Name: ___________________________ ECE 3600 homework # 1 **Due: Fri, 8/28/20** a

Base your answers on class lecture & discussion, books and/or internet research. Some possible sources:
http://www.nerc.com/
http://en.wikipedia.org/wiki/Electricity_generation
http://www.energy.gov/energysources/electricpower.htm
http://en.wikipedia.org/wiki/Relative_cost_of_electricity_generated_by_different_sources

1. What is the name of the organization which ensures the reliability of power in North America?

2. Electric Utilities have been forced to break up into two separate companies responsible for:

   a.

   b.

3. What does deregulation provide for independent power producers (IPPs)?

4. The current bottleneck to overall system capacity.

5. What are the advantages of a highly interconnected system? (List at least 2)

6. Rank the sources of electrical energy in the US (highest to lowest %)
   1.
   2.
   3.
   4.
   5.

7. List 3 of the "Other" sources.
   1.
   2.
   3.

8. Rank the sources of electrical energy in the US by environmental and social negatives (worst to best). Assume "Other" is all the 3 you listed above. Consider petroleum just a little worse than natural gas (due to the danger of spills). Also give (in your opinion) the worst environment or social negative of each. Your answers here may be subjective.
   1.
   2.
   3.
   4.

9. Rank the sources of electrical energy in the US cost per kWh.

   List Nat gas twice, once for single cycle and once for combined-cycle. Choose one of the "Other" that you listed above. Initial costs are amortized over the life of the generation facility. You will have to make some guesses and may qualify your answers.

   1. (cheapest)
   2.
   3.
   4.
   5.
   6. (most expensive).

10. Give the approximate efficiencies of each type of power plant:

    a. Hydroelectric

    b. Rankin-cycle steam turbine plants, regardless of the source of heat.
    (coal, oil, gas-steam, nuclear, solar-steam, geothermal)

    c. Single-cycle gas turbine

    d. Combined-cycle gas turbine

11. In nuclear fission reactions, what is particle is crucial to the chain reaction and is used to control the reaction rate?

12. a) Why can't a wind turbine's coefficient of performance (conversion of wind energy to rotational mechanical energy) be 100%?

    b) What two things can be controlled to maximize the coefficient of performance?

    c) What is the biggest single problem of wind power?

13. a) Do photovoltaic cells produce AC or DC power?

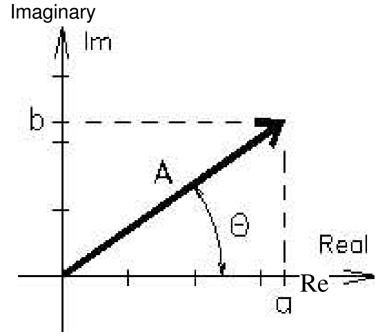
    b) What is the biggest single problem of photovoltaic cells?

14. What is cogeneration?

15. Some power sources are used to supply base loads and some are used to supply peak loads. Give some reasons to differentiate the sources in this way.

    <u>Base loads</u> <u>Peak loads</u>

Complex Numbers



$j = \sqrt{-1}$ the imaginary number

Rectangular Form $\mathbf{A} = a + b \cdot j$

$Re(\mathbf{A}) = a \qquad Im(\mathbf{A}) = b$

Polar Form $\mathbf{A} = A \cdot e^{j \cdot \theta}$

$Re(\mathbf{A}) = A \cdot \cos(\theta) \qquad Im(\mathbf{A}) = A \cdot \sin(\theta)$

Conversions $A = |\mathbf{A}| = \sqrt{a^2 + b^2} \qquad \theta = \arg(\mathbf{A}) = \operatorname{atan}\left(\frac{b}{a}\right)$

$a = A \cdot \cos(\theta) \qquad b = A \cdot \sin(\theta)$

$\mathbf{A} = A \cdot e^{j \cdot \theta} = A \cdot \cos(\theta) + A \cdot \sin(\theta) \cdot j \qquad \mathbf{A} = a + b \cdot j = \left(\sqrt{a^2 + b^2}\right) \cdot e^{j \cdot \operatorname{atan}\left(\frac{b}{a}\right)}$

Special Cases $j := \sqrt{-1} = e^{j \cdot 90 \cdot deg} \qquad \frac{1}{j} = -j = e^{-j \cdot 90 \cdot deg} \qquad e^{j \cdot 0 \cdot deg} = 1 \qquad e^{-j \cdot 180 \cdot deg} = e^{-j \cdot 180 \cdot deg} = -1$

$j \cdot e^{j \cdot \theta} = e^{j \cdot (\theta + 90 \cdot deg)}$

Define a 2nd number: rect: $\mathbf{D} = c + d \cdot j$ polar: $\mathbf{D} = D \cdot e^{j \cdot \phi}$

Equality $\mathbf{A} = \mathbf{D}$ if and only if $a = c$ and $b = d$ OR $A = D$ and $\theta = \phi$

Addition and Subtraction

$\mathbf{A} + \mathbf{D} = (a + b \cdot j) + (c + d \cdot j) = (a + c) + (b + d) \cdot j$

$\mathbf{A} - \mathbf{D} = (a + b \cdot j) - (c + d \cdot j) = (a - c) + (b - d) \cdot j$

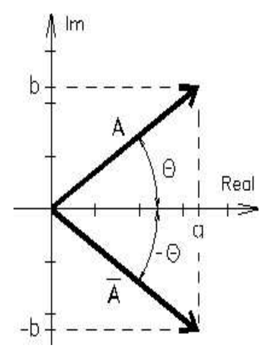
Convert polars to rectangular form first

Multiplication and Division $\mathbf{A} \cdot \mathbf{D} = (a + b \cdot j) \cdot (c + d \cdot j) = (a \cdot c - b \cdot d) + (b \cdot c + a \cdot d) \cdot j$

Rectangular: $\frac{\mathbf{A}}{\mathbf{D}} = \frac{a + b \cdot j}{c + d \cdot j} = \frac{a + b \cdot j}{c + d \cdot j} \cdot \frac{c - d \cdot j}{c - d \cdot j} = \frac{a \cdot c + b \cdot d}{c^2 + d^2} + \frac{b \cdot c - a \cdot d}{c^2 + d^2} \cdot j$

Polar: $\mathbf{A} \cdot \mathbf{D} = A \cdot e^{j \cdot \theta} \cdot D \cdot e^{j \cdot \phi} = A \cdot D \cdot e^{j \cdot (\theta + \phi)}$

$\frac{\mathbf{A}}{\mathbf{D}} = \frac{A \cdot e^{j \cdot \theta}}{D \cdot e^{j \cdot \phi}} = \frac{A}{D} \cdot e^{j \cdot (\theta - \phi)}$



