ECE 3600 Lecture 3 & 4 notes Introduction to AC Power, RMS

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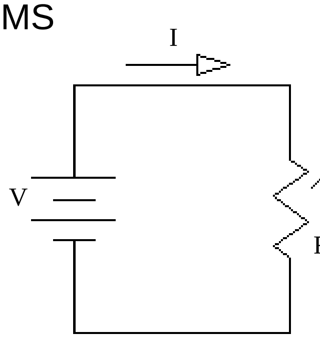
3/31/09,
2/20/10,
8/29/11
corrected
p,4 '14

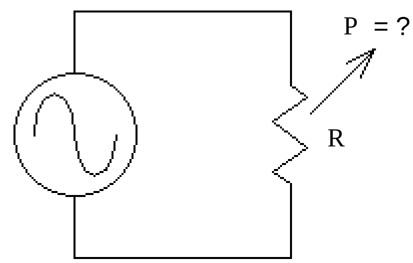
RMS

DC Power

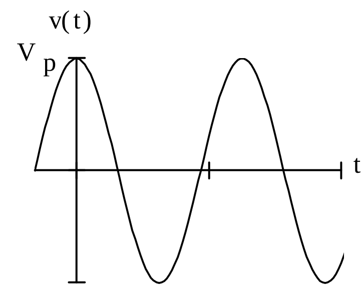


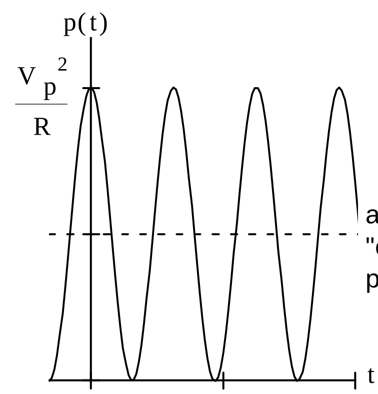
$$P = V \cdot I = \frac{V^2}{R} = I^2 \cdot R$$

AC Power



$$v(t) = V_p \cdot \cos(\omega \cdot t)$$





average or "effective" power = $\dfrac{\left(\dfrac{V_p^2}{R}\right)}{2}$

Couldn't we define an "effective" voltage that would allow us to use the same relationships for AC power as used for DC power?

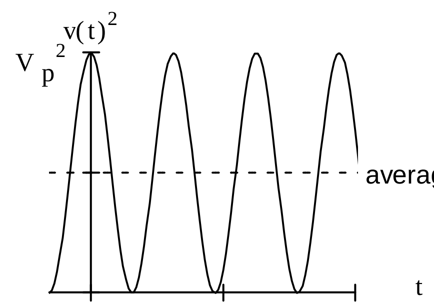
$$P_{ave} = \frac{\left(\dfrac{V_p^2}{R}\right)}{2} = \frac{\left(\dfrac{V_p^2}{2}\right)}{R} = \frac{\left(\dfrac{V_p}{\sqrt{2}}\right)^2}{R}$$

$$V_{eff} = \sqrt{\left(\frac{V_p}{\sqrt{2}}\right)^2} = \frac{V_p}{\sqrt{2}} = V_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T (v(t))^2 \, dt}$$

Square

Mean (average)

Root



average = $\dfrac{V_p^2}{2}$

RMS **R**oot of the **M**ean of the **S**quare

Use RMS in power calculations

Sinusoids

$$V_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T (v(t))^2 \, dt} = \sqrt{\frac{1}{T} \cdot \int_0^T \left(V_p \cdot \cos(\omega \cdot t)\right)^2 dt} = \sqrt{\frac{1}{T} \cdot \int_0^T V_p^2 \cdot \left(\frac{1}{2} + \frac{1}{2} \cdot \cos(2 \cdot \omega \cdot t)\right) dt}$$

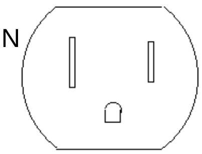
$$= \frac{V_p}{\sqrt{2}} \cdot \sqrt{\frac{1}{T} \cdot \int_0^T (1) \, dt + \frac{1}{T} \cdot \int_0^T \cos(2 \cdot \omega \cdot t) \, dt} = \frac{V_p}{\sqrt{2}} \cdot \sqrt{1 + 0}$$

