{mA} = 14.1 \text{mA} / -62.29^\circ$ b) $(13.8 + 15.1j) \cdot \text{mA} = 20.4 \text{mA} / 47.63^\circ$ c) $4.4 \text{mA} / 15.2^\circ$

5. a) $259 - 160j$ b) The current leads the voltage c) 20°

6. a) $19.5 \cdot \Omega$ b) $11.2 \cdot \Omega$ c) inductor $42.3 \cdot \text{mH}$

7. a) $60 / 36.87^\circ \text{ mA}$ b) $11.54 / 21^\circ \text{ V}$ c) i)

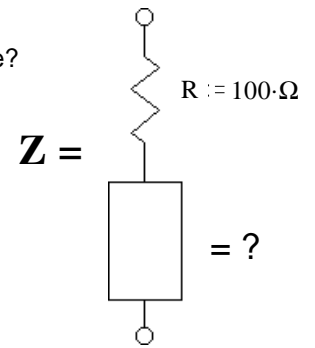
8. a) $172 / 53.4^\circ \Omega$ b) phase angle > 0 , resistor and inductor c) i) d) ii)

9. $657 \Omega / 67.4^\circ$

1. $Z = |Z| \cdot e^{-j \cdot 30 \cdot \text{deg}}$ We don't know its magnitude, but its phase angle is -30° .

Z is made of a 100Ω resistor in series with one other part. What is the part? type and value?

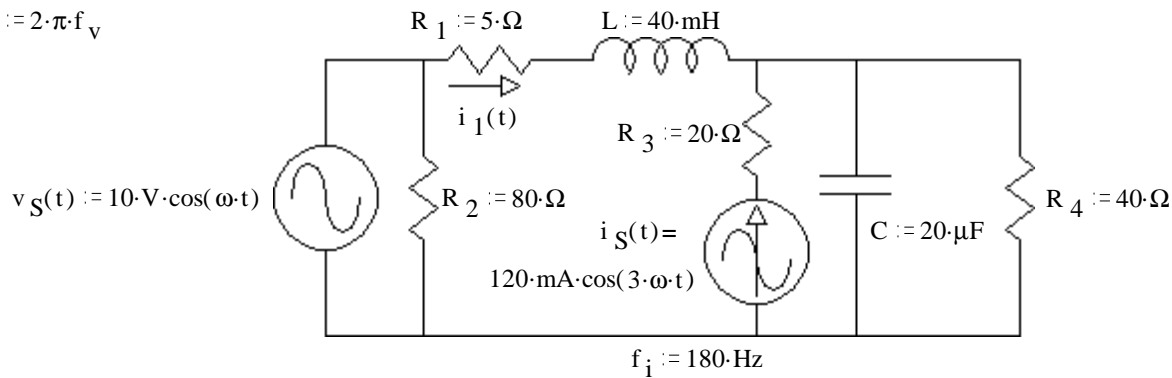
$f := 60 \cdot \text{Hz}$ $\omega := 2 \cdot \pi \cdot 60 \cdot \text{Hz}$



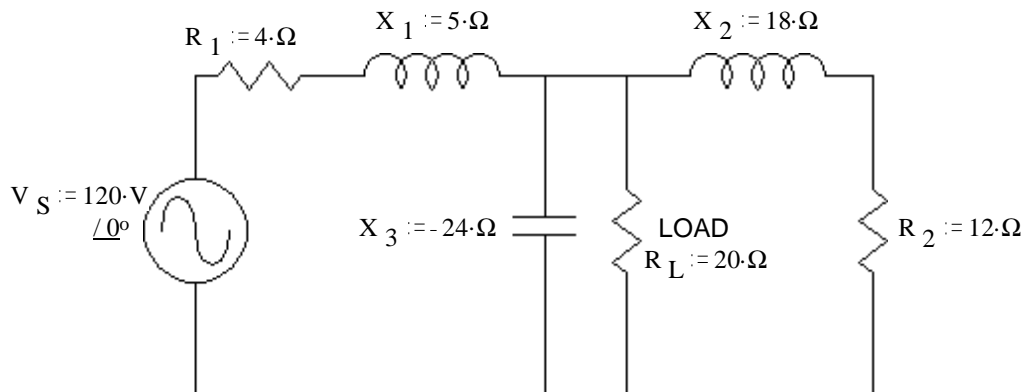
2. The circuit shown has two sources. The frequency of the current source is the third harmonic of the voltage source. Using superposition, find the current $i_1(t)$. Be sure to redraw the circuit twice as part of your solution. $i_1(t) = ?$

$f_v := 60 \cdot \text{Hz}$

$\omega := 2 \cdot \pi \cdot f_v$



3. a) In the circuit below R_L is the load resistor. Find and draw the Thevenin equivalent of the rest of the circuit.



b) Use the Thevenin equivalent to find the current through the load resistor and the voltage across the load resistor.

c) Find a replacement for R_L in order to maximize the power delivered to R_L .

d) Find the new current and voltage for the load resistor.

Answers

1. $45.9 \cdot \mu\text{F}$

2. $i_1(t) := 239 \cdot \text{mA} \cdot \cos(\omega \cdot t - 5.5 \cdot \text{deg}) + 96.1 \cdot \text{mA} \cdot \cos(3 \cdot \omega \cdot t + 94.7 \cdot \text{deg})$