Powers $\mathbf{A}^n = A^n \cdot e^{j \cdot n \cdot \theta} = A^n \cdot \cos(n \cdot \theta) + A^n \cdot \sin(n \cdot \theta) \cdot j$ Convert rectangulars first, usually

Conjugates

complex number	Conjugate	
$\mathbf{A} = a + b \cdot j$	$\overline{\mathbf{A}} = a - b \cdot j$	$\overline{\overline{\mathbf{A}}} = \mathbf{A}$
$\mathbf{A} = A \cdot e^{j \cdot \theta}$	$\overline{\mathbf{A}} = A \cdot e^{-j \cdot \theta}$	
$\mathbf{F} = \frac{3 + 5 \cdot j}{(2 - 6 \cdot j) \cdot e^{j \cdot 40 \cdot deg}}$	$\overline{\mathbf{F}} = \frac{3 - 5 \cdot j}{(2 + 6 \cdot j) \cdot e^{-j \cdot 40 \cdot deg}}$	

Euler's equation $e^{j \cdot \alpha} = \cos(\alpha) + j \cdot \sin(\alpha)$ OR: $\cos(\alpha) = \frac{e^{j \cdot \alpha} + e^{-j \cdot \alpha}}{2} \qquad \sin(\alpha) = \frac{e^{j \cdot \alpha} - e^{-j \cdot \alpha}}{2 \cdot j}$

$e^{j \cdot (\omega \cdot t + \theta)} = \cos(\omega \cdot t + \theta) + j \cdot \sin(\omega \cdot t + \theta)$

$Re\left[e^{j \cdot (\omega \cdot t + \theta)}\right] = \cos(\omega \cdot t + \theta)$

If we freeze this at time t=0, then we can represent $\cos(\omega \cdot t + \theta)$ by $e^{j \cdot \theta}$

Calculus Remember, when we write $e^{j \cdot \theta}$, we really mean $e^{j \cdot (\omega \cdot t + \theta)}$

$$\frac{d}{dt}\mathbf{A} = \frac{d}{dt}\left(A \cdot e^{j \cdot \theta}\right) = j \cdot \omega \cdot A \cdot e^{j \cdot \theta} = \omega \cdot A \cdot e^{j \cdot (\theta + 90 \cdot deg)}$$

$$\int \mathbf{A}\, dt = \int A \cdot e^{j \cdot \theta}\, dt = \frac{1}{j \cdot \omega} \cdot A \cdot e^{j \cdot \theta} = \frac{1}{\omega} \cdot A \cdot e^{j \cdot (\theta - 90 \cdot deg)}$$

Review of Phasors

ECE 3600

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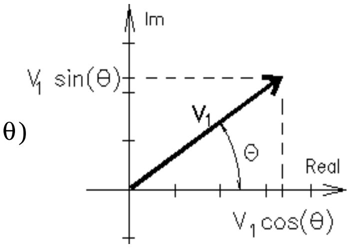
9/3/08
rev,

For steady-state sinusoidal response ONLY

Phasors



Time domain

$v(t) = \sqrt{2}\cdot V_1\cdot\cos(377\cdot t + \theta)$

Phasor, frequency domain (RMS)

$\mathbf{V_1} = V_1\cdot e^{j\cdot\theta} = V_1\ \underline{/\theta} = V_1\cdot\cos(\theta) + j\cdot V_1\cdot\sin(\theta)$

Impedances,

Inductor

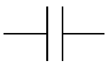
$v_L = L\cdot\frac{d}{dt}i_L = L\cdot\frac{d}{dt}I_p\cdot e^{j\cdot(\omega\cdot t+\theta)} = j\cdot\omega\cdot L\cdot\left[I_p\cdot e^{j\cdot(\omega\cdot t+\theta)}\right]$

$\mathbf{V_L}(\omega) = j\cdot\omega\cdot L\cdot\mathbf{I}(\omega)$

AC impedance

$\mathbf{Z_L} = j\cdot\omega\cdot L$

Capacitor



$i_C = C\cdot\frac{d}{dt}v_C = C\cdot\frac{d}{dt}V_p\cdot e^{j\cdot(\omega\cdot t+\theta)} = j\cdot\omega\cdot C\cdot\left[V_p\cdot e^{j\cdot(\omega\cdot t+\theta)}\right]$

$\mathbf{I_C}(\omega) = j\cdot\omega\cdot C\cdot\mathbf{V}(\omega)$ $\quad\mathbf{V_C}(\omega) = \frac{1}{j\cdot\omega\cdot C}\cdot\mathbf{I}(\omega)$

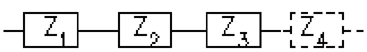
$\mathbf{Z_C} = \frac{1}{j\cdot\omega\cdot C} = \frac{-j}{\omega\cdot C}$

Resistor

$v_R = i_R\cdot R$ $\quad\mathbf{V_R}(\omega) = R\cdot\mathbf{I}(\omega)$ $\quad\mathbf{Z_R} = R$

You can use impedances just like resistances as long as you deal with the complex arithmetic.
ALL the DC circuit analysis techniques will work with AC.

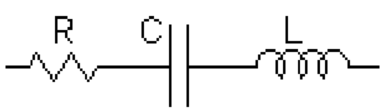
series:



$\mathbf{Z_{eq}} = \mathbf{Z_1} + \mathbf{Z_2} + \mathbf{Z_3} + \ldots$

$f := 60\cdot Hz$ $\quad\omega := 2\cdot\pi\cdot f$ $\quad\omega = 377\cdot\frac{rad}{sec}$

Example:



$R := 20\cdot\Omega$ $\quad C := 60\cdot\mu F$ $\quad L := 80\cdot mH$

$j\cdot\omega\cdot L = 30.159j\cdot\Omega$

$\frac{1}{j\cdot\omega\cdot C} = -44.21j\cdot\Omega$

$\mathbf{Z_{eq}} := R + \frac{1}{j\cdot\omega\cdot C} + j\cdot\omega\cdot L = 20\cdot\Omega - 44.21\cdot j\cdot\Omega + 30.16\cdot j\cdot\Omega = 20 - 14.05j\ \cdot\Omega$

$\sqrt{(20\cdot\Omega)^2 + (14.05\cdot\Omega)^2} = 24.44\cdot\Omega$ $\quad\operatorname{atan}\left(\frac{-14.05\cdot\Omega}{20\cdot\Omega}\right) = -35.09\cdot deg$

$\mathbf{Z_{eq}} = 24.44\Omega\ \underline{/-35.1}^o$

If: $\mathbf{V} := 120\cdot V\cdot e^{j\cdot 0\cdot deg}$ $\quad\mathbf{I} := \frac{\mathbf{V}}{\mathbf{Z_{eq}}} = \frac{120\cdot V}{24.44\cdot\Omega} = 4.91\cdot A$ $\quad\underline{/}\ \ 0 - -35.1 = 35.1\ \ deg$