Common household power

$f = 60 \cdot Hz$

$\omega = 377 \cdot \frac{rad}{sec}$

$T = 16.67 \cdot ms$

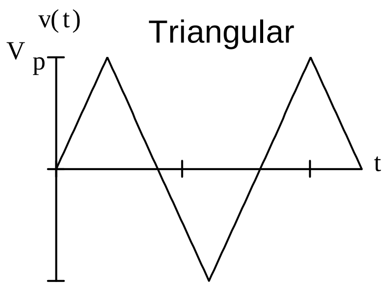


$V_{rms} := 120 \cdot V$

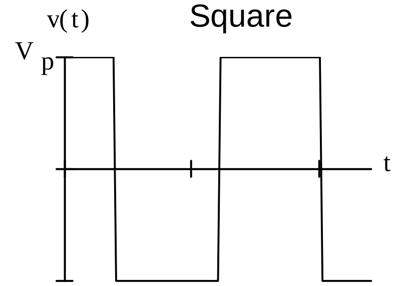
$V_p = V_{rms} \cdot \sqrt{2} = 170 \cdot V$

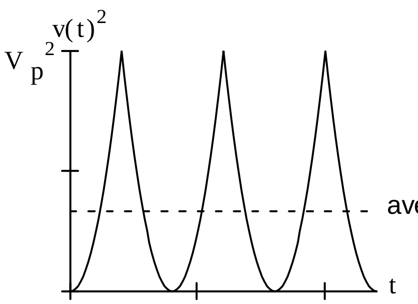
What about other wave shapes??

Triangular

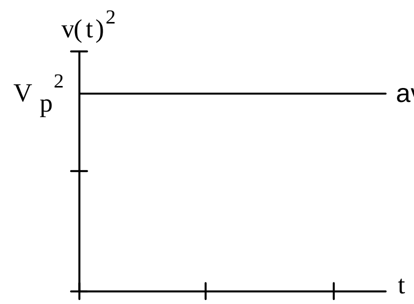


Square





average = $\frac{V_p^2}{3}$



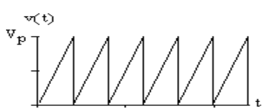
average = V_p^2

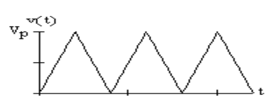
$V_{rms} = \frac{V_p}{\sqrt{3}}$

$V_{rms} = V_p$

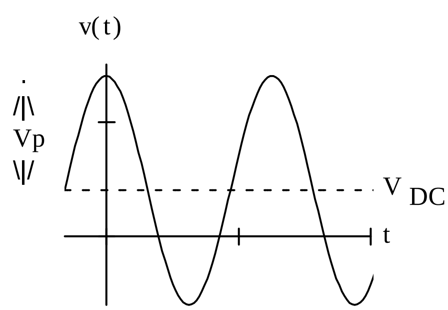
Works for all types of triangular and sawtooth waveforms

Same for DC





How about AC + DC ?



$$V_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T (v(t))^2\, dt}$$

$$= \sqrt{\frac{1}{T} \cdot \int_0^T \left(V_p \cdot \cos(\omega \cdot t) + V_{DC}\right)^2 dt}$$

$$= \sqrt{\frac{1}{T} \cdot \int_0^T \left[\left(V_p \cdot \cos(\omega \cdot t)\right)^2 + 2 \cdot \left(V_p \cdot \cos(\omega \cdot t)\right) \cdot V_{DC} + V_{DC}^2\right] dt}$$

$$= \sqrt{\frac{1}{T} \cdot \int_0^T \left(V_p \cdot \cos(\omega \cdot t)\right)^2 dt + \frac{1}{T} \cdot \int_0^T 2 \cdot \left(V_p \cdot \cos(\omega \cdot t)\right) \cdot V_{DC}\, dt + \frac{1}{T} \cdot \int_0^T V_{DC}^2\, dt}$$

