

Name: _____

ECE 3600 homework # 1

Due: Fri, 8/26/22

a

Base your answers on class lecture & discussion, books and/or internet research. Some possible sources:

<http://www.nerc.com/>

http://en.wikipedia.org/wiki/Electricity_generation

<http://www.energy.gov/energysources/electricpower.htm>

http://en.wikipedia.org/wiki/Relative_cost_of_electricity_generated_by_different_sources

1. What is the name of the organization which ensures the reliability of power in North America?

2. Electric Utilities have been forced to break up into two separate companies responsible for:

a.

b.

3. What does deregulation provide for independent power producers (IPPs)?

4. The current bottleneck to overall system capacity.

5. What are the advantages of a highly interconnected system? (List at least 2)

6. Rank the sources of electrical energy in the US (highest to lowest %) 1.

2.

3.

4.

5.

7. List 3 of the "Other" sources. 1.

2.

3.

8. Rank the sources of electrical energy in the US by environmental and social negatives 1.

(worst to best). Assume "Other" is all the 3 you listed above. Consider petroleum just a little worse than natural gas (due to the

2.

danger of spills). Also give (in your opinion) the worst environment or social negative of each. Your answers here may be subjective.

3.

4.

9. Rank the sources of electrical energy in the US cost per kWh.

List Nat gas twice, once for single cycle and once for combined-cycle. Choose one of the "Other" that you listed above. Initial costs are amortized over the life of the generation facility. You will have to make some guesses and may qualify your answers.

1. (cheapest)
- 2.
- 3.
- 4.
- 5.
6. (most expensive).

10. Give the approximate efficiencies of each type of power plant:

- a. Hydroelectric
- b. Rankin-cycle steam turbine plants, regardless of the source of heat.
(coal, oil, gas-steam, nuclear, solar-steam, geothermal)
- c. Single-cycle gas turbine
- d. Combined-cycle gas turbine

11. In nuclear fission reactions, what is particle is crucial to the chain reaction and is used to control the reaction rate?

12. a) Why can't a wind turbine's coefficient of performance (conversion of wind energy to rotational mechanical energy) be 100%?

b) What two things can be controlled to maximize the coefficient of performance?

c) What is the biggest single problem of wind power?

13. a) Do photovoltaic cells produce AC or DC power?

b) What is the biggest single problem of photovoltaic cells?

14. What is cogeneration?

15. Some power sources are used to supply base loads and some are used to supply peak loads. Give some reasons to differentiate the sources in this way.

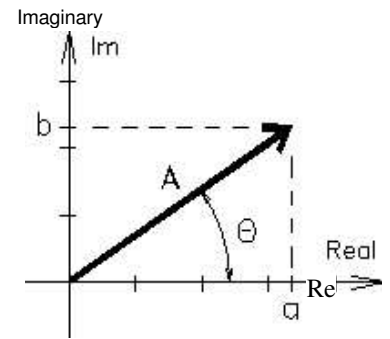
Base loads

Peak loads

Complex Numbers

ECE 3600

$j = \sqrt{-1}$ the imaginary number



Rectangular Form $A = a + b \cdot j$

$\text{Re}(A) = a$ $\text{Im}(A) = b$

Polar Form $A = A \cdot e^{j\theta}$

$\text{Re}(A) = A \cdot \cos(\theta)$ $\text{Im}(A) = A \cdot \sin(\theta)$

Conversions $A = |A| = \sqrt{a^2 + b^2}$ $\theta = \arg(A) = \text{atan}\left(\frac{b}{a}\right)$

$a = A \cdot \cos(\theta)$ $b = A \cdot \sin(\theta)$

$A = A \cdot e^{j\theta} = A \cdot \cos(\theta) + A \cdot \sin(\theta) \cdot j$ $A = a + b \cdot j = \left(\sqrt{a^2 + b^2}\right) \cdot e^{j \cdot \text{atan}\left(\frac{b}{a}\right)}$

Special Cases $j := \sqrt{-1} = e^{j \cdot 90\text{-deg}}$ $\frac{1}{j} = -j = e^{-j \cdot 90\text{-deg}}$ $e^{j \cdot 0\text{-deg}} = 1$ $e^{-j \cdot 180\text{-deg}} = e^{-j \cdot 180\text{-deg}} = -1$
 $j \cdot e^{j\theta} = e^{j(\theta + 90\text{-deg})}$

Define a 2nd number: rect: $D = c + d \cdot j$ polar: $D = D \cdot e^{j\phi}$

Equality $A = D$ if and only if $a = c$ and $b = d$ OR $A = D$ and $\theta = \phi$

Addition and Subtraction $A + D = (a + b \cdot j) + (c + d \cdot j) = (a + c) + (b + d) \cdot j$ Convert polars to rectangular form first
 $A - D = (a + b \cdot j) - (c + d \cdot j) = (a - c) + (b - d) \cdot j$

Multiplication and Division $A \cdot D = (a + b \cdot j) \cdot (c + d \cdot j) = (a \cdot c - b \cdot d) + (b \cdot c + a \cdot d) \cdot j$

Rectangular: $\frac{A}{D} = \frac{a + b \cdot j}{c + d \cdot j} = \frac{a + b \cdot j \cdot c - d \cdot j}{c + d \cdot j \cdot c - d \cdot j} = \frac{a \cdot c + b \cdot d}{c^2 + d^2} + \frac{b \cdot c - a \cdot d}{c^2 + d^2} \cdot j$

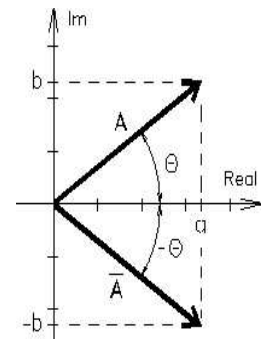
Polar: $A \cdot D = A \cdot e^{j\theta} \cdot D \cdot e^{j\phi} = A \cdot D \cdot e^{j(\theta + \phi)}$

$\frac{A}{D} = \frac{A \cdot e^{j\theta}}{D \cdot e^{j\phi}} = \frac{A}{D} \cdot e^{j(\theta - \phi)}$

