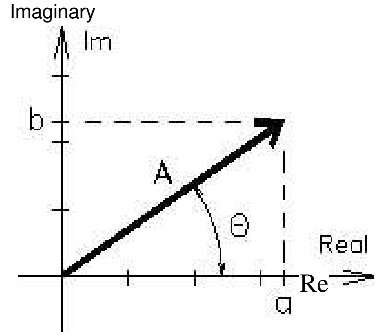
Complex Numbers

ECE 3600



$j = \sqrt{-1}$ the imaginary number

Rectangular Form $\mathbf{A} = a + b \cdot j$

$Re(\mathbf{A}) = a \qquad Im(\mathbf{A}) = b$

Polar Form $\mathbf{A} = A \cdot e^{j \cdot \theta}$

$Re(\mathbf{A}) = A \cdot \cos(\theta) \qquad Im(\mathbf{A}) = A \cdot \sin(\theta)$

Conversions $A = |\mathbf{A}| = \sqrt{a^2 + b^2} \qquad \theta = \arg(\mathbf{A}) = \operatorname{atan}\left(\frac{b}{a}\right)$

$a = A \cdot \cos(\theta) \qquad b = A \cdot \sin(\theta)$

$\mathbf{A} = A \cdot e^{j \cdot \theta} = A \cdot \cos(\theta) + A \cdot \sin(\theta) \cdot j \qquad \mathbf{A} = a + b \cdot j = \left(\sqrt{a^2 + b^2}\right) \cdot e^{j \cdot \operatorname{atan}\left(\frac{b}{a}\right)}$

Special Cases $j := \sqrt{-1} = e^{j \cdot 90 \cdot deg} \qquad \frac{1}{j} = -j = e^{-j \cdot 90 \cdot deg} \qquad e^{j \cdot 0 \cdot deg} = 1 \qquad e^{-j \cdot 180 \cdot deg} = e^{-j \cdot 180 \cdot deg} = -1$

$j \cdot e^{j \cdot \theta} = e^{j \cdot (\theta + 90 \cdot deg)}$

Define a 2nd number: rect: $\mathbf{D} = c + d \cdot j$ polar: $\mathbf{D} = D \cdot e^{j \cdot \phi}$

Equality $\mathbf{A} = \mathbf{D}$ if and only if $a = c$ and $b = d$ OR $A = D$ and $\theta = \phi$

Addition and Subtraction

$\mathbf{A} + \mathbf{D} = (a + b \cdot j) + (c + d \cdot j) = (a + c) + (b + d) \cdot j$

$\mathbf{A} - \mathbf{D} = (a + b \cdot j) - (c + d \cdot j) = (a - c) + (b - d) \cdot j$

Convert polars to rectangular form first

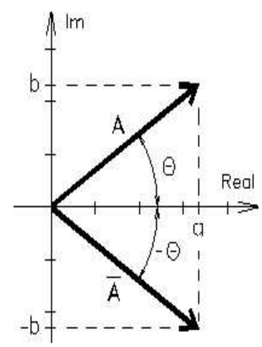
Multiplication and Division $\mathbf{A} \cdot \mathbf{D} = (a + b \cdot j) \cdot (c + d \cdot j) = (a \cdot c - b \cdot d) + (b \cdot c + a \cdot d) \cdot j$

Rectangular: $\frac{\mathbf{A}}{\mathbf{D}} = \frac{a + b \cdot j}{c + d \cdot j} = \frac{a + b \cdot j}{c + d \cdot j} \cdot \frac{c - d \cdot j}{c - d \cdot j} = \frac{a \cdot c + b \cdot d}{c^2 + d^2} + \frac{b \cdot c - a \cdot d}{c^2 + d^2} \cdot j$

Polar: $\mathbf{A} \cdot \mathbf{D} = A \cdot e^{j \cdot \theta} \cdot D \cdot e^{j \cdot \phi} = A \cdot D \cdot e^{j \cdot (\theta + \phi)}$

$\frac{\mathbf{A}}{\mathbf{D}} = \frac{A \cdot e^{j \cdot \theta}}{D \cdot e^{j \cdot \phi}} = \frac{A}{D} \cdot e^{j \cdot (\theta - \phi)}$

Powers $\mathbf{A}^n = A^n \cdot e^{j \cdot n \cdot \theta} = A^n \cdot \cos(n \cdot \theta) + A^n \cdot \sin(n \cdot \theta) \cdot j$ Convert rectangulars first, usually



Conjugates

complex number	Conjugate	
$\mathbf{A} = a + b \cdot j$	$\overline{\mathbf{A}} = a - b \cdot j$	$\overline{\overline{\mathbf{A}}} = \mathbf{A}$
$\mathbf{A} = A \cdot e^{j \cdot \theta}$	$\overline{\mathbf{A}} = A \cdot e^{-j \cdot \theta}$	
$\mathbf{F} = \frac{3 + 5 \cdot j}{(2 - 6 \cdot j) \cdot e^{j \cdot 40 \cdot deg}}$	$\overline{\mathbf{F}} = \frac{3 - 5 \cdot j}{(2 + 6 \cdot j) \cdot e^{-j \cdot 40 \cdot deg}}$	

Euler's equation $e^{j \cdot \alpha} = \cos(\alpha) + j \cdot \sin(\alpha)$ OR: $\cos(\alpha) = \frac{e^{j \cdot \alpha} + e^{-j \cdot \alpha}}{2} \qquad \sin(\alpha) = \frac{e^{j \cdot \alpha} - e^{-j \cdot \alpha}}{2 \cdot j}$

$e^{j \cdot (\omega \cdot t + \theta)} = \cos(\omega \cdot t + \theta) + j \cdot \sin(\omega \cdot t + \theta)$

$Re\left[e^{j \cdot (\omega \cdot t + \theta)}\right] = \cos(\omega \cdot t + \theta)$

If we freeze this at time t=0, then we can represent $\cos(\omega \cdot t + \theta)$ by $e^{j \cdot \theta}$

Calculus Remember, when we write $e^{j \cdot \theta}$, we really mean $e^{j \cdot (\omega \cdot t + \theta)}$

$\frac{d}{dt}\mathbf{A} = \frac{d}{dt}\left(A \cdot e^{j \cdot \theta}\right) = j \cdot \omega \cdot A \cdot e^{j \cdot \theta} = \omega \cdot A \cdot e^{j \cdot (\theta + 90 \cdot deg)}$

$\int \mathbf{A}\, dt = \int A \cdot e^{j \cdot \theta}\, dt = \frac{1}{j \cdot \omega} \cdot A \cdot e^{j \cdot \theta} = \frac{1}{\omega} \cdot A \cdot e^{j \cdot (\theta - 90 \cdot deg)}$

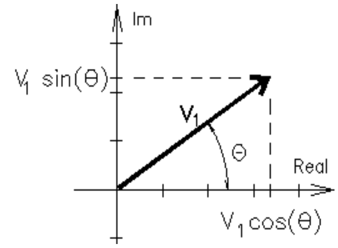
Review of Phasors

ECE 3600

A.Stolp
9/3/08
rev,

For steady-state sinusoidal response ONLY

Phasors



<u>Time domain</u>

$v(t) = V_1 \cdot \cos(377 \cdot t + \theta)$

<u>Phasor, frequency domain</u>

$\mathbf{V_1} = V_1 \cdot e^{j \cdot \theta} = V_1 \underline{/\theta} = V_1 \cdot \cos(\theta) + j \cdot V_1 \cdot \sin(\theta)$

Impedances,

Inductor

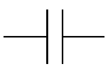
$v_L = L \cdot \frac{d}{dt} i_L = L \cdot \frac{d}{dt} I_p \cdot e^{j \cdot (\omega \cdot t + \theta)} = j \cdot \omega \cdot L \cdot \left[I_p \cdot e^{j \cdot (\omega \cdot t + \theta)} \right]$

$\mathbf{V_L}(\omega) = j \cdot \omega \cdot L \cdot \mathbf{I}(\omega)$

<u>AC impedance</u>

$\mathbf{Z_L} = j \cdot \omega \cdot L$

Capacitor



$i_C = C \cdot \frac{d}{dt} v_C = C \cdot \frac{d}{dt} V_p \cdot e^{j \cdot (\omega \cdot t + \theta)} = j \cdot \omega \cdot C \cdot \left[V_p \cdot e^{j \cdot (\omega \cdot t + \theta)} \right]$

$\mathbf{I_C}(\omega) = j \cdot \omega \cdot C \cdot \mathbf{V}(\omega)$ $\quad$ $\mathbf{V_C}(\omega) = \frac{1}{j \cdot \omega \cdot C} \cdot \mathbf{I}(\omega)$

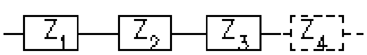
$\mathbf{Z_C} = \frac{1}{j \cdot \omega \cdot C} = \frac{-j}{\omega \cdot C}$

Resistor

$v_R = i_R \cdot R$ $\quad$ $\mathbf{V_R}(\omega) = R \cdot \mathbf{I}(\omega)$ $\quad$ $\mathbf{Z_R} = R$

You can use impedances just like resistances as long as you deal with the complex arithmetic.
ALL the DC circuit analysis techniques will work with AC.