- - - zero over one period - - -

$$= \sqrt{V_{rmsAC}^2 + 0 + V_{DC}^2} \qquad = \sqrt{V_{rmsAC}^2 + V_{DC}^2}$$

sinusoid: $V_{rms} = \frac{V_p}{\sqrt{2}}$ $\qquad I_{rms} = \frac{I_p}{\sqrt{2}}$

triangular: $V_{rms} = \frac{V_p}{\sqrt{3}}$ $\qquad I_{rms} = \frac{I_p}{\sqrt{3}}$

square: $V_{rms} = V_p$ $\qquad I_{rms} = I_p$

waveform + DC $\quad V_{rms} = \sqrt{V_{rmsAC}^2 + V_{DC}^2}$

<u>rectified average</u> $\quad V_{ra} = \frac{1}{T}\cdot\int_0^T |v(t)|\,dt$

$V_{ra} = \frac{2}{\pi}\cdot V_p \qquad I_{ra} = \frac{2}{\pi}\cdot I_p$

$V_{ra} = \frac{1}{2}\cdot V_p \qquad I_{ra} = \frac{1}{2}\cdot I_p$

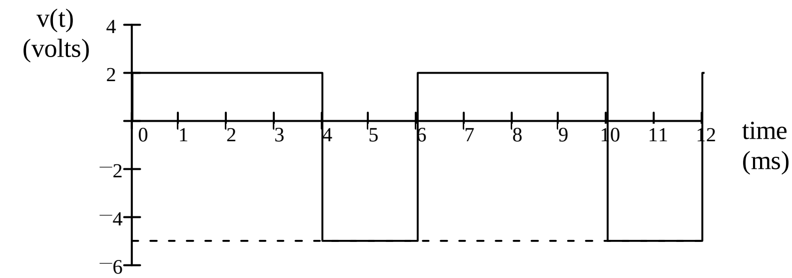
$V_{ra} = V_{rms} = V_p \qquad I_{ra} = I_{rms} = I_p$

Most AC meters don't measure true RMS. Instead, they measure V_{ra}, display $1.11V_{ra}$, and call it RMS. That works for sine waves but not for any other waveform.

Use RMS in power calculations

Some waveforms don't fall into these forms, then you have to perform the math from scratch

For waveform shown

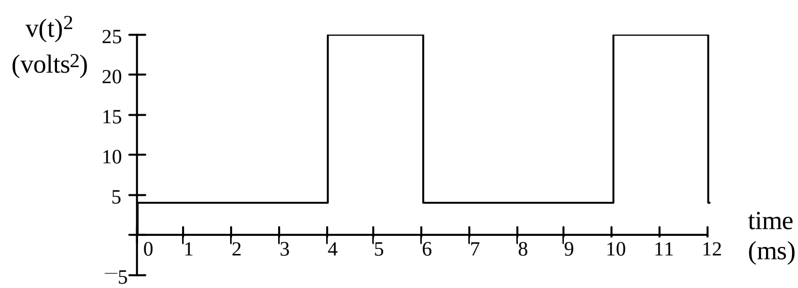


The average DC (V_{DC}) value

$$\frac{2\cdot V\cdot(4\cdot ms) + (-5\cdot V)\cdot(2\cdot ms)}{6\cdot ms} = -0.333\cdot V$$

The RMS (effective) value

Graphical way



$$\frac{4\cdot V^2\cdot(4\cdot ms) + 25\cdot V^2\cdot(2\cdot ms)}{6\cdot ms} = 11\cdot V^2$$

$V_{RMS} := \sqrt{11\cdot V^2} \qquad V_{RMS} = 3.32\cdot V$

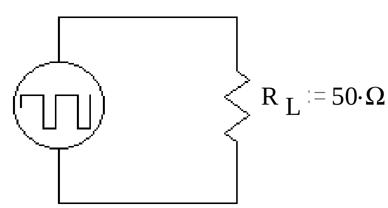
OR...

$$V_{RMS} = \sqrt{\frac{1}{T}\cdot\int_0^T (v(t))^2\,dt}$$

$$= \sqrt{\frac{1}{6\cdot ms}\cdot\left[\int_{0\cdot ms}^{4\cdot ms}(2\cdot V)^2\,dt + \int_{4\cdot ms}^{6\cdot ms}(-5\cdot V)^2\,dt\right]} = \sqrt{\frac{1}{6\cdot ms}\cdot\left[4\cdot ms\cdot(2\cdot V)^2 + 2\cdot ms\cdot(-5\cdot V)^2\right]} = 3.32\cdot V$$

The voltage is hooked to a resistor, as shown, for 6 seconds.