$4.91\cdot\cos(35.1\cdot deg) = 4.017$ $\quad 4.91\cdot\sin(35.1\cdot deg) = 2.823$ $\quad\mathbf{I} = 4.017 + 2.822j\ \cdot A$

slight roundoff error

Voltage divider:

$$\mathbf{V}_{Zn} = \mathbf{V}_{total} \cdot \frac{\mathbf{Z}_n}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \ldots}$$

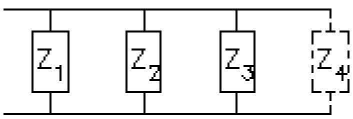
Eg: $\mathbf{V}_C := \mathbf{V} \cdot \frac{\frac{1}{j\cdot\omega\cdot C}}{\mathbf{Z}_{eq}} = 120\cdot V\cdot e^{j\cdot 0\cdot deg} \cdot \frac{44.21\cdot e^{-j\cdot 90\cdot deg}\cdot\Omega}{24.44\cdot e^{-j\cdot 35.1\cdot deg}\cdot\Omega}$

$120\cdot V\cdot\frac{44.21\cdot\Omega}{24.44\cdot\Omega} = 217.07 \cdot V$ $\quad \angle\ 0 + -90 - -35.1 = -54.9$ deg

$\mathbf{V}_C = 217.1V\ \angle -54.9^o$ $\quad \mathbf{V}_C = 124.771 - 177.604j \cdot V$

$217.1\cdot\cos(-54.9\cdot deg) = 124.8$ $\quad 217.1\cdot\sin(-54.9\cdot deg) = -177.6$

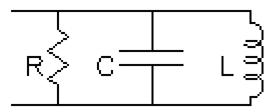
parallel:



$$\mathbf{Z}_{eq} = \frac{1}{\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \ldots}$$

Example:

$f := 60\cdot Hz$ $\quad \omega := 2\cdot\pi\cdot f$ $\quad \omega = 377 \cdot \frac{rad}{sec}$



$L := 80\cdot mH$

$j\cdot\omega\cdot L = 30.159j \cdot\Omega$

$R := 20\cdot\Omega$ $\quad C := 60\cdot\mu F$ $\quad \frac{1}{\omega\cdot L} = 3.316\cdot 10^{-2} \cdot\frac{1}{\Omega}$

$\frac{1}{j\cdot\omega\cdot C} = -44.21j \cdot\Omega$ $\quad \omega\cdot C = 2.262\cdot 10^{-2} \cdot\frac{1}{\Omega}$

$$\mathbf{Z}_{eq} := \frac{1}{\frac{1}{R} + \frac{1}{\left(\frac{1}{j\cdot\omega\cdot C}\right)} + \frac{1}{j\cdot\omega\cdot L}} = \frac{1}{\frac{1}{R} + j\cdot\omega\cdot C - \frac{j}{\omega\cdot L}} = \frac{1}{\frac{1}{20\cdot\Omega} + 2.262\cdot 10^{-2}\cdot j\cdot\frac{1}{\Omega} - 3.316\cdot 10^{-2}\cdot j\cdot\frac{1}{\Omega}}$$

$$= \frac{1}{\left(5\cdot 10^{-2} - 1.054\cdot 10^{-2}\cdot j\right)\cdot\frac{1}{\Omega}} \cdot \frac{5\cdot 10^{-2} + 1.054\cdot 10^{-2}\cdot j}{\left(5\cdot 10^{-2} + 1.054\cdot 10^{-2}\cdot j\right)} = 19.149 + 4.037j \cdot\Omega$$

$\sqrt{\left(5\cdot 10^{-2}\cdot\frac{1}{\Omega}\right)^2 + \left(1.054\cdot 10^{-2}\cdot\frac{1}{\Omega}\right)^2} = 5.11\cdot 10^{-2} \cdot\frac{1}{\Omega}$ $\quad \mathrm{atan}\left(\frac{1.054\cdot 10^{-2}\cdot\Omega}{5\cdot 10^{-2}\cdot\Omega}\right) = 11.9 \cdot deg$

$\frac{1}{5.11\cdot 10^{-2}\cdot\frac{1}{\Omega}} = 19.569 \cdot\Omega$ $\quad \mathbf{Z}_{eq} = 19.57\Omega\ \angle 11.9^o$

If: $\mathbf{V} := 120\cdot V\cdot e^{j\cdot 0\cdot deg}$ $\quad \mathbf{I} := \frac{\mathbf{V}}{\mathbf{Z}_{eq}} = \frac{120\cdot V}{19.57\cdot\Omega} = 6.132 \cdot A$ $\quad \angle\ 0 - 11.9 = -11.9$ deg

$6.132\cdot\cos(-11.9\cdot deg) = 6$ $\quad 6.132\cdot\sin(-11.9\cdot deg) = -1.264$ $\quad \mathbf{I} = 6 - 1.265j \cdot A$

slight roundoff error

Current divider:

$$\mathbf{I}_{Zn} = \mathbf{I}_{total} \cdot \frac{\frac{1}{\mathbf{Z}_n}}{\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \ldots}$$

Eg: $\mathbf{I}_L := \mathbf{I} \cdot \frac{\frac{1}{j\cdot\omega\cdot L}}{\frac{1}{R} + j\cdot\omega\cdot C + \frac{1}{j\cdot\omega\cdot L}} = \mathbf{I}\cdot\frac{\mathbf{Z}_{eq}}{j\cdot\omega\cdot L}$

$= 6.132\cdot A\cdot e^{j\cdot 11.9\cdot deg} \cdot \frac{19.57\cdot e^{-j\cdot 11.9\cdot deg}\cdot\Omega}{30.159\cdot e^{j\cdot 90\cdot deg}\cdot\Omega}$

$\mathbf{I}_L = 6.132\cdot A\cdot\frac{19.57\cdot\Omega}{30.159\cdot\Omega} = 3.979 \cdot A$ $\quad \angle\ 11.9 + -11.9 - 90 = -90$ deg $\quad \mathbf{I}_L = -3.979\cdot 10^3 j \cdot mA$

Duh... $\frac{\mathbf{V}}{j\cdot\omega\cdot L} = -3.979\cdot 10^3 j \cdot mA$

ECE 3600 Phasor Examples

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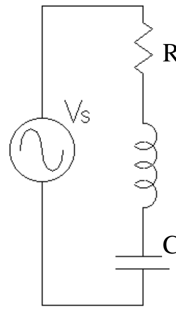
9/3/08
rev

Ex 1. Find $\mathbf{V_R}$, $\mathbf{V_L}$, and $\mathbf{V_C}$ in polar phasor form. $f := 2 \cdot kHz$