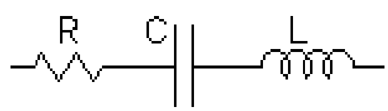
series:



$\mathbf{Z_{eq}} = \mathbf{Z_1} + \mathbf{Z_2} + \mathbf{Z_3} + \ldots$

$f := 60 \cdot Hz$ $\quad$ $\omega := 2 \cdot \pi \cdot f$ $\quad$ $\omega = 377 \cdot \frac{rad}{sec}$

Example:



$R := 20 \cdot \Omega$ $\quad$ $C := 60 \cdot \mu F$ $\quad$ $L := 80 \cdot mH$

$j \cdot \omega \cdot L = 30.159j \cdot \Omega$

$\frac{1}{j \cdot \omega \cdot C} = -44.21j \cdot \Omega$

$\mathbf{Z_{eq}} := R + \frac{1}{j \cdot \omega \cdot C} + j \cdot \omega \cdot L = 20 \cdot \Omega - 44.21 \cdot j \cdot \Omega + 30.16 \cdot j \cdot \Omega = 20 - 14.05j \cdot \Omega$

$\sqrt{(20 \cdot \Omega)^2 + (14.05 \cdot \Omega)^2} = 24.44 \cdot \Omega$ $\quad$ $\text{atan}\left(\frac{-14.05 \cdot \Omega}{20 \cdot \Omega}\right) = -35.09 \cdot deg$

$\mathbf{Z_{eq}} = 24.44\Omega \underline{/-35.1}^{o}$

If: $\quad$ $\mathbf{V} := 120 \cdot V \cdot e^{j \cdot 0 \cdot deg}$ $\quad$ $\mathbf{I} := \frac{\mathbf{V}}{\mathbf{Z_{eq}}} = \frac{120 \cdot V}{24.44 \cdot \Omega} = 4.91 \cdot A$ $\quad$ $\underline{/}\ 0 - -35.1 = 35.1$ deg

$4.91 \cdot \cos(35.1 \cdot deg) = 4.017$ $\quad$ $4.91 \cdot \sin(35.1 \cdot deg) = 2.823$ $\quad$ $\mathbf{I} = 4.017 + 2.822j \cdot A$

slight roundoff error

Voltage divider:

$$\mathbf{V}_{Zn} = \mathbf{V}_{total} \cdot \frac{\mathbf{Z}_n}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \ldots}$$

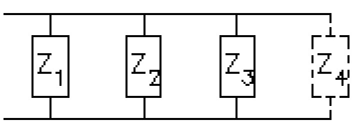
Eg: $\mathbf{V}_C := \mathbf{V} \cdot \dfrac{\frac{1}{j \cdot \omega \cdot C}}{\mathbf{Z}_{eq}} = 120 \cdot V \cdot e^{j \cdot 0 \cdot deg} \cdot \dfrac{44.21 \cdot e^{-j \cdot 90 \cdot deg} \cdot \Omega}{24.44 \cdot e^{-j \cdot 35.1 \cdot deg} \cdot \Omega}$

$120 \cdot V \cdot \dfrac{44.21 \cdot \Omega}{24.44 \cdot \Omega} = 217.07 \cdot V$ $\quad \angle \; 0 + -90 - -35.1 = -54.9 \text{ deg}$

$\mathbf{V}_C = 217.1V \; \underline{/-54.9^o}$ $\quad \mathbf{V}_C = 124.771 - 177.604j \cdot V$

$217.1 \cdot \cos(-54.9 \cdot deg) = 124.8$ $\quad 217.1 \cdot \sin(-54.9 \cdot deg) = -177.6$

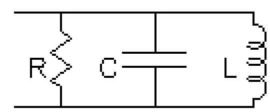
parallel:



$$\mathbf{Z}_{eq} = \frac{1}{\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \ldots}$$

Example:

$f := 60 \cdot Hz$ $\quad \omega := 2 \cdot \pi \cdot f$ $\quad \omega = 377 \cdot \dfrac{rad}{sec}$



$L := 80 \cdot mH$

$j \cdot \omega \cdot L = 30.159j \cdot \Omega$

$R := 20 \cdot \Omega$ $\quad C := 60 \cdot \mu F$ $\quad \dfrac{1}{\omega \cdot L} = 3.316 \cdot 10^{-2} \cdot \dfrac{1}{\Omega}$

$\dfrac{1}{j \cdot \omega \cdot C} = -44.21j \cdot \Omega$ $\quad \omega \cdot C = 2.262 \cdot 10^{-2} \cdot \dfrac{1}{\Omega}$

$$\mathbf{Z}_{eq} := \frac{1}{\frac{1}{R} + \frac{1}{\left(\frac{1}{j \cdot \omega \cdot C}\right)} + \frac{1}{j \cdot \omega \cdot L}} = \frac{1}{\frac{1}{R} + j \cdot \omega \cdot C - \frac{j}{\omega \cdot L}} = \frac{1}{\frac{1}{20 \cdot \Omega} + 2.262 \cdot 10^{-2} \cdot j \cdot \frac{1}{\Omega} - 3.316 \cdot 10^{-2} \cdot j \cdot \frac{1}{\Omega}} = \frac{1}{\left(5 \cdot 10^{-2} - 1.054 \cdot 10^{-2} \cdot j\right) \cdot \frac{1}{\Omega}}$$

$$= \frac{1}{\left(5 \cdot 10^{-2} - 1.054 \cdot 10^{-2} \cdot j\right) \cdot \frac{1}{\Omega}} \cdot \frac{5 \cdot 10^{-2} + 1.054 \cdot 10^{-2} \cdot j}{\left(5 \cdot 10^{-2} + 1.054 \cdot 10^{-2} \cdot j\right)} = 19.149 + 4.037j \cdot \Omega$$

OR, If you want a polar result, it's actually easier to change the denominator to polar and then do polar division.

$\sqrt{\left(5 \cdot 10^{-2} \cdot \frac{1}{\Omega}\right)^2 + \left(1.054 \cdot 10^{-2} \cdot \frac{1}{\Omega}\right)^2} = 5.11 \cdot 10^{-2} \cdot \dfrac{1}{\Omega}$ $\quad \operatorname{atan}\left(\dfrac{-1.054 \cdot 10^{-2} \cdot \Omega}{5 \cdot 10^{-2} \cdot \Omega}\right) = -11.9 \cdot deg$

$\dfrac{1}{5.11 \cdot 10^{-2} \cdot \frac{1}{\Omega}} = 19.569 \cdot \Omega$ $\quad \angle \; 0 - -11.9 = 11.9 \text{ deg}$ $\quad \mathbf{Z}_{eq} = 19.57\Omega \; \underline{/11.9^o}$

If: $\mathbf{V} := 120 \cdot V \cdot e^{j \cdot 0 \cdot deg}$ $\quad \mathbf{I} := \dfrac{\mathbf{V}}{\mathbf{Z}_{eq}} = \dfrac{120 \cdot V}{19.57 \cdot \Omega} = 6.132 \cdot A$ $\quad \angle \; 0 - 11.9 = -11.9 \text{ deg}$

$6.132 \cdot \cos(-11.9 \cdot deg) = 6$ $\quad 6.132 \cdot \sin(-11.9 \cdot deg) = -1.264$ $\quad \mathbf{I} = 6 - 1.265j \cdot A$

slight roundoff error

Current divider:

$$\mathbf{I}_{Zn} = \mathbf{I}_{total} \cdot \frac{\frac{1}{\mathbf{Z}_n}}{\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \ldots}$$

Eg: $\mathbf{I}_L := \mathbf{I} \cdot \dfrac{\frac{1}{j \cdot \omega \cdot L}}{\frac{1}{R} + j \cdot \omega \cdot C + \frac{1}{j \cdot \omega \cdot L}} = \mathbf{I} \cdot \dfrac{\left(\frac{1}{j \cdot \omega \cdot L}\right)}{\left(\frac{1}{\mathbf{Z}_{eq}}\right)} = \mathbf{I} \cdot \dfrac{\mathbf{Z}_{eq}}{j \cdot \omega \cdot L}$