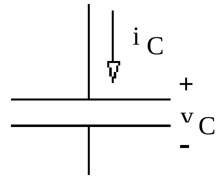
The energy is transfered to the resistor during that 6 seconds:

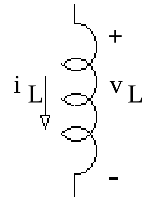


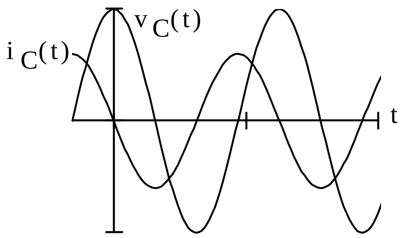
$P_L := \frac{V_{RMS}^2}{R_L} \qquad P_L = 0.22\cdot W$

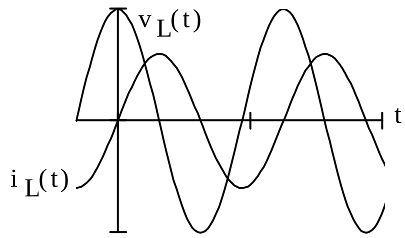
$W_L := P_L\cdot 6\cdot sec \qquad W_L = 1.32\cdot joule$ All converted to heat

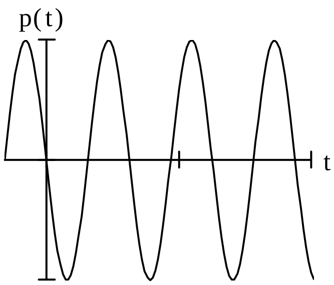
Capacitors and Inductors

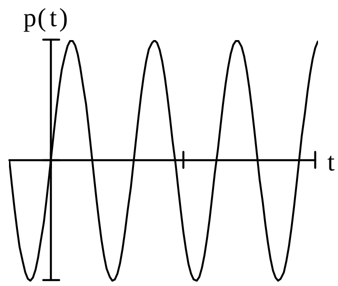












Average power is ZERO $P = 0$

Average power is ZERO $P = 0$

Capacitors and Inductors DO NOT dissipate (real) average power.

Reactive power is negative

$$Q_C = -I_{Crms} \cdot V_{Crms}$$

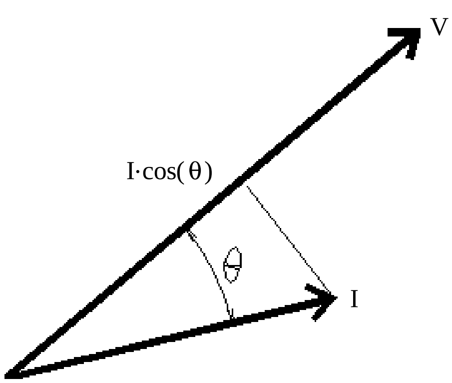
$$= -I_{Crms}^2 \cdot \frac{1}{\omega \cdot C} = -V_{Crms}^2 \cdot \omega \cdot C$$

Reactive power is positive

$$Q_L = I_{Lrms} \cdot V_{Lrms}$$

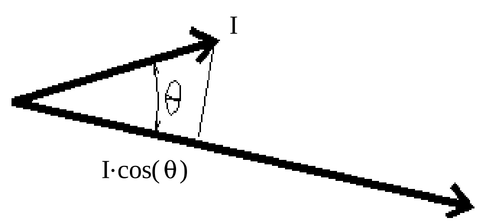
$$= I_{Lrms}^2 \cdot \omega \cdot L = \frac{V_{Lrms}^2}{\omega \cdot L}$$

If current and voltage are not in phase, only the in-phase part of the current matters for the power-- DOT PRODUCT



"Lagging" power

Inductor dominates



"Leading" Power

Capacitor dominates

Real Power

$P = I_{Rrms}^{2} \cdot R = \dfrac{V_{Rrms}^{2}}{R}$ for Resistors ONLY ! !

other wise....