$\mathbf{V}(j\omega)$

$6 \cdot V \cdot e^{j0}$

$f := 2 \cdot kHz$



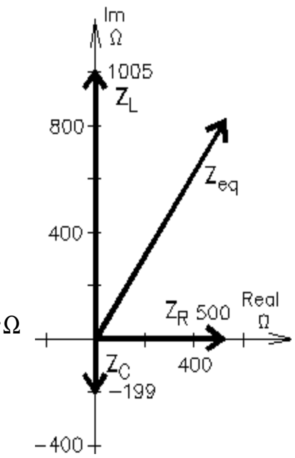
$R := 500 \cdot \Omega$ $\qquad \omega := 2 \cdot \pi \cdot f \qquad \omega = 1.257 \cdot 10^{4} \cdot \frac{rad}{sec}$

$L := 80 \cdot mH \qquad \mathbf{Z_L} := j \cdot \omega \cdot L \qquad \mathbf{Z_L} = 1.005j \cdot k\Omega$

$C := 0.4 \cdot \mu F \qquad \mathbf{Z_C} := \frac{1}{j \cdot \omega \cdot C} \qquad \mathbf{Z_C} = -0.199j \cdot k\Omega$

$\mathbf{Z_{eq}} := R + j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C} \qquad \mathbf{Z_{eq}} = 500 + 806.366j \cdot \Omega$

$\sqrt{500^2 + 806^2} = 948.491 \qquad \operatorname{atan}\left(\frac{806}{500}\right) = 58.187 \cdot deg \qquad \mathbf{Z_{eq}} = 948.5\Omega \ \underline{/ \ 58.2}^{o}$



find the current: $\mathbf{I} := \frac{6 \cdot V \cdot e^{j \cdot 0}}{\mathbf{Z_{eq}}}$ magnitude: $\frac{6 \cdot V}{948.5 \cdot \Omega} = 6.326 \cdot mA$ angle: $0 \cdot deg - 58.2 \cdot deg = -58.2 \cdot deg$

$\mathbf{I} = 6.326mA \ \underline{/ -58.2}^{o}$

	<u>find the magnitude</u>	<u>find the angle</u>	
$\mathbf{V_R} := \mathbf{I} \cdot R$	$6.326 \cdot mA \cdot 500 \cdot \Omega = 3.163 \cdot V$	$-58.2 \cdot deg + 0 \cdot deg = -58.2 \cdot deg$	$\mathbf{V_R} = 3.163V \ \underline{/ -58.2}^{o}$
$\mathbf{V_L} := \mathbf{I} \cdot \mathbf{Z_L}$	$6.326 \cdot mA \cdot 1005 \cdot \Omega = 6.358 \cdot V$	$-58.2 \cdot deg + 90 \cdot deg = 31.8 \cdot deg$	$\mathbf{V_L} = 6.358V \ \underline{/ \ 31.8}^{o}$
$\mathbf{V_C} := \mathbf{I} \cdot \mathbf{Z_C}$	$6.326 \cdot mA \cdot (-199) \cdot \Omega = -1.259 \cdot V$	$-58.2 \cdot deg + (90) \cdot deg = 31.8 \cdot deg$	$\mathbf{V_C} = -1.259V \ \underline{/ \ 31.8}^{o}$
OR:	$6.326 \cdot mA \cdot (199) \cdot \Omega = 1.259 \cdot V$	$-58.2 \cdot deg + (-90) \cdot deg = -148.2 \cdot deg$	$\mathbf{V_C} = 1.259V \ \underline{/ -148.2}^{o}$

OR, you can also find these voltages directly, using a voltage divider. I.E. to find $\mathbf{V_C}$ directly:

$$\mathbf{V_C} := \frac{\frac{1}{j \cdot \omega \cdot C}}{R + j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C}} \cdot 6 \cdot V = \frac{1}{R \cdot (j \cdot \omega \cdot C) + j \cdot \omega \cdot L \cdot (j \cdot \omega \cdot C) + 1} \cdot 6 \cdot V = \frac{1}{R \cdot (j \cdot \omega \cdot C) - \omega^2 \cdot L \cdot C + 1} \cdot 6 \cdot V$$

$$= \frac{1}{\left(1 - \omega^2 \cdot L \cdot C\right) + j \cdot \omega \cdot R \cdot C} \cdot 6 \cdot V \qquad \left(1 - \omega^2 \cdot L \cdot C\right) = -4.053 \qquad j \cdot \omega \cdot R \cdot C = 2.513j$$

$$= \frac{6 \cdot V}{-4.053 + 2.513 \cdot j} \cdot \frac{(-4.053 - 2.513 \cdot j)}{(-4.053 - 2.513 \cdot j)} = \frac{6 \cdot V \cdot (-4.053 - 2.513 \cdot j)}{(-4.053)^2 + 2.513^2}$$

$6 \cdot V \cdot (-4.053 - 2.513 \cdot j) = -24.318 - 15.078j \cdot V$

$(-4.053)^2 + 2.513^2 = 22.742$

$$= \left(\frac{-24.318}{22.742} - \frac{15.078 \cdot j}{22.742}\right) \cdot V = -1.069 - 0.663j \cdot V$$

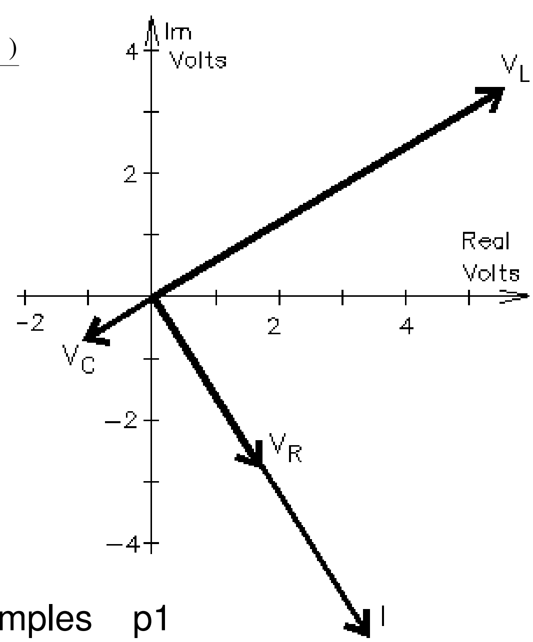
magnitude: $\sqrt{1.069^2 + 0.663^2} = 1.258$

angle: $\operatorname{atan}\left(\frac{-0.663}{-1.069}\right) = 31.81 \cdot deg$

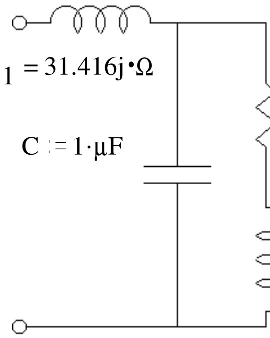
but this is actually in the third quadrant, so modify your calculator's results:

$31.81 \cdot deg - 180 \cdot deg = -148.19 \cdot deg$

$= 1.258V \ \underline{/ -148.2}^{o}$



$L_1 := 2 \cdot mH$



$j \cdot \omega \cdot L_1 = 31.416j \cdot \Omega$ $\quad R := 200 \cdot \Omega$ $\quad C := 1 \cdot \mu F$

$L_2 := 8 \cdot mH$ $\quad j \cdot \omega \cdot L_2 = 125.664j \cdot \Omega$

Ex 2. a) Find $\mathbf{Z_{eq}}$. $\quad f := 2.5 \cdot kHz \quad \omega := 2 \cdot \pi \cdot f \quad \omega = 1.571 \cdot 10^4 \cdot \frac{rad}{sec}$

Left branch

$$\mathbf{Z_l} := \frac{1}{j \cdot \omega \cdot C} \qquad \mathbf{Z_l} = -63.662j \cdot \Omega$$

Right branch

$$\mathbf{Z_r} := j \cdot \omega \cdot L_2 + R \qquad \mathbf{Z_r} = 200 + 125.664j \cdot \Omega$$

$$\mathbf{Z_{eq}} := j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + \frac{1}{\frac{1}{j \cdot \omega \cdot C}}} = j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C} = j \cdot \omega \cdot L_1 + \frac{R + j \cdot \omega \cdot L_2}{1 + j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2)}$$

$$= j \cdot \omega \cdot L_1 + \frac{R + j \cdot \omega \cdot L_2}{1 - \omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R}$$

Sometimes it's worth simplifying a little before puttiing in numbers.

$$\mathbf{Z_{eq}} = 31.416 \cdot j \cdot \Omega + \frac{(200 + 125.664 \cdot j) \cdot \Omega}{-0.974 + 3.142 \cdot j} \cdot \left(\frac{-0.974 - 3.142 \cdot j}{-0.974 - 3.142 \cdot j}\right) = 31.416 \cdot j \cdot \Omega + \frac{(200 + 125.664 \cdot j) \cdot (-0.974 - 3.142 \cdot j)}{0.974^2 + 3.142^2}$$

$$= 31.416 \cdot j \cdot \Omega + \frac{((200 \cdot (-0.974) - 125.664 \cdot (-3.142)) + (125.664 \cdot (-0.974) - 200 \cdot 3.142) \cdot j) \cdot \Omega}{0.974^2 + 3.142^2}$$

$$= 31.416 \cdot j \cdot \Omega + \frac{(200.036288 - 750.796736 \cdot j) \cdot \Omega}{10.82084} = 31.416 \cdot j \cdot \Omega + 18.486 \cdot \Omega - 69.384 \cdot j \cdot \Omega = 18.486 - 37.968j \cdot \Omega$$

$$\sqrt{18.49^2 + 37.97^2} = 42.233 \qquad \operatorname{atan}\left(\frac{-37.97}{18.49}\right) = -64.036 \cdot deg \qquad \mathbf{Z_{eq}} = 42.24\Omega \ \underline{/-64.04^o}$$

b) $\mathbf{V_{in}} := 12 \cdot V \cdot e^{j \cdot 20 \cdot deg}$ $\quad$ Find $\mathbf{I_{L1}}$, $\mathbf{V_C}$ $\quad \mathbf{I_{L1}} := \frac{\mathbf{V_{in}}}{\mathbf{Z_{eq}}} \quad \frac{12 \cdot V}{42.24 \cdot \Omega} = 284.091 \cdot mA \qquad 20 \cdot deg - (-64.04) \cdot deg = 84.04 \cdot deg$

$$\mathbf{I_{L1}} = 284mA \ \underline{/ 84.04^o} = 284 \cdot mA \cdot e^{j \cdot 84.04 \cdot deg} \qquad \mathbf{I_{L1}} = 29.485 + 282.569j \cdot mA$$

$$\mathbf{V_C} := \mathbf{I_{L1}} \cdot (18.486 - 69.384 \cdot j) \cdot \Omega \qquad 284 \cdot mA \cdot \sqrt{18.486^2 + 69.384^2} \cdot \Omega = 20.392 \cdot V$$

$$84.04 \cdot deg + \operatorname{atan}\left(\frac{-69.384}{18.486}\right) = 8.959 \cdot deg \qquad \mathbf{V_C} = 20.4V \ \underline{/ 8.96^o}$$

To find V_C directly:

You could then use another voltage divider to find $\mathbf{V_R}$ or $\mathbf{V_{L2}}$.

$$\mathbf{V_C} := \frac{\frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C}}{j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C}} \cdot \mathbf{V_{in}} = \frac{1}{j \cdot \omega \cdot L_1 \cdot \left(\frac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C\right) + 1} \cdot \mathbf{V_{in}} = \frac{1}{\frac{j \cdot \omega \cdot L_1}{R + j \cdot \omega \cdot L_2} - \omega^2 \cdot L_1 \cdot C + 1}$$

$$= \frac{1}{\frac{j \cdot \omega \cdot L_1}{R + j \cdot \omega \cdot L_2} - \omega^2 \cdot L_1 \cdot C + 1} \cdot \mathbf{V_{in}} = \frac{1}{\frac{j \cdot \omega \cdot L_1 \cdot (R - j \cdot \omega \cdot L_2)}{R^2 + (\omega \cdot L_2)^2} - \omega^2 \cdot L_1 \cdot C + 1} \cdot \mathbf{V_{in}}$$

$$= \frac{1}{\left[\frac{\omega^2 \cdot L_1 \cdot L_2}{R^2 + (\omega \cdot L_2)^2} - \omega^2 \cdot L_1 \cdot C + 1\right] + j \cdot \frac{\omega \cdot L_1 \cdot R}{R^2 + (\omega \cdot L_2)^2}} \cdot \mathbf{V_{in}} = \frac{12 \cdot V \cdot e^{j \cdot 20 \, deg}}{0.58816 \cdot e^{j \cdot 11.039 \, deg}} = \frac{12 \cdot V}{0.58816} \ \underline{/ 20 - 11.039^o}$$

$$= 20.4V \ \underline{/ 8.96^o} \qquad \text{Same}$$

$$\mathbf{V_C} = 20.153 + 3.178j \cdot V$$

Ex 2. Continued Find $\mathbf{I_{L2}}$.

