ECE 3600 3-Phase Examples

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Ex. 1 A Y-connected load is connected to 208-V 3-phase.
It draws 1.2kW of power at a power factor of 75%, leading. $P_{3\phi} := 1.2\cdot kW$ $pf := 0.75$

a) Find the apparent power and the reactive power.

$S_{3\phi} := \frac{P_{3\phi}}{pf}$ $S_{3\phi} = 1.6\cdot kVA$ $Q_{3\phi} := -\sqrt{S_{3\phi}^2 - P_{3\phi}^2}$ $Q_{3\phi} = -1.058\cdot kVAR$

Negative because the power factor is leading.

b) Find the line current.

$I_L = I_\phi = \frac{S_\phi}{V_\phi}$

$S_\phi := \frac{S_{3\phi}}{3}$ $V_\phi := \frac{208\cdot V}{\sqrt{3}}$ $V_\phi = 120.089\cdot V$ $I_L := \frac{S_\phi}{V_\phi}$ $I_L = 4.441\cdot A$

c) Find the values of the load components, assumming they are connected in series.
The components must be a resistor and a capacitor because
there is some real power and the power factor is leading.

$P_\phi := \frac{P_{3\phi}}{3}$ $R_L := \frac{P_\phi}{I_\phi^2} = \frac{P_\phi}{I_L^2} = 20.28\cdot\Omega$

$Q_\phi := \frac{Q_{3\phi}}{3}$ $Z_{CL} := \frac{Q_\phi}{I_L^2}$ $Z_{CL} = -17.885\cdot\Omega = -\frac{1}{\omega\cdot C}$

assume $\omega = 377\cdot\frac{rad}{sec}$ $C := -\frac{1}{\omega\cdot Z_{CL}}$ $C = 148.3\cdot\mu F$

d) Find the values of the load components, assumming they are connected in parallel.
Still a resistor and a capacitor.

$R_L := \frac{V_\phi^2}{P_\phi}$ $R_L = 36.053\cdot\Omega$

$Z_{CL} := \frac{V_\phi^2}{Q_\phi}$ $Z_{CL} = -40.881\cdot\Omega = -\frac{1}{\omega\cdot C}$

assume $\omega = 377\cdot\frac{rad}{sec}$ $C := -\frac{1}{\omega\cdot Z_{CL}}$ $C = 64.9\cdot\mu F$

e) Correct the power factor with Y-connected components.
Need inductors

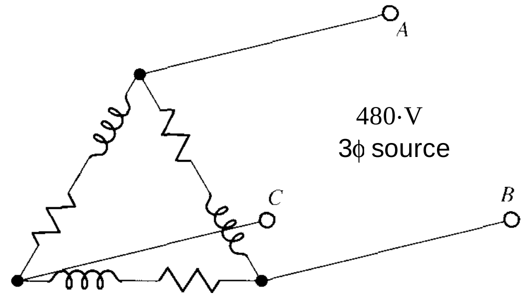
$Q_{\phi Ind} := -Q_\phi = \frac{V_\phi^2}{\omega\cdot L_Y}$ $L_Y := \frac{V_\phi^2}{\omega\cdot -Q_\phi}$ $L_Y = 108.4\cdot mH$

f) Correct the power factor with Δ-connected components.

$\omega\cdot L_\Delta = \mathbf{Z}_\Delta = 3\cdot\mathbf{Z}_y = 3\cdot\omega\cdot L_Y$

$3\cdot L_Y = 325.3\cdot mH$

Ex. 2 From F08, exam 1, Find the following:



All $\mathbf{Z} := (30 + 12\cdot j)\cdot\Omega$

a) The line current that would be measured by an ammeter.

$V_{LL} := 480\cdot V$ $\qquad \mathbf{Z}_{\Delta} := (30 + 12\cdot j)\cdot\Omega$

$I_{AB} := \left|\dfrac{V_{LL}}{\mathbf{Z}_{\Delta}}\right|$ $\qquad I_{AB} = 14.856\cdot A$

$I_A := \sqrt{3}\cdot I_{AB}$ $\qquad I_A = 25.73\cdot A$

b) The power consumed by the three-phase load.

$\theta := \operatorname{atan}\left(\dfrac{12}{30}\right)$ $\qquad \theta = 21.801\cdot deg$ $\qquad pf := \cos(\theta)$ $\qquad pf = 0.928$

$S_{3\phi} := 3\cdot V_{LL}\cdot I_{AB}$ $\qquad S_{3\phi} = 21.39\cdot kVA$

$P_{3\phi} := S_{3\phi}\cdot pf \;=\; 3\cdot I_{AB}^{\,2}\cdot 30\cdot\Omega = 19.86\cdot kW$

alternate way

c) The value of Y-connected impedances that would result in exactly the same line currents and same pf.

$\mathbf{Z}_{\mathbf{Y}} = \dfrac{\mathbf{Z}_{\Delta}}{3} = 10 + 4j \;\cdot\Omega$

d) The value of Y-connected capacitors that would correct the pf.