Powers $A^n = A^n \cdot e^{j \cdot n \cdot \theta} = A^n \cdot \cos(n \cdot \theta) + A^n \cdot \sin(n \cdot \theta) \cdot j$ Convert rectangulars first, usually

Conjugates complex number Conjugate
 $A = a + b \cdot j$ $\overline{A} = a - b \cdot j$ $\overline{\overline{A}} = A$

$A = A \cdot e^{j\theta}$ $\overline{A} = A \cdot e^{-j\theta}$
 $F = \frac{3 + 5 \cdot j}{(2 - 6 \cdot j) \cdot e^{j \cdot 40\text{-deg}}}$ $\overline{F} = \frac{3 - 5 \cdot j}{(2 + 6 \cdot j) \cdot e^{-j \cdot 40\text{-deg}}}$



Euler's equation $e^{j\alpha} = \cos(\alpha) + j \cdot \sin(\alpha)$ OR: $\cos(\alpha) = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$ $\sin(\alpha) = \frac{e^{j\alpha} - e^{-j\alpha}}{2 \cdot j}$

$e^{j(\omega \cdot t + \theta)} = \cos(\omega \cdot t + \theta) + j \cdot \sin(\omega \cdot t + \theta)$

$\text{Re}\left[e^{j(\omega \cdot t + \theta)}\right] = \cos(\omega \cdot t + \theta)$

If we freeze this at time $t=0$, then we can represent $\cos(\omega \cdot t + \theta)$ by $e^{j\theta}$

Calculus Remember, when we write $e^{j\theta}$, we really mean $e^{j(\omega \cdot t + \theta)}$

$\frac{d}{dt} A = \frac{d}{dt} (A \cdot e^{j\theta}) = j \cdot \omega \cdot A \cdot e^{j\theta} = \omega \cdot A \cdot e^{j(\theta + 90\text{-deg})}$

$\int A dt = \int A \cdot e^{j\theta} dt = \frac{1}{j \cdot \omega} \cdot A \cdot e^{j\theta} = \frac{1}{\omega} \cdot A \cdot e^{j(\theta - 90\text{-deg})}$

Review of Phasors

ECE 3600

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9/3/08
rev.

For steady-state sinusoidal response ONLY

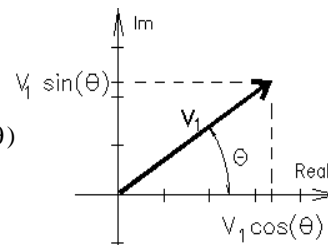
Phasors

Time domain

$$v(t) = \sqrt{2} \cdot V_1 \cdot \cos(377 \cdot t + \theta)$$

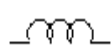
Phasor, frequency domain (RMS)

$$\mathbf{V}_1 = V_1 \cdot e^{j\theta} = V_1 \angle \theta = V_1 \cdot \cos(\theta) + j \cdot V_1 \cdot \sin(\theta)$$



Impedances,

Inductor



$$v_L = L \cdot \frac{d}{dt} i_L = L \cdot \frac{d}{dt} I_p \cdot e^{j(\omega t + \theta)} = j \cdot \omega \cdot L \cdot [I_p \cdot e^{j(\omega t + \theta)}]$$

$$\mathbf{V}_L(\omega) = j \cdot \omega \cdot L \cdot \mathbf{I}(\omega)$$

AC impedance

$$\mathbf{Z}_L = j \cdot \omega \cdot L$$

Capacitor



$$i_C = C \cdot \frac{d}{dt} v_C = C \cdot \frac{d}{dt} V_p \cdot e^{j(\omega t + \theta)} = j \cdot \omega \cdot C \cdot [V_p \cdot e^{j(\omega t + \theta)}]$$

$$\mathbf{I}_C(\omega) = j \cdot \omega \cdot C \cdot \mathbf{V}(\omega)$$

$$\mathbf{V}_C(\omega) = \frac{1}{j \cdot \omega \cdot C} \cdot \mathbf{I}(\omega)$$

$$\mathbf{Z}_C = \frac{1}{j \cdot \omega \cdot C} = \frac{-j}{\omega \cdot C}$$

Resistor



$$v_R = i_R \cdot R$$

$$\mathbf{V}_R(\omega) = R \cdot \mathbf{I}(\omega)$$

$$\mathbf{Z}_R = R$$

You can use impedances just like resistances as long as you deal with the complex arithmetic.
ALL the DC circuit analysis techniques will work with AC.

series:



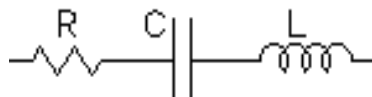
$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \dots$$

$$f := 60 \text{ Hz}$$

$$\omega := 2 \cdot \pi \cdot f$$

$$\omega = 377 \frac{\text{rad}}{\text{sec}}$$

Example:



$$R := 20 \cdot \Omega$$

$$L := 80 \text{ mH}$$

$$C := 60 \cdot \mu\text{F}$$

$$j \cdot \omega \cdot L = 30.159j \cdot \Omega$$

$$\frac{1}{j \cdot \omega \cdot C} = -44.21j \cdot \Omega$$

$$\mathbf{Z}_{eq} := R + \frac{1}{j \cdot \omega \cdot C} + j \cdot \omega \cdot L = 20 \cdot \Omega - 44.21j \cdot \Omega + 30.16j \cdot \Omega = 20 - 14.05j \cdot \Omega$$

$$\sqrt{(20 \cdot \Omega)^2 + (14.05 \cdot \Omega)^2} = 24.44 \cdot \Omega$$

$$\text{atan}\left(\frac{-14.05 \cdot \Omega}{20 \cdot \Omega}\right) = -35.09 \cdot \text{deg}$$

$$\mathbf{Z}_{eq} = 24.44 \Omega \angle -35.1^\circ$$

$$\text{If: } \mathbf{V} := 120 \cdot \text{V} \cdot e^{j \cdot 0 \cdot \text{deg}}$$

$$\mathbf{I} := \frac{\mathbf{V}}{\mathbf{Z}_{eq}} = \frac{120 \cdot \text{V}}{24.44 \cdot \Omega} = 4.91 \cdot \text{A} \quad \angle 0 - -35.1 = 35.1 \text{ deg}$$