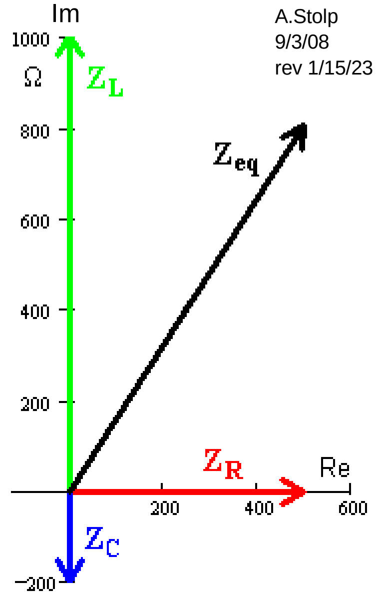
$= 6.132 \cdot A \cdot e^{j \cdot -11.9 \cdot deg} \cdot \dfrac{19.57 \cdot e^{j \cdot 11.9 \cdot deg} \cdot \Omega}{30.159 \cdot e^{j \cdot 90 \cdot deg} \cdot \Omega}$

$\mathbf{I}_L = 6.132 \cdot A \cdot \dfrac{19.57 \cdot \Omega}{30.159 \cdot \Omega} = 3.979 \cdot A$ $\quad \angle \; -11.9 + 11.9 - 90 = -90 \text{ deg}$ $\quad \mathbf{I}_L = -3.979 \cdot 10^3 j \cdot mA$

Duh... $\dfrac{\mathbf{V}}{j \cdot \omega \cdot L} = -3.979 \cdot 10^3 j \cdot mA$

ECE 3600 Phasor Examples

A.Stolp
9/3/08
rev 1/15/23

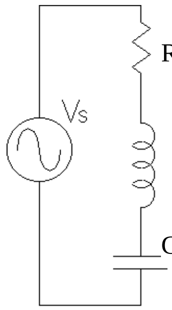


Ex. 1 Find $\mathbf{V_R}$, $\mathbf{V_L}$, and $\mathbf{V_C}$ in polar phasor form. $f := 2\cdot kHz$

$\mathbf{V}(j\omega)$

$6\cdot V\cdot e^{j0}$

$f := 2\cdot kHz$



$R := 500\cdot\Omega \qquad \omega := 2\cdot\pi\cdot f \qquad \omega = 12566\cdot\frac{rad}{sec}$

$L := 80\cdot mH \qquad \mathbf{Z_L} := j\cdot\omega\cdot L \qquad \mathbf{Z_L} = 1.005j\cdot k\Omega$

$C := 0.4\cdot\mu F \qquad \mathbf{Z_C} := \frac{1}{j\cdot\omega\cdot C} \qquad \mathbf{Z_C} = -0.199j\cdot k\Omega$

$\mathbf{Z_{eq}} := R + j\cdot\omega\cdot L + \frac{1}{j\cdot\omega\cdot C} \qquad \mathbf{Z_{eq}} = 500 + 806.366j\ \cdot\Omega$

$\sqrt{500^2 + 806^2} = 948.491 \quad \operatorname{atan}\left(\frac{806}{500}\right) = 58.187\cdot deg \qquad \mathbf{Z_{eq}} = 948.5\Omega\ \underline{/\ 58.2^o}$

find the current: $\mathbf{I} := \frac{6\cdot V\cdot e^{j\cdot 0}}{\mathbf{Z_{eq}}}$ magnitude: $\frac{6\cdot V}{948.5\cdot\Omega} = 6.326\cdot mA$ angle: $0\cdot deg - 58.2\cdot deg = -58.2\cdot deg$

$\mathbf{I} = 6.326mA\ \underline{/\text{-}58.2^o}$

	<u>find the magnitude</u>	<u>find the angle</u>	
$\mathbf{V_R} := \mathbf{I}\cdot R$	$6.326\cdot mA\cdot 500\cdot\Omega = 3.163\cdot V$	$-58.2\cdot deg + 0\cdot deg = -58.2\cdot deg$	$\mathbf{V_R} = 3.163V\ \underline{/\text{-}58.2^o}$
$\mathbf{V_L} := \mathbf{I}\cdot\mathbf{Z_L}$	$6.326\cdot mA\cdot 1005\cdot\Omega = 6.358\cdot V$	$-58.2\cdot deg + 90\cdot deg = 31.8\cdot deg$	$\mathbf{V_L} = 6.358V\ \underline{/\ 31.8^o}$
$\mathbf{V_C} := \mathbf{I}\cdot\mathbf{Z_C}$	$6.326\cdot mA\cdot(-199)\cdot\Omega = -1.259\cdot V$	$-58.2\cdot deg + (90)\cdot deg = 31.8\cdot deg$	$\mathbf{V_C} = \text{-}1.259V\ \underline{/\ 31.8^o}$
OR:	$6.326\cdot mA\cdot(199)\cdot\Omega = 1.259\cdot V$	$-58.2\cdot deg + (-90)\cdot deg = -148.2\cdot deg$	$\mathbf{V_C} = 1.259V\ \underline{/\text{-}148.2^o}$

OR, you can also find these voltages directly, using a voltage divider. I.E. to find $\mathbf{V_C}$ directly:

$$\mathbf{V_C} := \frac{\frac{1}{j\cdot\omega\cdot C}}{R + j\cdot\omega\cdot L + \frac{1}{j\cdot\omega\cdot C}}\cdot 6\cdot V = \frac{1}{R\cdot(j\cdot\omega\cdot C) + j\cdot\omega\cdot L\cdot(j\cdot\omega\cdot C) + 1}\cdot 6\cdot V = \frac{1}{R\cdot(j\cdot\omega\cdot C) - \omega^2\cdot L\cdot C + 1}\cdot 6\cdot V$$

$$= \frac{1}{\left(1 - \omega^2\cdot L\cdot C\right) + j\cdot\omega\cdot R\cdot C}\cdot 6\cdot V \qquad \left(1 - \omega^2\cdot L\cdot C\right) = -4.053 \qquad j\cdot\omega\cdot R\cdot C = 2.513j$$

$$= \frac{6\cdot V}{-4.053 + 2.513\cdot j}\cdot\frac{(-4.053 - 2.513\cdot j\ )}{(-4.053 - 2.513\cdot j\ )} = \frac{6\cdot V\cdot(-4.053 - 2.513\cdot j\ )}{(-4.053)^2 + 2.513^2}$$

$6\cdot V\cdot(-4.053 - 2.513\cdot j\ ) = -24.318 - 15.078j\ \cdot V$

$(-4.053)^2 + 2.513^2 = 22.742$

$$= \left(\frac{-24.318}{22.742} - \frac{15.078\cdot j}{22.742}\right)\cdot V = -1.069 - 0.663j\ \cdot V$$

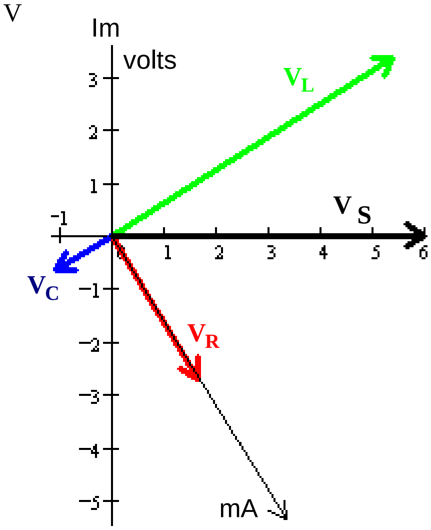
magnitude: $\sqrt{1.069^2 + 0.663^2} = 1.258$

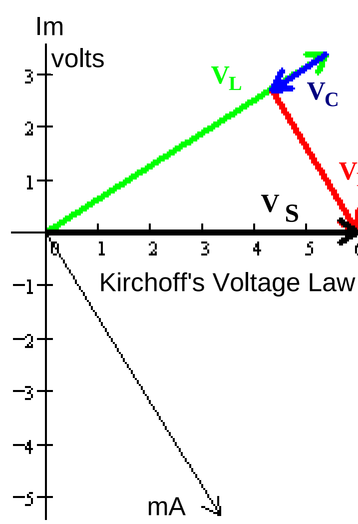
angle: $\operatorname{atan}\left(\frac{-0.663}{-1.069}\right) = 31.81\cdot deg$

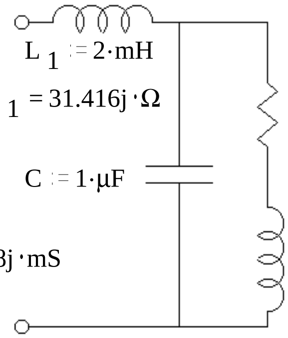
but this is actually in the third quadrant,
so modify your calculator's results:

$31.81\cdot deg - 180\cdot deg = -148.19\cdot deg$

$= 1.258V\ \underline{/\text{-}148.2^o}$







$L_1 := 2\cdot mH$ $\quad j\cdot\omega\cdot L_1 = 31.416j\cdot\Omega$ $\quad R := 200\cdot\Omega$ $\quad C := 1\cdot\mu F$ $\quad L_2 := 8\cdot mH$ $\quad j\cdot\omega\cdot L_2 = 125.664j\cdot\Omega$

Ex. 2 a) Find $\mathbf{Z_{eq}}$. $\quad f := 2.5\cdot kHz \quad \omega := 2\cdot\pi\cdot f \quad \omega = 15708\cdot\frac{rad}{sec}$

$$\mathbf{Z_{eq}} = j\cdot\omega\cdot L_1 + \frac{1}{\frac{1}{R + j\cdot\omega\cdot L_2} + \frac{1}{\frac{1}{j\cdot\omega\cdot C}}} = j\cdot\omega\cdot L_1 + \frac{1}{\frac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C}$$

$j\cdot\omega\cdot C = 15.708j\cdot mS$

$$\mathbf{Z_{eq}} := j\cdot\omega\cdot L_1 + \frac{1}{\frac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C} = 31.416\cdot j\cdot\Omega + \frac{1}{\frac{1}{(200 + 125.664\cdot j)\cdot\Omega} + 15.708\cdot j\cdot mS}$$

$$= 31.416\cdot j\cdot\Omega + \frac{1}{(3.585 - 2.252\cdot j + 15.708\cdot j)\cdot mS} = 31.416\cdot j\cdot\Omega + (18.487 - 69.391\cdot j)\cdot\Omega = 18.487 - 37.975j\cdot\Omega$$

$$\left|\mathbf{Z_{eq}}\right| = 42.238\cdot\Omega \qquad \arg\left(\mathbf{Z_{eq}}\right) = -64.043\cdot deg$$

b) $\mathbf{V_{in}} := 12\cdot V\cdot e^{j\cdot 20\cdot deg}$ $\quad$ Find $\mathbf{I_{L1}}$, $\mathbf{V_C}$ $\quad \mathbf{I_{L1}} := \frac{\mathbf{V_{in}}}{\mathbf{Z_{eq}}}$ $\quad \frac{12\cdot V}{42.24\cdot\Omega} = 284.1\cdot mA$ $\quad 20\cdot deg - (-64.04)\cdot deg = 84.04\cdot deg$

$\mathbf{I_{L1}} = 284.1mA\ \underline{/\ 84.04^o} = 284.1\cdot mA\cdot e^{j\cdot 84.04\cdot deg}$ $\qquad \mathbf{I_{L1}} = 29.485 + 282.569j\cdot mA$

$\mathbf{V_C} := \mathbf{I_{L1}}\cdot(18.486 - 69.384\cdot j)\cdot\Omega$ $\qquad 284.1\cdot mA\cdot\sqrt{18.486^2 + 69.384^2}\cdot\Omega = 20.4\cdot V$ $\qquad 84.04\cdot deg + \operatorname{atan}\left(\frac{-69.384}{18.486}\right) = 8.959\cdot deg$

$\mathbf{V_C} = 20.4V\ \underline{/\ 8.96^o}$

To find $\mathbf{V_C}$ directly:

$$\mathbf{V_C} := \frac{\frac{1}{\frac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C}}{j\cdot\omega\cdot L_1 + \frac{1}{\frac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C}}\cdot\mathbf{V_{in}} = \frac{1}{j\cdot\omega\cdot L_1\cdot\left(\frac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C\right) + 1}\cdot\mathbf{V_{in}}$$

$\mathbf{V_C} = 20.153 + 3.178j\cdot V$

You could then use another voltage divider to find $\mathbf{V_R}$ or $\mathbf{V_{L2}}$.

c) Find $\mathbf{I_{L2}}$

$$\mathbf{I_{L2}} := \frac{\mathbf{V_C}}{R + j\cdot\omega\cdot L_2} = \frac{20.4\cdot V\cdot e^{j\cdot 8.96\cdot deg}}{236.202\cdot\Omega\cdot e^{j\cdot 32.142\cdot deg}} = \frac{20.4\cdot V}{236.202\cdot\Omega}\ \underline{/\ 8.96 - 32.142^o} = 86.4mA\ \underline{/ -23.18}$$

Or, directly by Current divider:

$$\mathbf{I_{L2}} := \frac{\frac{1}{R + j\cdot\omega\cdot L_2}}{j\cdot\omega\cdot C + \frac{1}{R + j\cdot\omega\cdot L_2}}\cdot\mathbf{I_{L1}} = \frac{1}{j\cdot\omega\cdot C\cdot\left(R + j\cdot\omega\cdot L_2\right) + 1}\cdot\mathbf{I_{L1}} = 79.404 - 34.001j\cdot mA$$

d) How about $\mathbf{I_C}$?

$$\mathbf{I_C} := \frac{\mathbf{V_C}}{\left(\frac{1}{j\cdot\omega\cdot C}\right)} = \mathbf{V_C}\cdot j\cdot\omega\cdot C = 20.4V\ \underline{/\ 8.96^o}\ \ 15.708mS\ \underline{/\ 90^o} = 320mA\ \underline{/\ 98.96^o}$$

Or, directly by Current divider: $\mathbf{I_C} := \frac{j\cdot\omega\cdot C}{j\cdot\omega\cdot C + \frac{1}{R + j\cdot\omega\cdot L_2}}\cdot\mathbf{I_{L1}}$

This current is greater than the input current. What's going on?

The angle between $\mathbf{I_C}$ & $\mathbf{I_{L2}}$ is big enough that they somewhat cancel each other out, partial resonance.

Check Kirchoff's Current Law: $\quad \mathbf{I_C} + \mathbf{I_{L2}} = 29.485 + 282.569j\cdot mA = \mathbf{I_{L1}} = 29.485 + 282.569j\cdot mA$

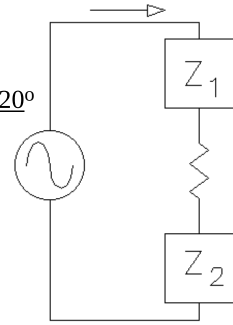
$i(t) = 25\cdot mA\cdot \cos\left(377\cdot\frac{rad}{sec}\cdot t + 10\cdot deg\right)$

Ex. 3 a) Find $\mathbf{Z_2}$.

$\mathbf{I} := 25\cdot mA\cdot e^{j\cdot 10\cdot deg}$

$\mathbf{V_{in}} := 10\cdot V\cdot e^{-j\cdot 20\cdot deg}$

$\mathbf{V_{in}}$ = 10V /-20°



$\mathbf{Z_1} := (80 - 60\cdot j)\cdot\Omega$

$R := 50\cdot\Omega$

$\mathbf{Z_2}$ = ?

$\mathbf{Z_T} := \frac{\mathbf{V_{in}}}{\mathbf{I}} = \frac{10\cdot V}{25\cdot mA}\ \underline{/\ \text{-}20\ \text{-}\ 10^o} = 400\Omega\ \underline{/\text{-}30^o}$

$\mathbf{Z_T} = 346.41 - 200j\ \cdot\Omega$

$\mathbf{Z_2} := \mathbf{Z_T} - R - \mathbf{Z_1} = (346.41 - 200\cdot j\ )\cdot\Omega - 50\cdot\Omega - (80 - 60\cdot j)\cdot\Omega = 216.41 - 140j\ \cdot\Omega$

b) Circle 1: i) The source current leads the source voltage <--- answer, because 10° > -20°.

ii) The source voltage leads the source current

Ex. 4 a) The impedance $\mathbf{Z_1}$ (above) is made of two components in series. What are they and what are their values?

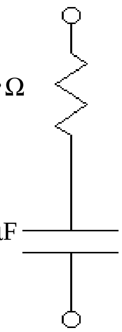
$\mathbf{Z_1} = 80 - 60j\ \cdot\Omega \qquad \omega := 377\cdot\frac{rad}{sec}$

Must have a resistor because there is a real part.

$R := \text{Re}(\mathbf{Z_1}) \qquad R = 80\cdot\Omega$

Must have a capacitor because the imaginary part is negative.

$\text{Im}(\mathbf{Z_1}) = -60\cdot\Omega = \frac{-1}{\omega\cdot C} \qquad C := \frac{-1}{\omega\cdot \text{Im}(\mathbf{Z_1})} \qquad C = 44.21\cdot\mu F$



b) The impedance $\mathbf{Z_1}$ is made of two components in parallel. What are they and what are their values?

$\mathbf{Z_1} = 80 - 60j\ \cdot\Omega$

Must have a resistor because there is a real part.

Must have a capacitor because the imaginary part is negative.