$Q_{1\phi} := \dfrac{1}{3}\cdot\sqrt{S_{3\phi}^{\,2} - P_{3\phi}^{\,2}}$ $\qquad Q_{1\phi} = 2.648\cdot kVAR$ $\qquad$ so we need: $Q_C := -Q_{1\phi}$

$Q_C = -2.648\cdot kVAR = -\dfrac{V_{LN}^{\,2}}{\left(\dfrac{1}{\omega\cdot C}\right)} = -V_{LN}^{\,2}\cdot\omega\cdot C$ $\qquad C := \dfrac{Q_C}{-\left(\dfrac{V_{LL}}{\sqrt{3}}\right)^2\cdot\omega}$ $\qquad C = 91.47\cdot\mu F$

Ex. 3 For the three-phase delta-connected load in fig P1 .7, The line-to-line voltage and line current are:

$\mathbf{V}_{\mathbf{AB}} := 480\cdot V \;\underline{/0^o}$ $\qquad \mathbf{I}_{\mathbf{A}} = 10A\underline{/-40^o}$

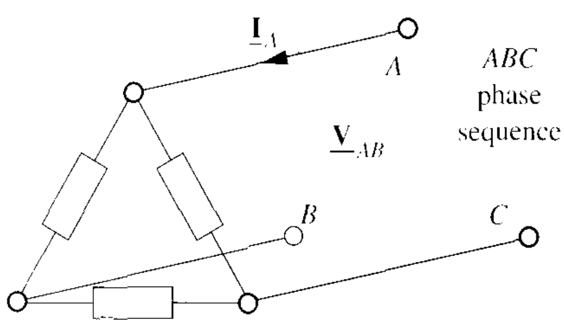


Figure P1.7

a) What is $\mathbf{V}_{\mathbf{CA}}$?

$\mathbf{V}_{\mathbf{CA}} := 480\cdot V \;\underline{/120^o} \;=\; 480\cdot V \;\underline{/-240^o}$

b) What is the phase current in the load? (As always, give the rms value.)

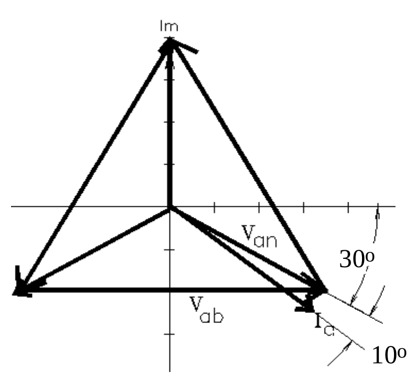
$\dfrac{10\cdot A}{\sqrt{3}} = 5.774\cdot A$

c) What is the time-average power into the load?

$\mathbf{V}_{\mathbf{AN}} := \dfrac{480\cdot V}{\sqrt{3}} \;\underline{/-30^o}$ $\qquad$ Since $\quad \mathbf{I}_{\mathbf{A}} = 10A\underline{/-40^o}$ $\qquad$ **I** lags **V** by 10^o

$\theta := 10\cdot deg$

$P_{3\phi} = 3\cdot\left(480\cdot V\cdot\dfrac{10\cdot A}{\sqrt{3}}\right)\cdot\cos(\theta) = 8.188\cdot kW$



d) What is the phase impedance? $\quad Z_{\Delta} := \dfrac{(480\cdot V)}{\left(\dfrac{10\cdot A}{\sqrt{3}}\right)}$ $\qquad Z_{\Delta} = 83.14\cdot\Omega \quad \underline{/10^o}$

Ex. 4 In the three-phase circuit shown in Fig. P1.9. find the following:

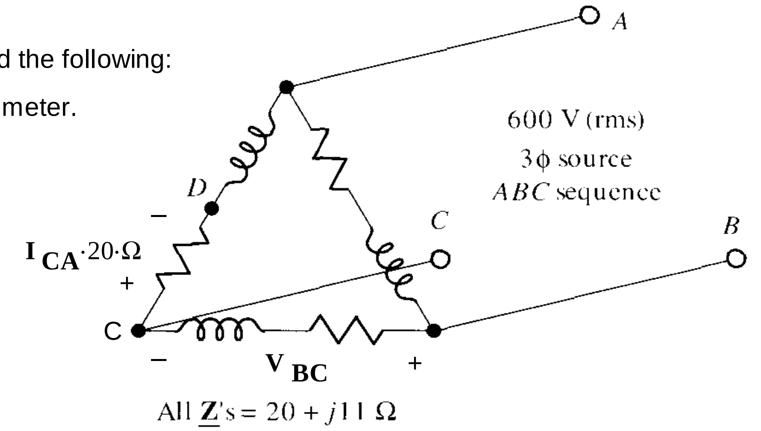


Figure P1.9

a) The line current that would be measured by an ammeter.