$$4.91 \cdot \cos(35.1 \cdot \text{deg}) = 4.017$$

$$4.91 \cdot \sin(35.1 \cdot \text{deg}) = 2.823$$

$$\mathbf{I} = 4.017 + 2.822j \cdot \text{A}$$

slight roundoff error

Voltage divider:

$$V_{Z_n} = V_{total} \cdot \frac{Z_n}{Z_1 + Z_2 + Z_3 + \dots}$$

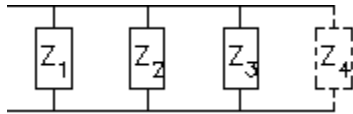
Eg: $V_C := V \cdot \frac{j \cdot \omega \cdot C}{Z_{eq}} = 120 \cdot V \cdot e^{j \cdot 0 \cdot \text{deg}} \cdot \frac{44.21 \cdot e^{-j \cdot 90 \cdot \text{deg}} \cdot \Omega}{24.44 \cdot e^{-j \cdot 35.1 \cdot \text{deg}} \cdot \Omega}$

$$120 \cdot V \cdot \frac{44.21 \cdot \Omega}{24.44 \cdot \Omega} = 217.07 \cdot V \quad \angle 0 + -90 - -35.1 = -54.9 \text{ deg}$$

$$V_C = 217.1V \angle -54.9^\circ \quad V_C = 124.771 - 177.604j \cdot V$$

$$217.1 \cdot \cos(-54.9 \cdot \text{deg}) = 124.8 \quad 217.1 \cdot \sin(-54.9 \cdot \text{deg}) = -177.6$$

parallel:



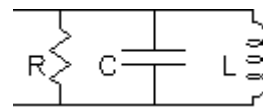
$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \dots}$$

Example:

$f := 60 \cdot \text{Hz}$

$\omega := 2 \cdot \pi \cdot f$

$\omega = 377 \cdot \frac{\text{rad}}{\text{sec}}$



$L := 80 \cdot \text{mH}$

$j \cdot \omega \cdot L = 30.159j \cdot \Omega$

$R := 20 \cdot \Omega$

$C := 60 \cdot \mu\text{F}$

$\frac{1}{\omega \cdot L} = 3.316 \cdot 10^{-2} \cdot \frac{1}{\Omega}$

$\frac{1}{j \cdot \omega \cdot C} = -44.21j \cdot \Omega$

$\omega \cdot C = 2.262 \cdot 10^{-2} \cdot \frac{1}{\Omega}$

$$Z_{eq} := \frac{1}{\frac{1}{R} + \frac{1}{\left(\frac{1}{j \cdot \omega \cdot C}\right)} + \frac{1}{j \cdot \omega \cdot L}} = \frac{1}{\frac{1}{R} + j \cdot \omega \cdot C - \frac{j}{\omega \cdot L}} = \frac{1}{\frac{1}{20 \cdot \Omega} + 2.262 \cdot 10^{-2} \cdot j \cdot \frac{1}{\Omega} - 3.316 \cdot 10^{-2} \cdot j \cdot \frac{1}{\Omega}}$$

$$= \frac{1}{\left(5 \cdot 10^{-2} - 1.054 \cdot 10^{-2} \cdot j\right) \cdot \frac{1}{\Omega} \cdot \left(5 \cdot 10^{-2} + 1.054 \cdot 10^{-2} \cdot j\right)} = 19.149 + 4.037j \cdot \Omega$$

$$\sqrt{\left(5 \cdot 10^{-2} \cdot \frac{1}{\Omega}\right)^2 + \left(1.054 \cdot 10^{-2} \cdot \frac{1}{\Omega}\right)^2} = 5.11 \cdot 10^{-2} \cdot \frac{1}{\Omega} \quad \text{atan}\left(\frac{1.054 \cdot 10^{-2} \cdot \Omega}{5 \cdot 10^{-2} \cdot \Omega}\right) = 11.9 \cdot \text{deg}$$

$$\frac{1}{5.11 \cdot 10^{-2} \cdot \frac{1}{\Omega}} = 19.569 \cdot \Omega$$

$Z_{eq} = 19.57 \Omega / 11.9^\circ$

ff: $V := 120 \cdot V \cdot e^{j \cdot 0 \cdot \text{deg}} \quad I := \frac{V}{Z_{eq}} = \frac{120 \cdot V}{19.57 \cdot \Omega} = 6.132 \cdot A \quad \angle 0 - 11.9 = -11.9 \text{ deg}$

$6.132 \cdot \cos(-11.9 \cdot \text{deg}) = 6$

$6.132 \cdot \sin(-11.9 \cdot \text{deg}) = -1.264$

$I = 6 - 1.265j \cdot A$

slight roundoff error

Current divider:

$$I_{Z_n} = I_{total} \cdot \frac{\frac{1}{Z_n}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \dots}$$

Eg: $I_L := I \cdot \frac{\frac{1}{j \cdot \omega \cdot L}}{\frac{1}{R} + j \cdot \omega \cdot C + \frac{1}{j \cdot \omega \cdot L}} = I \cdot \frac{Z_{eq}}{j \cdot \omega \cdot L}$

$$= 6.132 \cdot A \cdot e^{j \cdot 11.9 \cdot \text{deg}} \cdot \frac{19.57 \cdot e^{-j \cdot 11.9 \cdot \text{deg}} \cdot \Omega}{30.159 \cdot e^{j \cdot 90 \cdot \text{deg}} \cdot \Omega}$$

$$I_L = 6.132 \cdot A \cdot \frac{19.57 \cdot \Omega}{30.159 \cdot \Omega} = 3.979 \cdot A$$

$\angle 11.9 + -11.9 - 90 = -90 \text{ deg}$

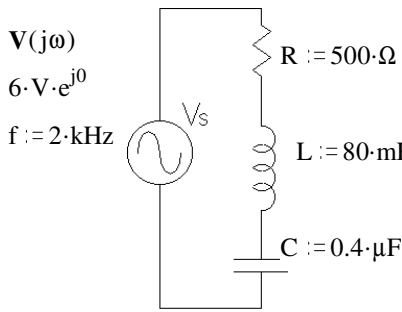
$I_L = -3.979 \cdot 10^3 j \cdot \text{mA}$