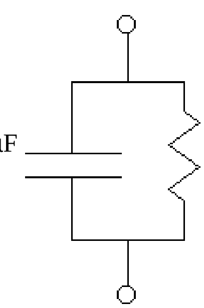
$$\mathbf{Z_1} = \frac{1}{\frac{1}{R} + j\cdot\omega\cdot C} \qquad \frac{1}{\mathbf{Z_1}} = \frac{1}{(80 - 60\cdot j\ )\cdot\Omega}\cdot\left(\frac{80+60\cdot j}{80+60\cdot j}\right) = \frac{80+60\cdot j}{80^2+60^2} = \frac{80+60\cdot j}{10,000}\cdot\frac{1}{\Omega}$$

$$80^2 + 60^2 = 10000$$

$$\frac{1}{\mathbf{Z_1}} = 8 + 6j\ \cdot mS = 0.008 + 0.006\cdot j\ \frac{1}{\Omega} = \frac{1}{R} + j\cdot\omega\cdot C$$

$\frac{1}{R} = 0.008\cdot\frac{1}{\Omega} \qquad R := \frac{1}{0.008\cdot\frac{1}{\Omega}} \qquad R = 125\cdot\Omega$

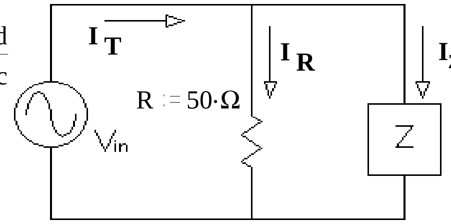
$\omega\cdot C = 0.006\cdot\frac{1}{\Omega} \qquad C := \frac{0.006\cdot\frac{1}{\Omega}}{\omega} \qquad C = 15.915\cdot\mu F$



$R = 125\cdot\Omega$

Ex. 5 a) Find $\mathbf{V_{in}}$ in polar form.

$\omega := 1000 \cdot \frac{rad}{sec}$



$\mathbf{I_Z} = 100 \underline{/\ 0^o}$ mA

$\mathbf{Z} := (40 + 60 \cdot j) \cdot \Omega$

$\mathbf{I_Z} := 100 \cdot mA \qquad \mathbf{Z} := (40 + 60 \cdot j) \cdot \Omega$

$\mathbf{V_{in}} := \mathbf{I_Z} \cdot \mathbf{Z} \qquad \mathbf{V_{in}} = (4 + 6j) \cdot V$

$\sqrt{4^2 + 6^2} = 7.211 \qquad \text{atan}\left(\frac{6}{4}\right) = 56.31 \cdot deg \qquad \mathbf{V_{in}} = 7.21V \underline{/ \text{-}56.3^o}$

b) Find $\mathbf{I_T}$ in polar form.

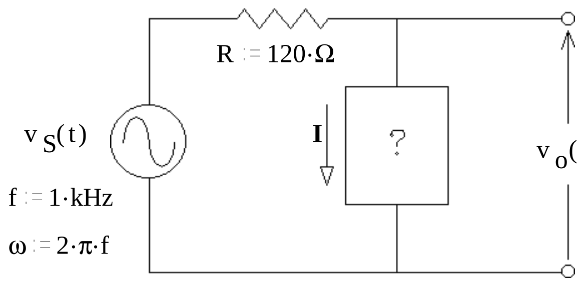
$\mathbf{I_R} := \frac{\mathbf{V_{in}}}{R} = \frac{(4 + 6 \cdot j) \cdot V}{50 \cdot \Omega} = \frac{4 \cdot V}{50 \cdot \Omega} + \frac{6 \cdot j \cdot V}{50 \cdot \Omega} = (80 + 120j) \cdot mA$

$\mathbf{I_T} := \mathbf{I_R} + \mathbf{I_Z} = (80 + 120 \cdot j) \cdot mA + 100 \cdot mA = (180 + 120j) \cdot mA$

$|\mathbf{I_T}| = 216.3 \cdot mA \qquad \arg(\mathbf{I_T}) = 33.69 \cdot deg \qquad \mathbf{I_T} = 216.3mA \underline{/\ 33.7^o}$

c) Circle 1: i) $\mathbf{I_T}$ leads $\mathbf{V_{in}}$ ii) $\mathbf{V_{in}}$ leads $\mathbf{I_T}$ answer ii), $56.3^o > 33.7^o$

Ex. 6 You need to design a circuit in which the the "output" voltage leads the input voltage ($v_S(t)$) by 30º of phase.



$f := 1 \cdot kHz$

$\omega := 2 \cdot \pi \cdot f$

$\omega = 6.283 \cdot 10^3 \cdot \frac{rad}{sec}$

a) What should go in the box: R, L, C?

$\mathbf{V_o} = \frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}} \cdot \mathbf{V_S}$

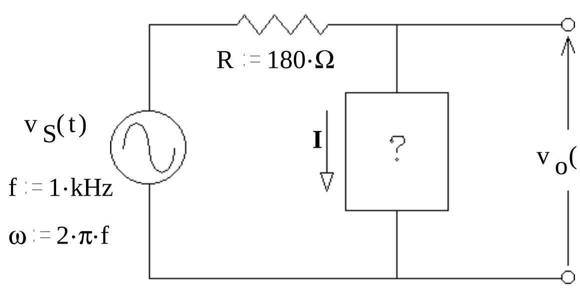
angle of $\frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}}$ is 30º.

This can only happen if the angle of $\mathbf{Z_{box}}$ is positive, so $\mathbf{Z_{box}}$ is a inductor

b) Find its value. $\mathbf{V_o} = \quad \mathbf{V_o} = \frac{j \cdot \omega \cdot L}{R + j \cdot \omega \cdot L} \cdot \mathbf{V_S}$ angle: $\frac{j \cdot \omega \cdot L}{R + j \cdot \omega \cdot L}$ is $90 - \text{atan}\left(\frac{\omega \cdot L}{R}\right) = 30^o$ so $\text{atan}\left(\frac{\omega \cdot L}{R}\right) = 60^o$.

$\frac{\omega \cdot L}{R} = \tan(60 \cdot deg) = 1.732 \qquad L := \frac{R \cdot 1.732}{\omega} \qquad L = 33.1 \cdot mH$

Ex. 7 You need to design a circuit in which the the "output" voltage lags the input voltage ($v_S(t)$) by 40º of phase.



$f := 1 \cdot kHz$

$\omega := 2 \cdot \pi \cdot f$

a) What should go in the box: R, L, C?

$\mathbf{V_o} = \frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}} \cdot \mathbf{V_S}$

angle of $\frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}}$ is -40º.

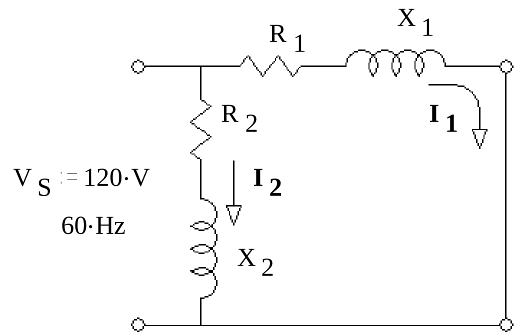
This can only happen if the angle of $\mathbf{Z_{box}}$ is negative, so $\mathbf{Z_{box}}$ is a capacitor

b) Find its value.

$\mathbf{V_o} = \frac{\frac{1}{j \cdot \omega \cdot C}}{R + \frac{1}{j \cdot \omega \cdot C}} \cdot \mathbf{V_S}$ angle: $\frac{\frac{1}{j \cdot \omega \cdot C}}{R + \frac{1}{j \cdot \omega \cdot C}}$ is $-90 - \text{atan}\left(\frac{-\frac{1}{\omega \cdot C}}{R}\right) = -90 - \text{atan}\left(-\frac{1}{\omega \cdot C \cdot R}\right)$ so $\text{atan}\left(-\frac{1}{\omega \cdot C \cdot R}\right) = -50^o$

$-\frac{1}{\omega \cdot C \cdot R} = \tan(-50 \cdot deg) = -1.192 \qquad C := \frac{1}{\omega \cdot R \cdot 1.192} \qquad C = 0.742 \cdot \mu F$

Ex. 8 The magnitudes of $\mathbf{I_1}$ and $\mathbf{I_2}$ are $3A$ and $2A$. They lag the supply voltage by 20° and 30°. respectively.



a) Find the values of R_1, R_2, X_1 and X_2.