$$\mathbf{Z_r} := R + j\cdot\omega\cdot L_2$$

$$\mathbf{Z_r} = 200 + 125.664j \;\cdot\Omega \qquad \sqrt{200^2 + 125.664^2} = 236.202 \qquad \operatorname{atan}\left(\frac{125.664}{200}\right) = 32.142\cdot\deg$$

$$\mathbf{I_{L2}} = \frac{20.4\cdot V\cdot e^{j\cdot 8.96\cdot\deg}}{236.202\cdot\Omega\cdot e^{j\cdot 32.142\cdot\deg}} = \frac{20.4\cdot V}{236.202\cdot\Omega}\ \underline{/\ 8.96 - 32.142^o} = 86.4\text{mA}\ \underline{/\text{-}23.18}$$

Or, directly by Current divider:

$$\mathbf{I_{L2}} := \frac{\dfrac{1}{R + j\cdot\omega\cdot L_2}}{j\cdot\omega\cdot C + \dfrac{1}{R + j\cdot\omega\cdot L_2}}\cdot\mathbf{I_{L1}} = \frac{1}{j\cdot\omega\cdot C\cdot\left(R + j\cdot\omega\cdot L_2\right) + 1}\cdot\mathbf{I_{L1}} = \frac{\mathbf{I_{L1}}}{1 - \omega^2\cdot C\cdot L_2 + j\cdot\omega\cdot C\cdot R}$$

denominator: $\sqrt{\left(1 - \omega^2\cdot C\cdot L_2\right)^2 + (\omega\cdot C\cdot R)^2} = 3.289 \qquad \operatorname{atan}\left(\dfrac{\omega\cdot C\cdot R}{1 - \omega^2\cdot C\cdot L_2}\right) + 180\cdot\deg = 107.224\cdot\deg$

$$\mathbf{I_{L2}} = \frac{284\cdot mA\cdot e^{j\cdot 84.04\cdot\deg}}{3.289\cdot e^{j\cdot 107.224\cdot\deg}} = \frac{284\cdot mA}{3.289}\ \underline{/\ 84.04 - 107.224^o} = 86.4\text{mA}\ \underline{/\text{-}23.18}$$

$$\mathbf{I_{L2}} = 79.404 - 34.001j \;\cdot mA$$

How about $\mathbf{I_C}$? $\mathbf{I_C} := \dfrac{\mathbf{V_C}}{\left(\dfrac{1}{j\cdot\omega\cdot C}\right)} = \mathbf{V_C}\cdot j\cdot\omega\cdot C = 20.4\text{V}\ \underline{/\ 8.96^o}\ \ 0.015708\ \underline{/\ 90^o}\ \dfrac{1}{\Omega} = 320\text{mA}\ \underline{/\ 98.96^o}$

Or, directly by Current divider:

$$\mathbf{I_C} := \frac{j\cdot\omega\cdot C}{j\cdot\omega\cdot C + \dfrac{1}{R + j\cdot\omega\cdot L_2}}\cdot\mathbf{I_{L1}} = \frac{j\cdot\omega\cdot C\cdot\left(R + j\cdot\omega\cdot L_2\right)}{j\cdot\omega\cdot C\cdot\left(R + j\cdot\omega\cdot L_2\right) + 1}\cdot\mathbf{I_{L1}} = \frac{-\omega^2\cdot C\cdot L_2 + j\cdot\omega\cdot C\cdot R}{1 - \omega^2\cdot C\cdot L_2 + j\cdot\omega\cdot C\cdot R}\cdot\mathbf{I_{L1}}$$

numerator: $\sqrt{\left(\omega^2\cdot C\cdot L_2\right)^2 + (\omega\cdot C\cdot R)^2} = 3.71 \qquad \operatorname{atan}\left(\dfrac{\omega\cdot C\cdot R}{-\omega^2\cdot C\cdot L_2}\right) + 180\cdot\deg = 122.142\cdot\deg$

Second quadrant

denominator is the same as above.

$$\mathbf{I_C} = \frac{3.71\cdot e^{j\cdot 122.14\cdot\deg}}{3.289\cdot e^{j\cdot 107.224\cdot\deg}}\cdot 284\cdot mA\cdot e^{j\cdot 84.04\cdot\deg} = \frac{3.71}{3.289}\cdot 284\cdot mA\ \underline{/\ 122.14 - 107.224 + 84.04^o} = 320\text{mA}\ \underline{/\ 98.96^o}$$

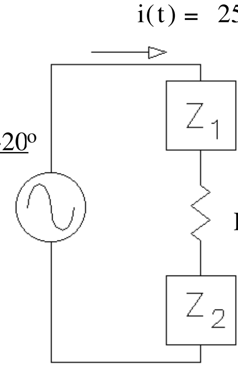
This current is greater than the input current. What's going on?

The angle between $\mathbf{I_C}$ & $\mathbf{I_{L2}}$ is big enough that they somewhat cancel each other out.

Check Kirchoff's Current Law: $\mathbf{I_C} + \mathbf{I_{L2}} = 29.485 + 282.569j \;\cdot mA \quad = \quad \mathbf{I_{L1}} = 29.485 + 282.569j \;\cdot mA$

Ex 3. a) Find $\mathbf{Z_2}$.

$$i(t) = 25\cdot mA\cdot\cos\left(377\cdot\frac{rad}{sec}\cdot t + 10\cdot\deg\right)$$



$\mathbf{V_{in}} = 10\text{V}\ \underline{/\text{-}20^o}$ $\qquad \mathbf{Z_1} := (120 - 60\cdot j)\cdot\Omega \qquad R := 50\cdot\Omega \qquad \mathbf{Z_2} = ?$

$$\mathbf{I} := 25\cdot mA\cdot e^{j\cdot 10\cdot\deg}$$

$$\mathbf{V_{in}} := 10\cdot V\cdot e^{-j\cdot 20\cdot\deg}$$

$$\mathbf{Z_T} := \frac{\mathbf{V_{in}}}{\mathbf{I}} = \frac{10\cdot V}{25\cdot mA}\ \underline{/\ \text{-}20 - 10^o} = 400\Omega\ \underline{/\text{-}30^o}$$

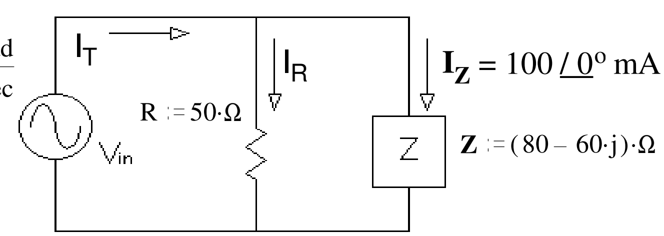
$$\mathbf{Z_T} = 346.41 - 200j \;\cdot\Omega$$

$$\mathbf{Z_2} := \mathbf{Z_T} - R - \mathbf{Z_1} = (346.41 - 200\cdot j)\cdot\Omega - 50\cdot\Omega - (120 - 60\cdot j)\cdot\Omega = 176.41 - 140j \;\cdot\Omega$$

b) Circle 1: i) The source current leads the source voltage <--- answer, because $10^o > -20^o$.

ii) The source voltage leads the source current

Ex 4. a) Find $\mathbf{V}_{in}$ in polar form.