Duh... $\frac{V}{j \cdot \omega \cdot L} = -3.979 \cdot 10^3 j \cdot \text{mA}$

ECE 3600 Phasor Examples

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9/3/08
rev

Ex 1. Find V_R , V_L , and V_C in polar phasor form. $f := 2 \cdot \text{kHz}$



$$\omega := 2 \cdot \pi \cdot f \quad \omega = 1.257 \cdot 10^4 \frac{\text{rad}}{\text{sec}}$$

$$Z_L := j \cdot \omega \cdot L \quad Z_L = 1.005j \cdot \text{k}\Omega$$

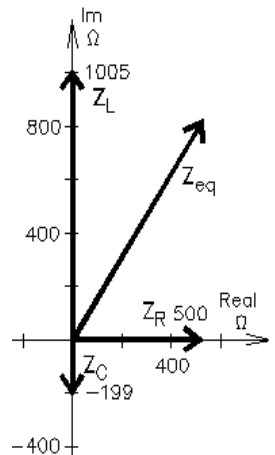
$$Z_C := \frac{1}{j \cdot \omega \cdot C} \quad Z_C = -0.199j \cdot \text{k}\Omega$$

$$Z_{eq} := R + j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C} \quad Z_{eq} = 500 + 806.366j \cdot \Omega$$

$$\sqrt{500^2 + 806^2} = 948.491$$

$$\text{atan}\left(\frac{806}{500}\right) = 58.187 \cdot \text{deg}$$

$$Z_{eq} = 948.5 \Omega / 58.2^\circ$$



find the current: $I := \frac{6 \cdot V \cdot e^{j0}}{Z_{eq}}$

magnitude: $\frac{6 \cdot V}{948.5 \Omega} = 6.326 \cdot \text{mA}$

angle: $0 \cdot \text{deg} - 58.2 \cdot \text{deg} = -58.2 \cdot \text{deg}$

$$I = 6.326 \text{mA} / -58.2^\circ$$

find the magnitude

find the angle

$$V_R := I \cdot R \quad 6.326 \cdot \text{mA} \cdot 500 \cdot \Omega = 3.163 \cdot \text{V}$$

$$-58.2 \cdot \text{deg} + 0 \cdot \text{deg} = -58.2 \cdot \text{deg}$$

$$V_R = 3.163 \text{V} / -58.2^\circ$$

$$V_L := I \cdot Z_L \quad 6.326 \cdot \text{mA} \cdot 1005 \cdot \Omega = 6.358 \cdot \text{V}$$

$$-58.2 \cdot \text{deg} + 90 \cdot \text{deg} = 31.8 \cdot \text{deg}$$

$$V_L = 6.358 \text{V} / 31.8^\circ$$

$$V_C := I \cdot Z_C \quad 6.326 \cdot \text{mA} \cdot (-199) \cdot \Omega = -1.259 \cdot \text{V}$$

$$-58.2 \cdot \text{deg} + (90) \cdot \text{deg} = 31.8 \cdot \text{deg}$$

$$V_C = -1.259 \text{V} / 31.8^\circ$$

OR: $6.326 \cdot \text{mA} \cdot (199) \cdot \Omega = 1.259 \cdot \text{V}$

$$-58.2 \cdot \text{deg} + (-90) \cdot \text{deg} = -148.2 \cdot \text{deg}$$

$$V_C = 1.259 \text{V} / -148.2^\circ$$

OR, you can also find these voltages directly, using a voltage divider. I.E. to find V_C directly:

$$V_C := \frac{\frac{1}{j \cdot \omega \cdot C}}{R + j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C}} \cdot 6 \cdot \text{V} = \frac{1}{R \cdot (j \cdot \omega \cdot C) + j \cdot \omega \cdot L \cdot (j \cdot \omega \cdot C) + 1} \cdot 6 \cdot \text{V} = \frac{1}{R \cdot (j \cdot \omega \cdot C) - \omega^2 \cdot L \cdot C + 1} \cdot 6 \cdot \text{V}$$

$$= \frac{1}{(1 - \omega^2 \cdot L \cdot C) + j \cdot \omega \cdot R \cdot C} \cdot 6 \cdot \text{V} \quad (1 - \omega^2 \cdot L \cdot C) = -4.053 \quad j \cdot \omega \cdot R \cdot C = 2.513j$$

$$= \frac{6 \cdot \text{V}}{-4.053 + 2.513j} \cdot \frac{(-4.053 - 2.513j)}{(-4.053 - 2.513j)} = \frac{6 \cdot \text{V} \cdot (-4.053 - 2.513j)}{(-4.053)^2 + 2.513^2}$$

$$6 \cdot \text{V} \cdot (-4.053 - 2.513j) = -24.318 - 15.078j \cdot \text{V}$$

$$(-4.053)^2 + 2.513^2 = 22.742$$

$$= \left(\frac{-24.318}{22.742} - \frac{15.078j}{22.742} \right) \cdot \text{V} = -1.069 - 0.663j \cdot \text{V}$$

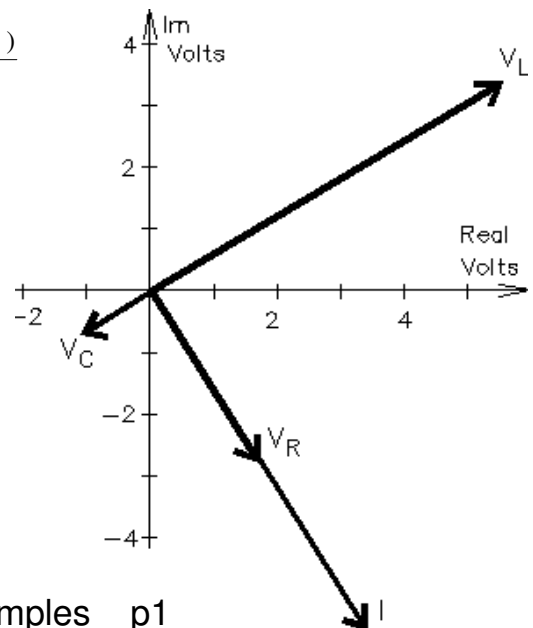
magnitude: $\sqrt{1.069^2 + 0.663^2} = 1.258$

angle: $\text{atan}\left(\frac{-0.663}{-1.069}\right) = 31.81 \cdot \text{deg}$

but this is actually in the third quadrant, so modify your calculator's results:

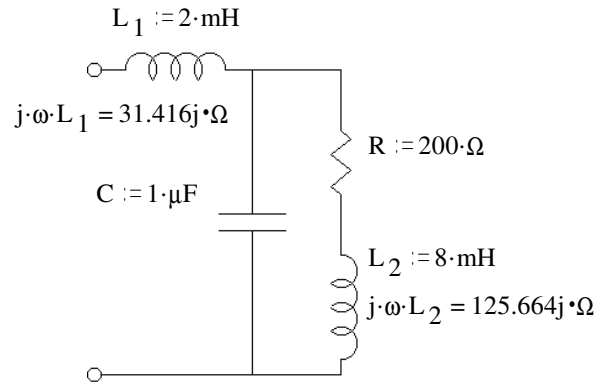
$$31.81 \cdot \text{deg} - 180 \cdot \text{deg} = -148.19 \cdot \text{deg}$$

$$= 1.258 \text{V} / -148.2^\circ$$



ECE 3600 Phasor Examples p2

Ex 2. a) Find Z_{eq} . $f := 2.5 \cdot \text{kHz}$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 1.571 \cdot 10^4 \frac{\text{rad}}{\text{sec}}$



Left branch

$$Z_L := \frac{1}{j \cdot \omega \cdot C} \quad Z_L = -63.662j \cdot \Omega$$

Right branch

$$Z_R := j \cdot \omega \cdot L_2 + R \quad Z_R = 200 + 125.664j \cdot \Omega$$

$$Z_{eq} := j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + \frac{1}{\frac{1}{j \cdot \omega \cdot C}}} = j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C} = j \cdot \omega \cdot L_1 + \frac{R + j \cdot \omega \cdot L_2}{1 + j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2)}$$

$$= j \cdot \omega \cdot L_1 + \frac{R + j \cdot \omega \cdot L_2}{1 - \omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R}$$

Sometimes it's worth simplifying a little before putting in numbers.

$$Z_{eq} = 31.416j \cdot \Omega + \frac{(200 + 125.664j) \cdot \Omega}{-0.974 + 3.142j} \cdot \left(\frac{-0.974 - 3.142j}{-0.974 - 3.142j} \right) = 31.416j \cdot \Omega + \frac{(200 + 125.664j) \cdot (-0.974 - 3.142j)}{0.974^2 + 3.142^2}$$

$$= 31.416j \cdot \Omega + \frac{((200 \cdot (-0.974)) - 125.664 \cdot (-3.142)) + (125.664 \cdot (-0.974) - 200 \cdot 3.142) \cdot j}{0.974^2 + 3.142^2} \cdot \Omega$$

$$= 31.416j \cdot \Omega + \frac{(200.036288 - 750.796736j) \cdot \Omega}{10.82084} = 31.416j \cdot \Omega + 18.486 \cdot \Omega - 69.384j \cdot \Omega = 18.486 - 37.968j \cdot \Omega$$

$$\sqrt{18.49^2 + 37.97^2} = 42.233 \quad \text{atan}\left(\frac{-37.97}{18.49}\right) = -64.036 \cdot \text{deg} \quad Z_{eq} = 42.24 \Omega \angle -64.04^\circ$$

b) $V_{in} := 12 \cdot V \cdot e^{j20 \cdot \text{deg}}$ Find I_{L1} , V_C $I_{L1} := \frac{V_{in}}{Z_{eq}} = \frac{12 \cdot V}{42.24 \cdot \Omega} = 284.091 \cdot \text{mA}$ $20 \cdot \text{deg} - (-64.04) \cdot \text{deg} = 84.04 \cdot \text{deg}$

$$I_{L1} = 284 \text{mA} \angle 84.04^\circ = 284 \cdot \text{mA} \cdot e^{j84.04 \cdot \text{deg}} \quad I_{L1} = 29.485 + 282.569j \cdot \text{mA}$$

$$V_C := I_{L1} \cdot (18.486 - 69.384j) \cdot \Omega \quad 284 \cdot \text{mA} \cdot \sqrt{18.486^2 + 69.384^2} \cdot \Omega = 20.392 \cdot V$$

$$84.04 \cdot \text{deg} + \text{atan}\left(\frac{-69.384}{18.486}\right) = 8.959 \cdot \text{deg} \quad V_C = 20.4V \angle 8.96^\circ$$

To find V_C directly:

$$V_C := \frac{\frac{1}{R + j \cdot \omega \cdot L_2}}{j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + \frac{1}{\frac{1}{j \cdot \omega \cdot C}}}} \cdot V_{in} = \frac{1}{j \cdot \omega \cdot L_1 \cdot \left(\frac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C \right) + 1} \cdot V_{in} = \frac{1}{\frac{j \cdot \omega \cdot L_1}{R + j \cdot \omega \cdot L_2} - \omega^2 \cdot L_1 \cdot C + 1}$$

$$= \frac{1}{\frac{j \cdot \omega \cdot L_1}{R + j \cdot \omega \cdot L_2} - \omega^2 \cdot L_1 \cdot C + 1} \cdot V_{in} = \frac{1}{\frac{j \cdot \omega \cdot L_1 \cdot (R - j \cdot \omega \cdot L_2)}{R^2 + (\omega \cdot L_2)^2} - \omega^2 \cdot L_1 \cdot C + 1} \cdot V_{in}$$

$$= \frac{1}{\left[\frac{\omega^2 \cdot L_1 \cdot L_2}{R^2 + (\omega \cdot L_2)^2} - \omega^2 \cdot L_1 \cdot C + 1 \right] + j \cdot \frac{\omega \cdot L_1 \cdot R}{R^2 + (\omega \cdot L_2)^2}} \cdot V_{in} = \frac{12 \cdot V \cdot e^{j20 \cdot \text{deg}}}{0.58816 \cdot e^{j11.039 \cdot \text{deg}}} = \frac{12 \cdot V}{0.58816} \angle 20 - 11.039^\circ$$

$$= 20.4V \angle 8.96^\circ \quad \text{Same}$$

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Ex 2. Continued Find I_{L2} .