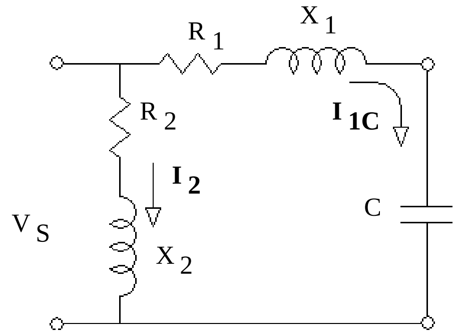
$$\mathbf{Z_1} := \frac{120 \cdot V}{3 \cdot A \cdot e^{-j \cdot 20 \cdot deg}} \qquad \mathbf{Z_1} = 37.588 + 13.681j \ \cdot \Omega$$

$$R_1 := \mathrm{Re}(\mathbf{Z_1}) \qquad R_1 = 37.588 \cdot \Omega$$

$$X_1 := \mathrm{Im}(\mathbf{Z_1}) \qquad X_1 = 13.681 \cdot \Omega$$

$$\mathbf{Z_2} := \frac{120 \cdot V}{2 \cdot A \cdot e^{-j \cdot 30 \cdot deg}} \qquad \mathbf{Z_2} = 51.962 + 30j \ \cdot \Omega \qquad R_2 := \mathrm{Re}(\mathbf{Z_2}) \qquad R_2 = 51.962 \cdot \Omega \qquad X_2 := \mathrm{Im}(\mathbf{Z_2}) \qquad X_2 = 30 \cdot \Omega$$

b) Add C to the circuit such that $\mathbf{I_{1C}}$ leads $\mathbf{I_2}$ by 90°. Find the value of C.
$\omega := 2 \cdot \pi \cdot 60 \cdot Hz$



$$\mathbf{I_{1C}} = \frac{120 \cdot V}{R_1 + j \cdot X_1 + j \cdot X_C}$$ needs to be at an angle of + 50°

So: $$\mathrm{atan}\left(\frac{X_1 + X_C}{R_1}\right) = -50 \cdot deg$$

$$\frac{X_1 + X_C}{R_1} = \tan(-50 \cdot deg)$$

$$X_C := R_1 \cdot \tan(-50 \cdot deg) - X_1 \qquad X_C = -58.476 \cdot \Omega = \frac{-1}{\omega \cdot C} \qquad C := \frac{-1}{\omega \cdot X_C} \qquad C = 45.4 \cdot \mu F$$

c) Change C so that the magnitudes of $\mathbf{I_{1C}}$ and $\mathbf{I_2}$ are the same. Find the new C.

$$\left|\mathbf{I_{1C}}\right| = \left|\frac{120 \cdot V}{R_1 + j \cdot X_1 + j \cdot X_C}\right|$$ needs to be $2A$ So: $$\left|R_1 + j \cdot X_1 + j \cdot X_C\right| = 60 \cdot \Omega$$

$$\sqrt{R_1^{\,2} + (X_1 + X_C)^2} = 60 \cdot \Omega$$

$$(X_1 + X_C) = \sqrt{(60 \cdot \Omega)^2 - R_1^{\,2}} = 46.767 \cdot \Omega$$

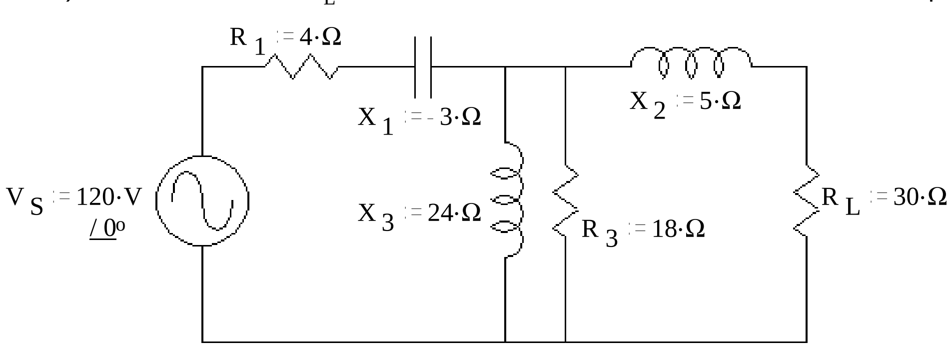
$$X_C := \sqrt{(60 \cdot \Omega)^2 - R_1^{\,2}} - X_1 \qquad X_C = 33.086 \cdot \Omega = \frac{-1}{\omega \cdot C}$$ NOT POSSIBLE

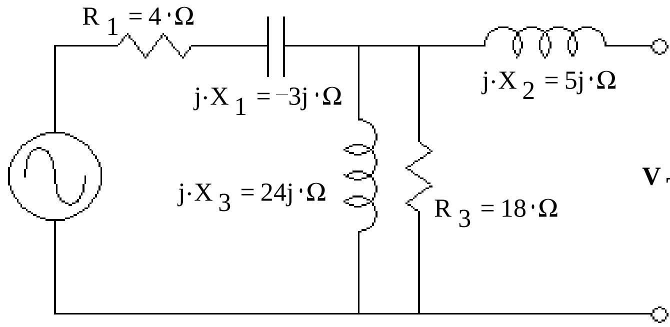
So: $$(X_1 + X_C) = -46.767 \cdot \Omega$$

And: $$X_C := -\sqrt{(60 \cdot \Omega)^2 - R_1^{\,2}} - X_1 \qquad X_C = -60.448 \cdot \Omega = \frac{-1}{\omega \cdot C} \qquad C := \frac{-1}{\omega \cdot X_C} \qquad C = 43.9 \cdot \mu F$$

You'll use a very similar method to find start- and run- capacitors for single-phase induction motors.

Ex. 9 a) In the circuit below R_L is the load resistor. Find and draw the Thevenin equivalent of the rest of the circuit.



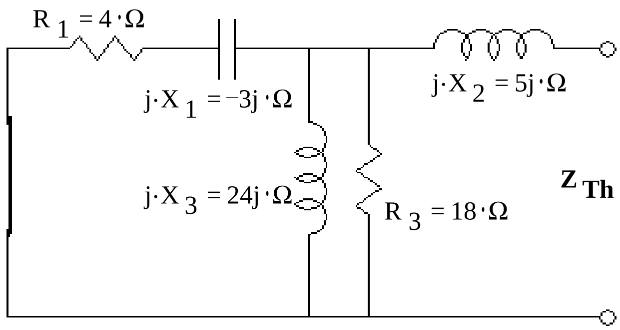


$$\mathbf{V_{Th}} := V_S \cdot \frac{\left(\dfrac{1}{\dfrac{1}{j \cdot X_3} + \dfrac{1}{R_3}}\right)}{R_1 + j \cdot X_1 + \left(\dfrac{1}{\dfrac{1}{j \cdot X_3} + \dfrac{1}{R_3}}\right)}$$

$|\mathbf{V_{Th}}| = 104.645 \cdot V$

$\arg(\mathbf{V_{Th}}) = 16.899 \cdot deg$

$\mathbf{V_{Th}} = 100.126 + 30.418j \cdot V$



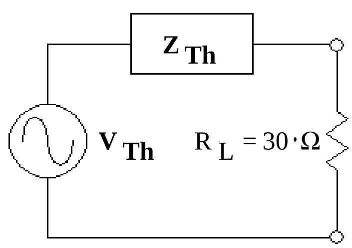
$$\mathbf{Z_{Th}} := \frac{1}{\dfrac{1}{R_1 + j \cdot X_1} + \dfrac{1}{j \cdot X_3} + \dfrac{1}{R_3}} + j \cdot X_2$$

$|\mathbf{Z_{Th}}| = 5.3962 \cdot \Omega$

$\arg(\mathbf{Z_{Th}}) = 40.587 \cdot deg$

$\mathbf{Z_{Th}} = 4.098 + 3.511j \cdot \Omega$

b) Use the Thevenin equivalent to find the current through the load resistor and the voltage across the load resistor.



$$\mathbf{I_{RL}} := \frac{\mathbf{V_{Th}}}{\mathbf{Z_{Th}} + R_L}$$

$\mathbf{I_{RL}} = 2.997 + 0.584j \cdot A$

$|\mathbf{I_{RL}}| = 3.053 \cdot A$ $\quad \arg(\mathbf{I_{RL}}) = 11.02 \cdot deg$

$\mathbf{V_{RL}} := \mathbf{I_{RL}} \cdot R_L$ $\quad \mathbf{V_{RL}} = 89.895 + 17.507j \cdot V$

$|\mathbf{V_{RL}}| = 91.584 \cdot V$ $\quad \arg(\mathbf{V_{RL}}) = 11.02 \cdot deg$

c) Find a replacement for R_L in order to maximize the power delivered to R_L.