$\mathbf{I_Z} := 100 \cdot mA$ $\quad \mathbf{Z} := (80 - 60 \cdot j) \cdot \Omega$ $\quad \omega := 1000 \cdot \frac{rad}{sec}$

$\mathbf{V_{in}} := \mathbf{I_Z} \cdot \mathbf{Z}$ $\quad \mathbf{V_{in}} = 8 - 6j \cdot V$

$\sqrt{8^2 + 6^2} = 10$ $\quad \operatorname{atan}\left(\frac{-6}{8}\right) = -36.87 \cdot deg$

$\mathbf{V_{in}} = 10V \underline{/ -36.9^o}$

b) Find $\mathbf{I_T}$ in polar form.

$$\mathbf{I_R} := \frac{\mathbf{V_{in}}}{R} = \frac{10 \cdot V}{50 \cdot \Omega} \underline{/ -36.9^o} = \frac{10 \cdot V}{50 \cdot \Omega} \cdot \cos(-36.9 \cdot deg) + j \cdot \frac{10 \cdot V}{50 \cdot \Omega} \cdot \sin(-36.9 \cdot deg) = 160 - 120i \cdot mA$$

$$\mathbf{I_T} := \mathbf{I_R} + \mathbf{I_Z} = (160 - 120 \cdot j) \cdot mA + 100 \cdot mA = 260 - 120j \cdot mA$$

$\sqrt{260^2 + 120^2} = 286.356$ $\quad \operatorname{atan}\left(\frac{-120}{260}\right) = -24.78 \cdot deg$ $\quad \mathbf{I_T} = 286mA \underline{/ -24.8^o}$

c) Circle 1: i) The source current leads the source voltage ii) The source voltage leads the source current

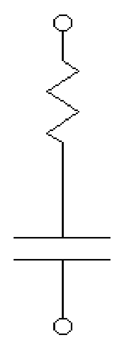
answer i), $-24.8^o > -36.9^o$

Ex 5. a) The impedance $\mathbf{Z}$ (above) is made of two components in series. What are they and what are their values?

$\mathbf{Z} = 80 - 60j \cdot \Omega$

Must have a resistor because there is a real part.

$R := \operatorname{Re}(\mathbf{Z})$ $\quad R = 80 \cdot \Omega$



Must have a capacitor because the imaginary part is negative.

$\operatorname{Im}(\mathbf{Z}) = -60 \cdot \Omega$ $\quad = \frac{-1}{\omega \cdot C}$ $\quad C := \frac{-1}{\omega \cdot \operatorname{Im}(\mathbf{Z})}$ $\quad C = 16.667 \cdot \mu F$

b) The impedance $\mathbf{Z}$ is made of two components in <u>parallel</u>. What are they and what are their values?

$\mathbf{Z} = 80 - 60j \cdot \Omega$

Must have a resistor because there is a real part.

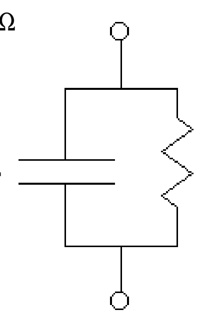
Must have a capacitor because the imaginary part is negative.

$$\mathbf{Z} = \frac{1}{\frac{1}{R} + j \cdot \omega \cdot C} \qquad \frac{1}{\mathbf{Z}} = \frac{1}{(80 - 60 \cdot j) \cdot \Omega} \cdot \left(\frac{80 + 60 \cdot j}{80 + 60 \cdot j}\right) = \frac{80 + 60 \cdot j}{80^2 + 60^2} = \frac{80 + 60 \cdot j}{10,000} \cdot \frac{1}{\Omega}$$

$$\frac{1}{\mathbf{Z}} = 8 \cdot 10^{-3} + 6 \cdot 10^{-3} j \cdot \Omega^{-1} = \frac{1}{R} + j \cdot \omega \cdot C$$

$\frac{1}{R} = .008 \cdot \frac{1}{\Omega}$ $\quad R := \frac{1}{.008 \cdot \frac{1}{\Omega}}$ $\quad R = 125 \cdot \Omega$

$\omega \cdot C = .006 \cdot \frac{1}{\Omega}$ $\quad C := \frac{.006 \cdot \frac{1}{\Omega}}{\omega}$ $\quad C = 6 \cdot \mu F$



ECE 3600 homework # 2 **Due: Tue, 9/1/20**

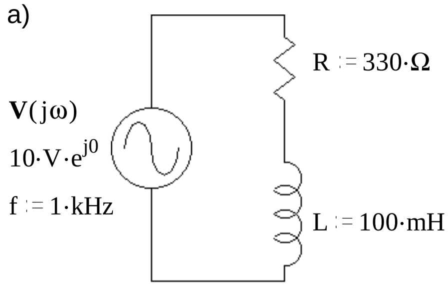
c

1. Express the impedance of a 5.2mH inductor at 60 Hz in polar form.

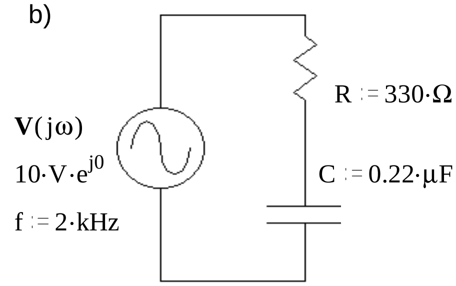
2. A capacitor impedance has a magnitude of 240Ω at a frequency of 1.8kHz. What is the value of capacitor?

3. Find $\mathbf{Z_{eq}}$ in each case.

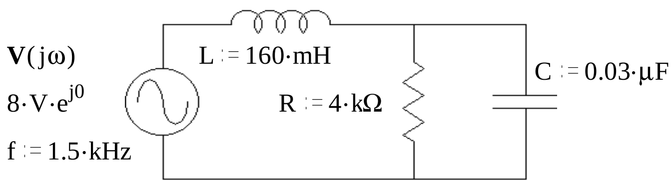
a)



b)



c)

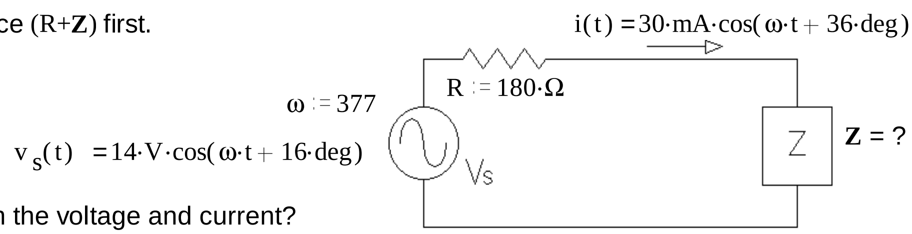


4. Find the current $\mathbf{I}(j\omega)$ in each case above.

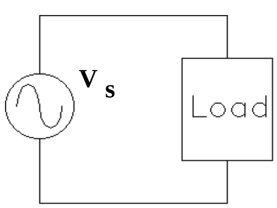
5. a) Find **Z**. Hint: Find the total impedance (R+**Z**) first.

   b) Which leads, current or voltage?