$P = V_{rms} \cdot I_{rms} \cdot \cos(\theta) = I_{rms}^{2} \cdot |\mathbf{Z}| \cdot \cos(\theta) = \dfrac{V_{rms}^{2}}{|\mathbf{Z}|} \cdot \cos(\theta)$ units: watts, kW, MW, etc.

P = "Real" Power (average) $= V_{rms} \cdot I_{rms} \cdot pf = I_{rms}^{2} \cdot |\mathbf{Z}| \cdot pf = \dfrac{V_{rms}^{2}}{|\mathbf{Z}|} \cdot pf$

Reactive Power

capacitors -> **-** Q $Q_C = I_{Crms}^{2} \cdot X_C = \dfrac{V_{Crms}^{2}}{X_C}$ $X_C = -\dfrac{1}{\omega \cdot C}$ and is a **negative** number

inductors -> + Q $Q_L = I_{Lrms}^{2} \cdot X_L = \dfrac{V_{Lrms}^{2}}{X_L}$ $X_L = \omega \cdot L$ and is a positive number

other wise....

Q = Reactive "power" $= V_{rms} \cdot I_{rms} \cdot \sin(\theta)$ units: VAR, kVAR, etc. "volt-amp-reactive"

Complex and Apparent Power

$\mathbf{S}$ = Complex "power" $= \mathbf{V}_{\mathbf{rms}} \cdot \overline{\mathbf{I}_{\mathbf{rms}}}$ (complex congugate) $= P + jQ = V_{rms} I_{rms} \underline{/\theta}$ units: VA, kVA, etc. "volt-amp"

NOT $I_{rms}^{2} \cdot \mathbf{Z}$ **NOR** $\dfrac{V_{rms}^{2}}{\mathbf{Z}}$

S = Apparent "power" $= |\mathbf{S}| = V_{rms} \cdot I_{rms} = \sqrt{P^2 + Q^2}$ units: VA, kVA, etc. "volt-amp"

Power factor

$pf = \cos(\theta)$ = power factor (sometimes expressed in %) $0 \leq pf \leq 1$

θ is the **phase angle** between the voltage and the current or the phase angle of the impedance. $\theta = \theta_Z$

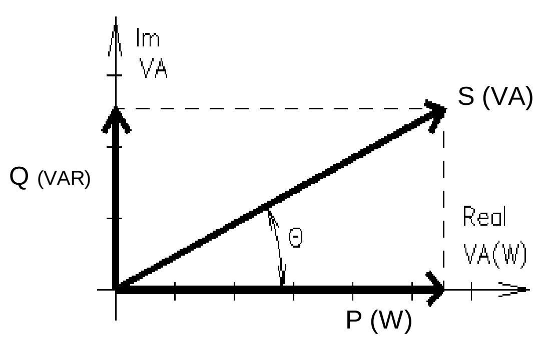
$\theta < 0$ Load is "Capacitive", power factor is "leading". This condition is very rare

$\theta > 0$ Load is "Inductive", power factor is "lagging". This condition is so common you can assume any power factor given is lagging unless specified otherwise. Transformers and motors make most loads inductive.

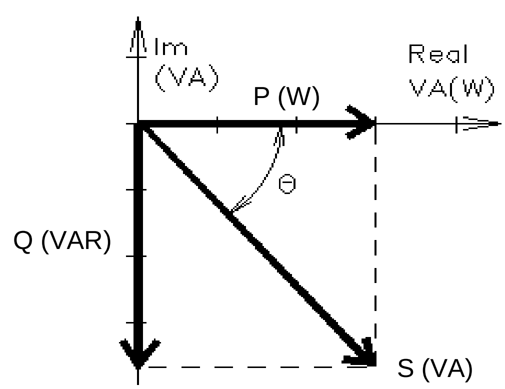
Industrial users are charged for the reactive power that they use, so power factor < 1 is a bad thing.

Power factor < 1 is also bad for the power company. To deliver the same power to the load, they have more line current (and thus more line losses).

Power factors are "corrected" by adding capacitors (or capacitve loads) in parallel with the inductive loads which cause the problems. (In the rare case that the load is capacitive, the pf would be corrected by an inductor.)



"Lagging" power



"Leading" Power