$R_L := |\mathbf{Z_{Th}}|$

$R_L = 5.396 \cdot \Omega$

d) Find the new current and voltage for the load resistor.

$$\mathbf{I_{RL}} := \frac{\mathbf{V_{Th}}}{\mathbf{Z_{Th}} + R_L}$$

$\mathbf{I_{RL}} = 10.32 - 0.612j \cdot A$ $\quad |\mathbf{I_{RL}}| = 10.338 \cdot A$ $\quad \arg(\mathbf{I_{RL}}) = -3.395 \cdot deg$

$\mathbf{V_{RL}} := \mathbf{I_{RL}} \cdot R_L$ $\quad \mathbf{V_{RL}} = 55.687 - 3.303j \cdot V$ $\quad |\mathbf{V_{RL}}| = 55.785 \cdot V$ $\quad \arg(\mathbf{V_{RL}}) = -3.395 \cdot deg$

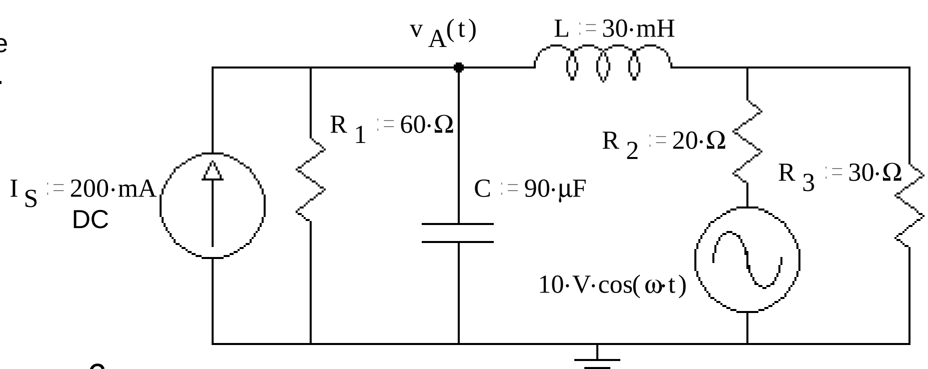
You'll use a Thevenin equivalent circuit to analyze induction motors.

Ex. 10 The circuit shown has two sources. The current source is DC and the voltage source is 60Hz.

Using superposition, find the nodal voltage $v_A(t)$. Be sure to redraw the circuit twice as part of your solution.

$v_A(t) = ?$

$\omega := 2 \cdot \pi \cdot 60 \cdot Hz$

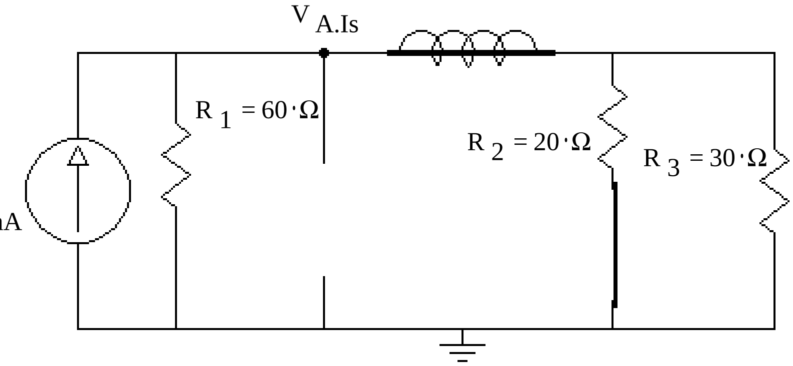


Eliminate voltage source

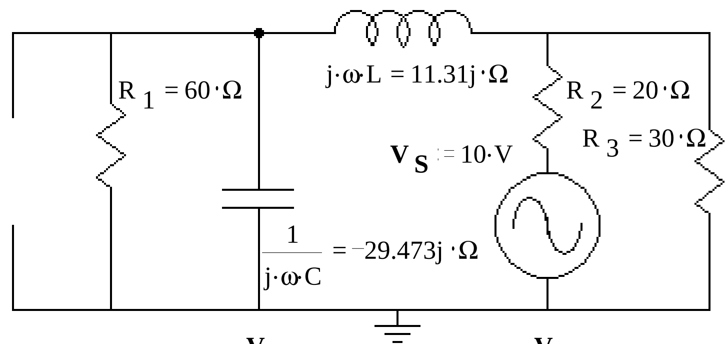
$$R_{eq} := \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} \qquad R_{eq} = 10\cdot\Omega$$

$$V_{A.Is} := I_S \cdot R_{eq}$$

$$V_{A.Is} = 2\cdot V$$



Eliminate current source
Let's use nodal analysis
node A



$$\mathbf{I_L} = \mathbf{I_1} + \mathbf{I_C}$$

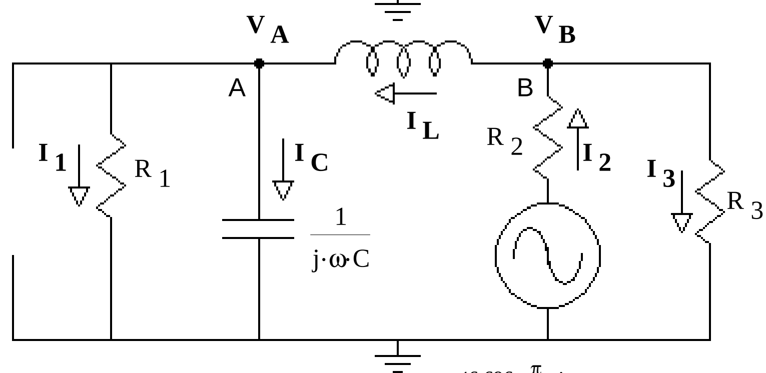
$$\frac{\mathbf{V_B}-\mathbf{V_A}}{j\cdot\omega\cdot L} = \frac{\mathbf{V_A}}{R_1} + \mathbf{V_A}\cdot j\cdot\omega\cdot C$$

$$\mathbf{V_B}-\mathbf{V_A} = \left(\frac{\mathbf{V_A}}{R_1} + \mathbf{V_A}\cdot j\cdot\omega\cdot C\right)\cdot(j\cdot\omega\cdot L)$$

$$j\cdot\omega\cdot L = 11.31j\cdot\Omega$$

$$\mathbf{V_B} = \left(\frac{\mathbf{V_A}}{R_1} + \mathbf{V_A}\cdot j\cdot\omega\cdot C\right)\cdot j\cdot\omega\cdot L + \mathbf{V_A}$$

$$j\cdot\omega\cdot C = 33.929j\cdot mS$$



node B

$$\mathbf{I_2} = \mathbf{I_L} + \mathbf{I_3}$$

$$\frac{\mathbf{V_S}-\mathbf{V_B}}{R_2} = \frac{\mathbf{V_B}-\mathbf{V_A}}{j\cdot\omega\cdot L} + \frac{\mathbf{V_B}}{R_3}$$

$$\frac{\mathbf{V_S}}{R_2} + \frac{\mathbf{V_A}}{j\cdot\omega\cdot L} = \mathbf{V_B}\cdot\left(\frac{1}{j\cdot\omega\cdot L}+\frac{1}{R_3}+\frac{1}{R_2}\right) = \mathbf{V_B}\cdot(83.333 - 88.419\cdot j)\cdot mS = \mathbf{V_B}\cdot 121.5\cdot mS\cdot e^{-46.696\cdot\frac{\pi}{180}\cdot j}$$

$$\mathbf{V_B} = \frac{\mathbf{V_S}}{R_2\cdot\left(\frac{1}{j\cdot\omega\cdot L}+\frac{1}{R_3}+\frac{1}{R_2}\right)} + \frac{\mathbf{V_A}}{j\cdot\omega\cdot L\cdot\left(\frac{1}{j\cdot\omega\cdot L}+\frac{1}{R_3}+\frac{1}{R_2}\right)} = \left(\frac{\mathbf{V_A}}{R_1} + \mathbf{V_A}\cdot j\cdot\omega\cdot C\right)\cdot j\cdot\omega\cdot L + \mathbf{V_A}$$