$$Z_R := R + j \cdot \omega \cdot L_2$$

$$Z_R = 200 + 125.664j \cdot \Omega \quad \sqrt{200^2 + 125.664^2} = 236.202 \quad \text{atan}\left(\frac{125.664}{200}\right) = 32.142 \cdot \text{deg}$$

$$I_{L2} = \frac{20.4 \cdot V \cdot e^{j \cdot 8.96 \cdot \text{deg}}}{236.202 \cdot \Omega \cdot e^{j \cdot 32.142 \cdot \text{deg}}} = \frac{20.4 \cdot V}{236.202 \cdot \Omega} \quad / \underline{8.96 - 32.142}^\circ = 86.4 \text{mA} \quad / \underline{-23.18}$$

Or, directly by
Current divider: $I_{L2} := \frac{1}{j \cdot \omega \cdot C + \frac{1}{R + j \cdot \omega \cdot L_2}} \cdot I_{L1} = \frac{1}{j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2) + 1} \cdot I_{L1} = \frac{I_{L1}}{1 - \omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R}$

denominator: $\sqrt{(1 - \omega^2 \cdot C \cdot L_2)^2 + (\omega \cdot C \cdot R)^2} = 3.289 \quad \text{atan}\left(\frac{\omega \cdot C \cdot R}{1 - \omega^2 \cdot C \cdot L_2}\right) + 180 \cdot \text{deg} = 107.224 \cdot \text{deg}$

$$I_{L2} = \frac{284 \cdot \text{mA} \cdot e^{j \cdot 84.04 \cdot \text{deg}}}{3.289 \cdot e^{j \cdot 107.224 \cdot \text{deg}}} = \frac{284 \cdot \text{mA}}{3.289} \quad / \underline{84.04 - 107.224}^\circ = 86.4 \text{mA} \quad / \underline{-23.18}$$

$$I_{L2} = 79.404 - 34.001j \cdot \text{mA}$$

How about I_C ? $I_C := \frac{V_C}{\left(\frac{1}{j \cdot \omega \cdot C}\right)} = V_C \cdot j \cdot \omega \cdot C = 20.4 \text{V} / \underline{8.96}^\circ \cdot 0.015708 / \underline{90}^\circ \cdot \frac{1}{\Omega} = 320 \text{mA} / \underline{98.96}^\circ$

Or, directly by
Current divider: $I_C := \frac{j \cdot \omega \cdot C}{j \cdot \omega \cdot C + \frac{1}{R + j \cdot \omega \cdot L_2}} \cdot I_{L1} = \frac{j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2)}{j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2) + 1} \cdot I_{L1} = \frac{-\omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R}{1 - \omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R} \cdot I_{L1}$

numerator: $\sqrt{(\omega^2 \cdot C \cdot L_2)^2 + (\omega \cdot C \cdot R)^2} = 3.71 \quad \text{atan}\left(\frac{\omega \cdot C \cdot R}{-\omega^2 \cdot C \cdot L_2}\right) + 180 \cdot \text{deg} = 122.142 \cdot \text{deg}$
denominator is the same as above. Second quadrant

$$I_C = \frac{3.71 \cdot e^{j \cdot 122.14 \cdot \text{deg}}}{3.289 \cdot e^{j \cdot 107.224 \cdot \text{deg}}} \cdot 284 \cdot \text{mA} \cdot e^{j \cdot 84.04 \cdot \text{deg}} = \frac{3.71}{3.289} \cdot 284 \cdot \text{mA} \quad / \underline{122.14 - 107.224 + 84.04}^\circ = 320 \text{mA} / \underline{98.96}^\circ$$

This current is greater than the input current. What's going on?

The angle between I_C & I_{L2} is big enough that they somewhat cancel each other out.

Check Kirchoff's Current Law: $I_C + I_{L2} = 29.485 + 282.569j \cdot \text{mA} = I_{L1} = 29.485 + 282.569j \cdot \text{mA}$

Ex 3. a) Find Z_2 .

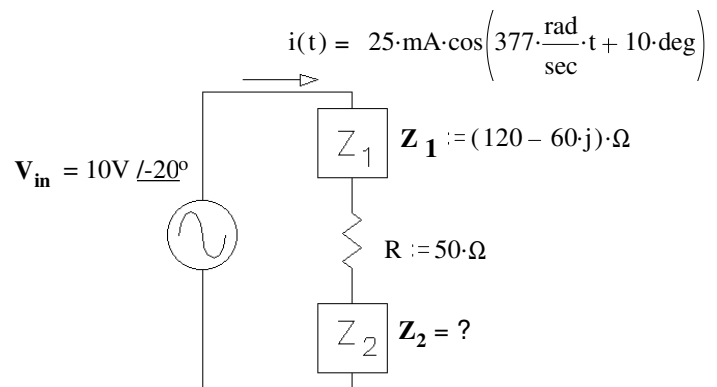
$$I := 25 \cdot \text{mA} \cdot e^{j \cdot 10 \cdot \text{deg}}$$

$$V_{in} := 10 \cdot V \cdot e^{-j \cdot 20 \cdot \text{deg}}$$

$$Z_T := \frac{V_{in}}{I} = \frac{10 \cdot V}{25 \cdot \text{mA}} \quad / \underline{-20 - 10}^\circ = 400 \Omega / \underline{-30}^\circ$$

$$Z_T = 346.41 - 200j \cdot \Omega$$

$$Z_2 := Z_T - R - Z_1 = (346.41 - 200j \cdot \Omega) - 50 \cdot \Omega - (120 - 60j \cdot \Omega) = 176.41 - 140j \cdot \Omega$$



b) Circle 1: i) The source current leads the source voltage

<--- answer, because $10^\circ > -20^\circ$.