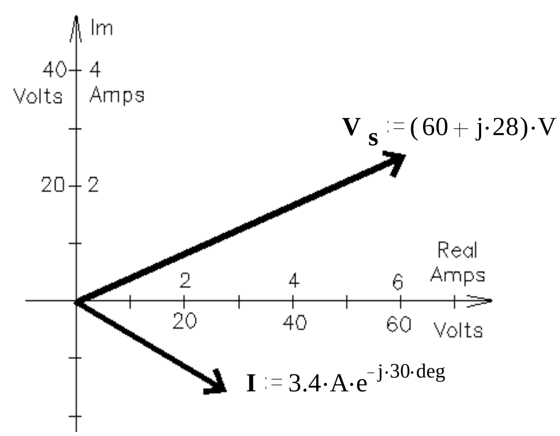
   c) By how much?
   I.E. what is the phase angle between the voltage and current?



6. The phasor diagram at right shows the voltage and current in the circuit below



Assume the load consists of a resistor in series with a reactive component and the frequency is 60 Hz.



a) What is the magnitude of the impedance?

b) What is the value of the resistor?

c) What is the reactive component (type and value)?

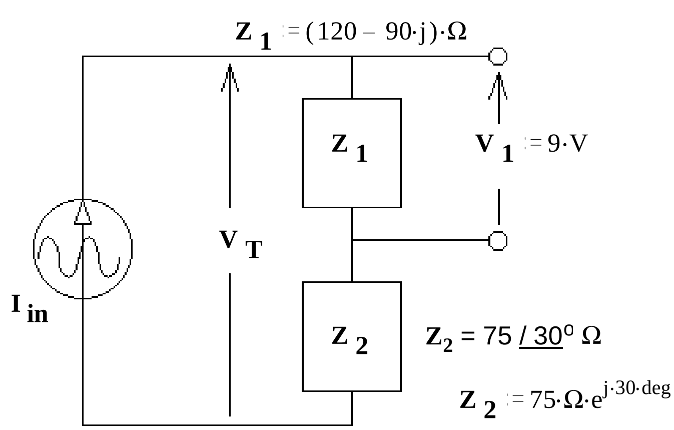
7. a) Find the AC current source, $\mathbf{I_{in}}$ in polar form.

   b) Find $\mathbf{V_T}$.

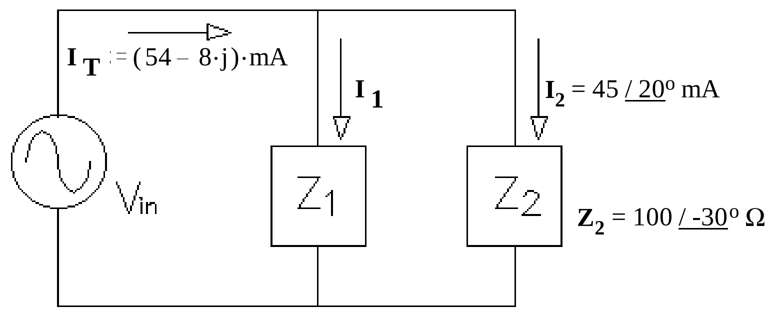
   c) Choose one:

      i) The source current leads the source voltage.

      ii) The source current lags the source voltage.



8. a) Find $\mathbf{Z_1}$.



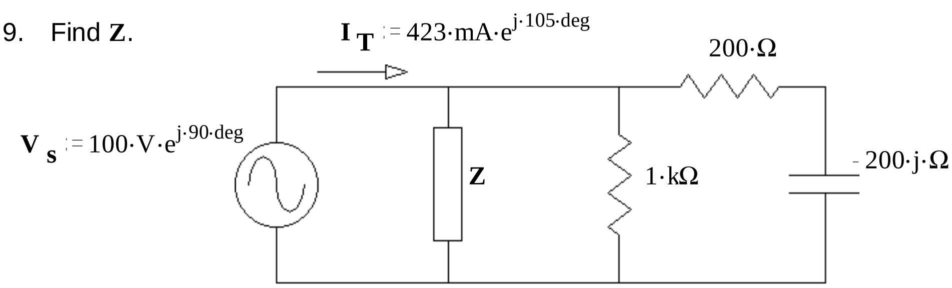
b) To make $\mathbf{Z_1}$ in the simplest way, what part(s) would you need? Just determine the needed part(s) from the list below and state why you made that choice, don't find the values.

resistor capacitor inductor power supply current source
Thevenin resistor Ideal transformer voltmeter ammeter scope

c) Choose one: i) $\mathbf{I_2}$ leads the source voltage ($\mathbf{V_{in}}$) ii) $\mathbf{I_2}$ lags the source voltage ($\mathbf{V_{in}}$)

d) Choose one: i) $\mathbf{I_1}$ leads $\mathbf{I_2}$ ii) $\mathbf{I_1}$ lags $\mathbf{I_2}$

9. Find $\mathbf{Z}$.



Answers

1. 1.96 Ω $\angle 90^o$
2. $0.368\cdot\mu F$
3. a) $(330 + 628.3\cdot j)\cdot\Omega$ = 709.7Ω $\angle 62.29^o$ b) $(330 - 361.7\cdot j)\cdot\Omega$ = 489.6Ω $\angle -47.63^o$ c) 1.82kΩ $\angle -15.2^o$
4. a) $(6.6 - 12.5\cdot j)\cdot mA$ = 14.1mA $\angle -62.29^o$ b) $(13.8 + 15.1\cdot j)\cdot mA$ = 20.4mA $\angle 47.63^o$ c) 4.4mA $\angle 15.2^o$
5. a) $259 - 160\cdot j$ b) The current leads the voltage c) 20º
6. a) $19.5\cdot\Omega$ b) $11.2\cdot\Omega$ c) inductor $42.3\cdot mH$
7. a) 60 $\angle 36.87^o$ mA b) 11.54 $\angle 21^o$ V c) i)
8. a) 172 $\angle 53.4^o$ Ω b) phase angle > 0, resistor and inductor c) i) d) ii)
9. 657 Ω $\angle 67.4^o$