Equate to node A equation:

$$\frac{\mathbf{V_S}}{R_2\cdot\left(\frac{1}{j\cdot\omega\cdot L}+\frac{1}{R_3}+\frac{1}{R_2}\right)} = \left(\frac{\mathbf{V_A}}{R_1} + \mathbf{V_A}\cdot j\cdot\omega\cdot C\right)\cdot j\cdot\omega\cdot L + \mathbf{V_A} - \frac{\mathbf{V_A}}{1 + j\cdot\omega\cdot L\cdot\left(\frac{1}{R_3}+\frac{1}{R_2}\right)}$$

$$= \mathbf{V_A}\cdot\left[\left(\frac{1}{R_1} + j\cdot\omega\cdot C\right)\cdot j\cdot\omega\cdot L + 1 - \frac{1}{1 + j\cdot\omega\cdot L\cdot\left(\frac{1}{R_3}+\frac{1}{R_2}\right)}\right]$$

$$1 + j\cdot\omega\cdot L\cdot\left(\frac{1}{R_3}+\frac{1}{R_2}\right) = 1 + 0.942j$$

$$\left(\frac{1}{R_1} + j\cdot\omega\cdot C\right)\cdot j\cdot\omega\cdot L = -0.384 + 0.188j$$

$$\mathbf{V_A} := \frac{\mathbf{V_S}}{R_2\cdot\left(\frac{1}{j\cdot\omega\cdot L}+\frac{1}{R_3}+\frac{1}{R_2}\right)}\cdot\frac{1}{\left[\left(\frac{1}{R_1} + j\cdot\omega\cdot C\right)\cdot j\cdot\omega\cdot L + 1 - \frac{1}{1 + j\cdot\omega\cdot L\cdot\left(\frac{1}{R_3}+\frac{1}{R_2}\right)}\right]}$$

$$= \frac{\mathbf{V_S}}{R_2\cdot 121.5\cdot mS\cdot e^{-j\cdot 46.696\cdot deg}\cdot\left[(-0.384 + 0.188\cdot j) + 1 - \frac{1}{1+0.942\cdot j}\right]}$$

$$\mathbf{V_A} = 4.796 - 3.5j\cdot V$$

$$\left|\mathbf{V_A}\right| = 5.938\cdot V \qquad \arg(\mathbf{V_A}) = -36.12\cdot deg$$

$$V_{A.Vs} = 5.938\cdot V\cdot\cos(377\cdot t - 36.1\cdot deg)$$

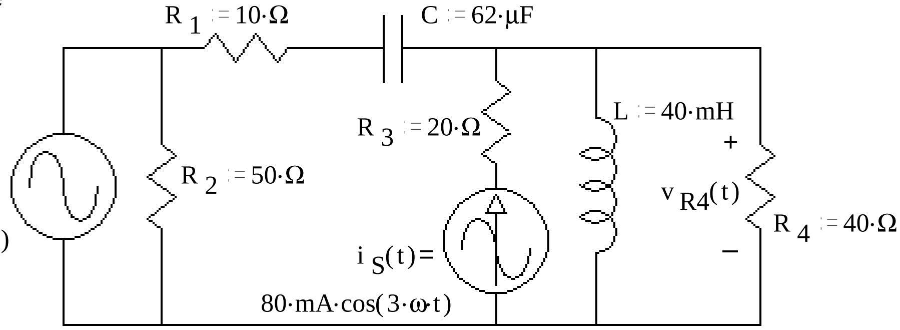
Add the results $v_A(t) = 2\cdot V + 5.938\cdot V\cdot\cos(377\cdot t - 36.1\cdot deg)$

Ex. 11 The circuit shown has two sources. The frequency of the current source is the third harmonic of the voltage source.

Using superposition, find the voltage across R_4. Be sure to redraw the circuit twice as part of your solution.

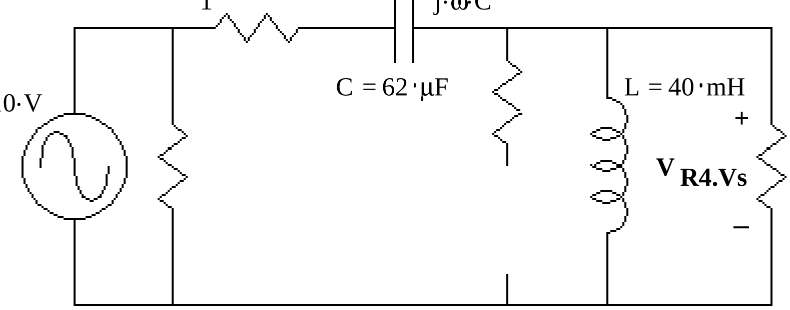
$v_{R4}(t) = ?$

$v_S(t) := 10 \cdot V \cdot \cos(\omega \cdot t)$
$f := 60 \cdot Hz$
$\omega := 2 \cdot \pi \cdot f$



Eliminate current source

$\mathbf{Z_C} = \frac{1}{j \cdot \omega \cdot C} = -42.784j \cdot \Omega$



$\mathbf{Z_L} = j \cdot \omega \cdot L = 15.08j \cdot \Omega$

$$\mathbf{V_{R4.Vs}} := \mathbf{V_S} \cdot \frac{\frac{1}{\left(\frac{1}{j \cdot \omega \cdot L} + \frac{1}{R_4}\right)}}{R_1 + \frac{1}{j \cdot \omega \cdot C} + \frac{1}{\left(\frac{1}{j \cdot \omega \cdot L} + \frac{1}{R_4}\right)}}$$

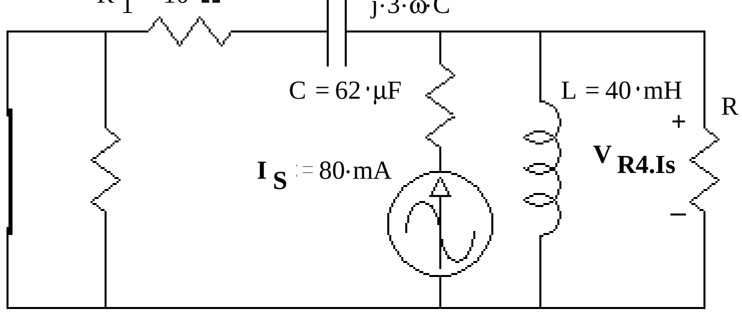
$\mathbf{V_{R4.Vs}} = -2.875 + 3.138j \cdot V$

$\left|\mathbf{V_{R4.Vs}}\right| = 4.256 \cdot V \quad \arg(\mathbf{V_{R4.Vs}}) = 132.5 \cdot deg$

$v_{R4.Vs}(t) := 4.256 \cdot V \cdot \cos(\omega \cdot t + 132.5 \cdot deg)$

Eliminate voltage source

$\mathbf{Z_C} = \frac{1}{j \cdot 3 \cdot \omega \cdot C} = -14.261j \cdot \Omega$



$\mathbf{Z_L} = j \cdot 3 \cdot \omega \cdot L = 45.239j \cdot \Omega$

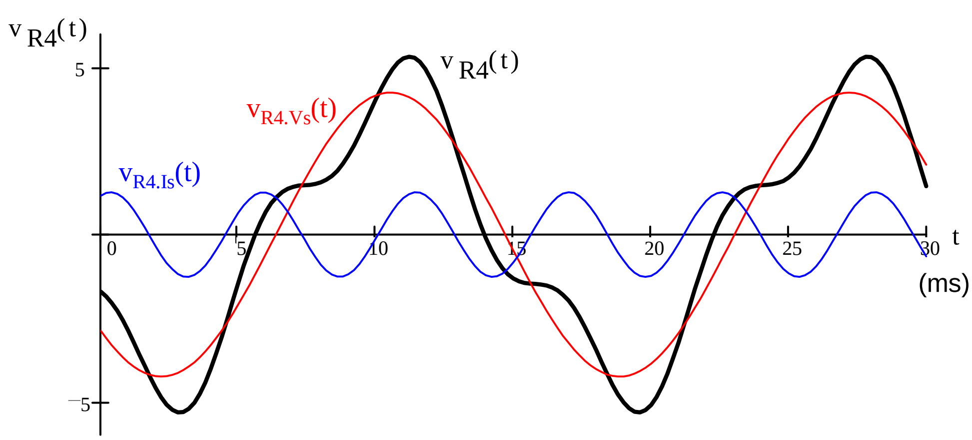
$$\mathbf{V_{R4.Is}} := \mathbf{I_S} \cdot \frac{1}{\left(\frac{1}{R_1 + \frac{1}{j \cdot 3 \cdot \omega \cdot C}} + \frac{1}{j \cdot 3 \cdot \omega \cdot L} + \frac{1}{R_4}\right)}$$

$\mathbf{V_{R4.Is}} = 1.165 - 0.501j \cdot V$

$\left|\mathbf{V_{R4.Is}}\right| = 1.268 \cdot V \quad \arg(\mathbf{V_{R4.Is}}) = -23.25 \cdot deg$

$v_{R4.Is}(t) := 1.268 \cdot V \cdot \cos(3 \cdot \omega \cdot t - 23.25 \cdot deg)$

Add the results $\quad v_{R4}(t) := 4.256 \cdot V \cdot \cos(\omega \cdot t + 132.5 \cdot deg) + 1.268 \cdot V \cdot \cos(3 \cdot \omega \cdot t - 23.25 \cdot deg)$ $\quad t := 0, .2 .. 30$



3rd harmonics like this are caused by iron cores used in transformers and motors.

Nodal anaysis is used in power flow calculations

A variation of superposition is used to analyze faults on transmission lines.