ii) The source voltage leads the source current

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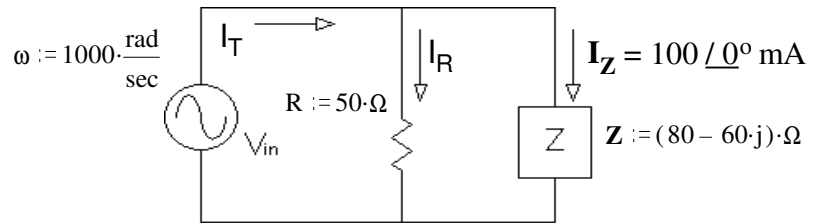
Ex 4. a) Find V_{in} in polar form.

$$I_Z := 100 \cdot \text{mA} \quad Z := (80 - 60j) \cdot \Omega$$

$$V_{in} := I_Z \cdot Z \quad V_{in} = 8 - 6j \cdot \text{V}$$

$$\sqrt{8^2 + 6^2} = 10 \quad \text{atan}\left(\frac{-6}{8}\right) = -36.87 \cdot \text{deg}$$

$$V_{in} = 10\text{V} \angle -36.9^\circ$$



b) Find I_T in polar form. $I_R := \frac{V_{in}}{R} = \frac{10 \cdot \text{V}}{50 \cdot \Omega} \angle -36.9^\circ = \frac{10 \cdot \text{V}}{50 \cdot \Omega} \cdot \cos(-36.9 \cdot \text{deg}) + j \cdot \frac{10 \cdot \text{V}}{50 \cdot \Omega} \cdot \sin(-36.9 \cdot \text{deg}) = 160 - 120j \cdot \text{mA}$

$$I_T := I_R + I_Z = (160 - 120j) \cdot \text{mA} + 100 \cdot \text{mA} = 260 - 120j \cdot \text{mA}$$

$$\sqrt{260^2 + 120^2} = 286.356 \quad \text{atan}\left(\frac{-120}{260}\right) = -24.78 \cdot \text{deg} \quad I_T = 286\text{mA} \angle -24.8^\circ$$

c) Circle 1: i) The source current leads the source voltage answer i), $-24.8^\circ > -36.9^\circ$ ii) The source voltage leads the source current

Ex 5. a) The impedance Z (above) is made of two components in series. What are they and what are their values?

$$Z = 80 - 60j \cdot \Omega$$

Must have a resistor because there is a real part.

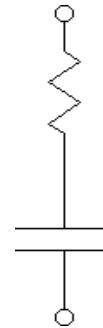
$$R := \text{Re}(Z)$$

$$R = 80 \cdot \Omega$$

Must have a capacitor because the imaginary part is negative.

$$\text{Im}(Z) = -60 \cdot \Omega = \frac{-1}{\omega \cdot C} \quad C := \frac{-1}{\omega \cdot \text{Im}(Z)}$$

$$C = 16.667 \cdot \mu\text{F}$$



b) The impedance Z is made of two components in parallel. What are they and what are their values?

$$Z = 80 - 60j \cdot \Omega$$

Must have a resistor because there is a real part.

Must have a capacitor because the imaginary part is negative.

$$Z = \frac{1}{\frac{1}{R} + j \cdot \omega \cdot C} \quad \frac{1}{Z} = \frac{1}{(80 - 60j) \cdot \Omega} \cdot \frac{(80 + 60j)}{(80 + 60j)} = \frac{80 + 60j}{80^2 + 60^2} = \frac{80 + 60j}{10,000} \cdot \frac{1}{\Omega}$$

$$\frac{1}{Z} = 8 \cdot 10^{-3} + 6 \cdot 10^{-3}j \cdot \Omega^{-1} = \frac{1}{R} + j \cdot \omega \cdot C$$

$$\frac{1}{R} = .008 \cdot \frac{1}{\Omega}$$

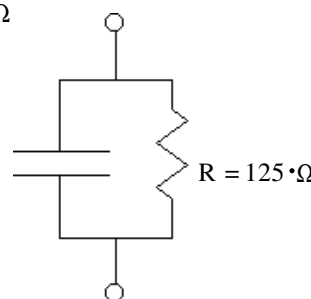
$$R := \frac{1}{.008 \cdot \frac{1}{\Omega}}$$

$$R = 125 \cdot \Omega$$

$$\omega \cdot C = .006 \cdot \frac{1}{\Omega}$$

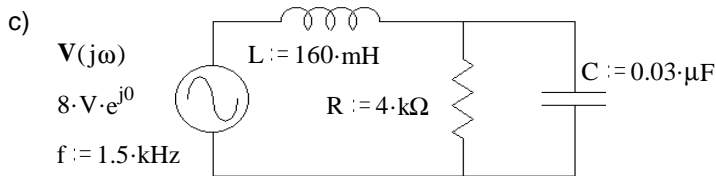
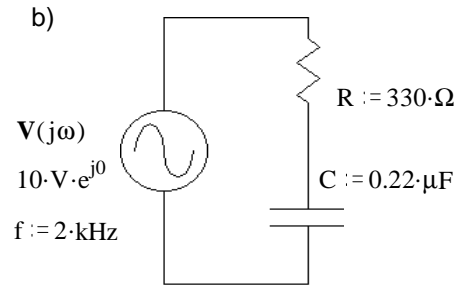
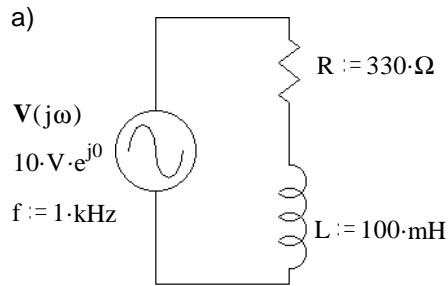
$$C := \frac{.006 \cdot \frac{1}{\Omega}}{\omega}$$

$$C = 6 \cdot \mu\text{F} \quad R = 125 \cdot \Omega$$



- Express the impedance of a 5.2mH inductor at 60 Hz in polar form.
- A capacitor impedance has a magnitude of 240Ω at a frequency of 1.8kHz. What is the value of capacitor?

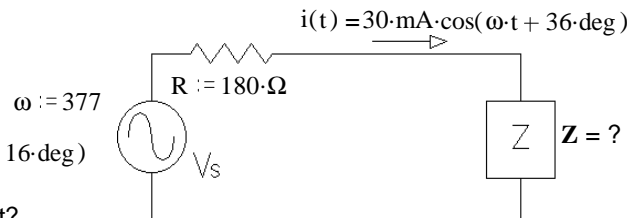
3. Find Z_{eq} in each case.



- Find the current $I(j\omega)$ in each case above.
- a) Find Z . Hint: Find the total impedance ($R+Z$) first.

b) Which leads, current or voltage?

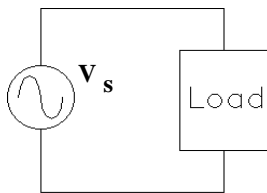
$v_s(t) = 14 \cdot V \cdot \cos(\omega \cdot t + 16 \cdot \text{deg})$



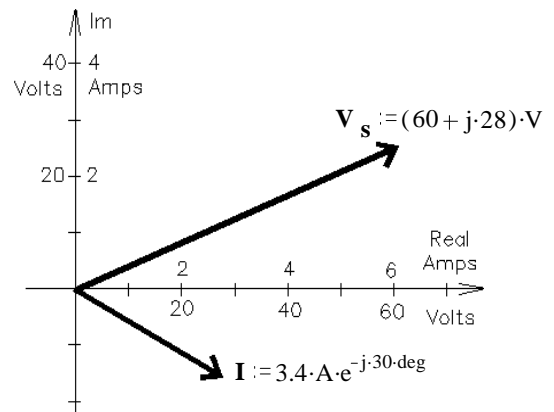
c) By how much?

I.E. what is the phase angle between the voltage and current?

6. The phasor diagram at right shows the voltage and current in the circuit below



Assume the load consists of a resistor in series with a reactive component and the frequency is 60 Hz.



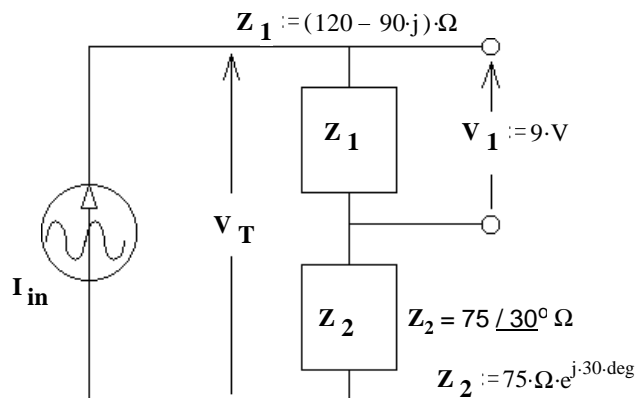
- What is the magnitude of the impedance?
- What is the value of the resistor?
- What is the reactive component (type and value)?

7. a) Find the AC current source, I_{in} in polar form.

b) Find V_T .

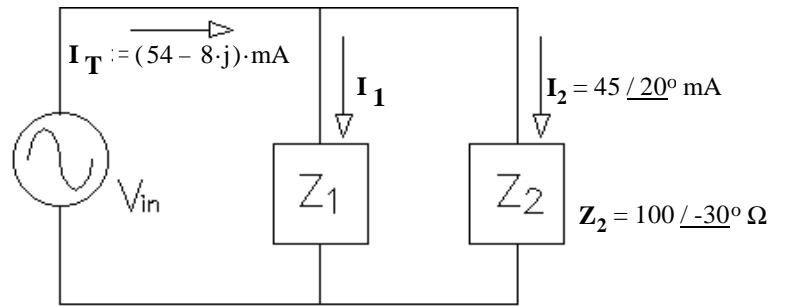
c) Choose one:

- The source current leads the source voltage.
- The source current lags the source voltage.



ECE 3600 homework # 2 p.2

8. a) Find Z_1 .

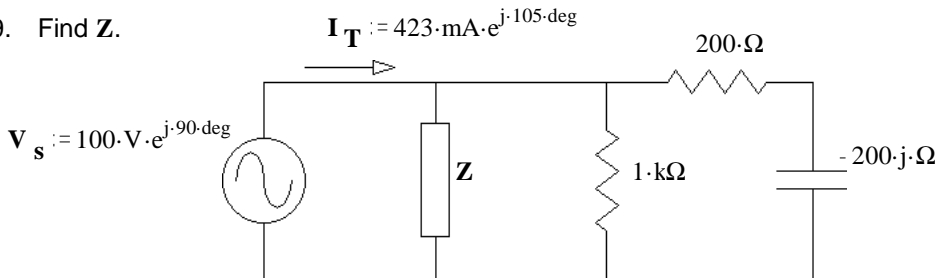


b) To make Z_1 in the simplest way, what part(s) would you need? Just determine the needed part(s) from the list below and state why you made that choice, don't find the values.

- | | | | | |
|-------------------|-------------------|-----------|--------------|----------------|
| resistor | capacitor | inductor | power supply | current source |
| Thevenin resistor | Ideal transformer | voltmeter | ammeter | scope |

- c) Choose one: i) I_2 leads the source voltage (V_{in}) ii) I_2 lags the source voltage (V_{in})
- d) Choose one: i) I_1 leads I_2 ii) I_1 lags I_2

9. Find Z .



Answers

1. $1.96 \Omega / 90^\circ$
2. $0.368 \mu F$
3. a) $(330 + 628.3j) \Omega = 709.7 \Omega / 62.29^\circ$ b) $(330 - 361.7j) \Omega = 489.6 \Omega / -47.63^\circ$ c) $1.82 k\Omega / -15.2^\circ$
4. a) $(6.6 - 12.5j) \text{ mA} = 14.1 \text{ mA} / -62.29^\circ$ b) $(13.8 + 15.1j) \text{ mA} = 20.4 \text{ mA} / 47.63^\circ$ c) $4.4 \text{ mA} / 15.2^\circ$
5. a) $259 - 160j$ b) The current leads the voltage c) 20°
6. a) 19.5Ω b) 11.2Ω c) inductor 42.3 mH
7. a) $60 / 36.87^\circ \text{ mA}$ b) $11.54 / 21^\circ \text{ V}$ c) i)
8. a) $172 / 53.4^\circ \Omega$ b) phase angle > 0 , resistor and inductor c) i) d) ii)
9. $657 \Omega / 67.4^\circ$