$V_{LL} := 600 \cdot V$ $\quad$ $\mathbf{Z}_{\Delta} := (20 + 11 \cdot j) \cdot \Omega$

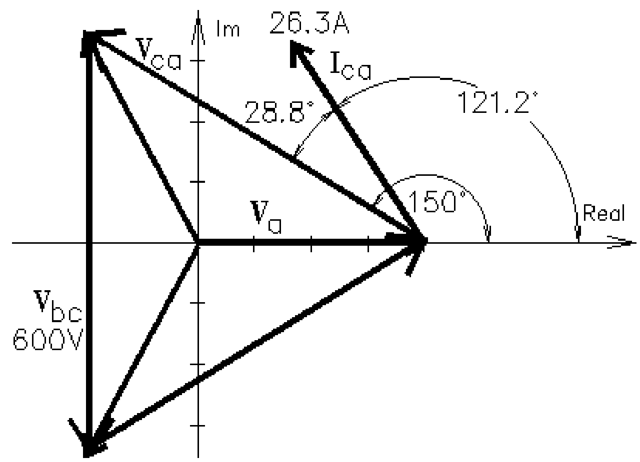
$I_{AB} := \left| \frac{V_{LL}}{\mathbf{Z}_{\Delta}} \right|$ $\quad$ $I_{AB} = 26.286 \cdot A$

$I_A := \sqrt{3} \cdot I_{AB}$ $\quad$ $I_A = 45.53 \cdot A$

b) The power factor of the three-phase load.

$\phi := \operatorname{atan}\left(\frac{11}{20}\right)$ $\quad$ $\phi = 28.811 \cdot \deg$ $\quad$ $pf = \cos(\phi) = 0.876$

c) The voltage that would be measured between B and D by a voltmeter.



Using $\mathbf{V}_A$ as reference (0^o):

$\mathbf{V}_{BC} := 600 \cdot V \cdot e^{-j \cdot 90 \cdot \deg}$

$\mathbf{I}_{CA} := 26.286 \cdot A \cdot e^{j \cdot (150 - 28.811) \cdot \deg}$

$\mathbf{V}_{CD} := \mathbf{I}_{CA} \cdot 20 \cdot \Omega$

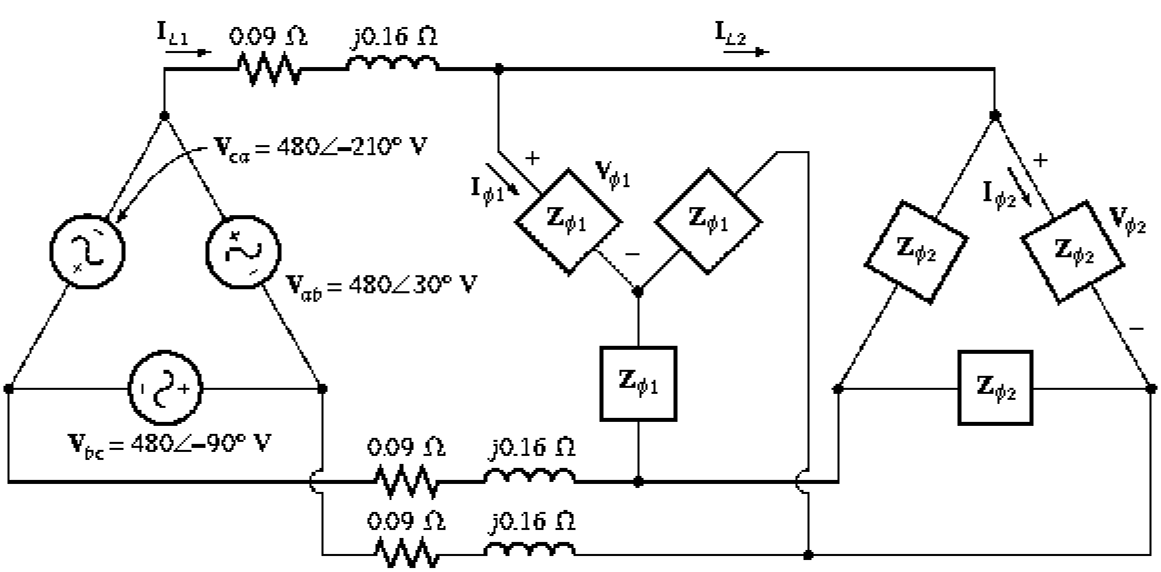
$\mathbf{V}_{CD} = -272.251 + 449.734j \cdot V$

$\mathbf{V}_{BD} := \mathbf{V}_{BC} + \mathbf{V}_{CD}$ $\quad$ $\mathbf{V}_{BD} = -272.251 - 150.266j \cdot V$ $\quad$ $\left|\mathbf{V}_{BD}\right| = 311 \cdot V$

(must be the sum, NOT the difference, see the + and - signs on the drwawing.)

Ex. 5 Textbook problem 2-2

Figure P2-1 shows a three-phase power system with two loads. The Δ-connected generator is producing a line voltage of 480 V, and the line impedance is 0.09 + j0.16 Ω. Load 1 is Y-connected, with a phase impedance of 2.5□ /36.87° Ω and load 2 is Δ-connected, with a phase impedance of 5 /-20° Ω.



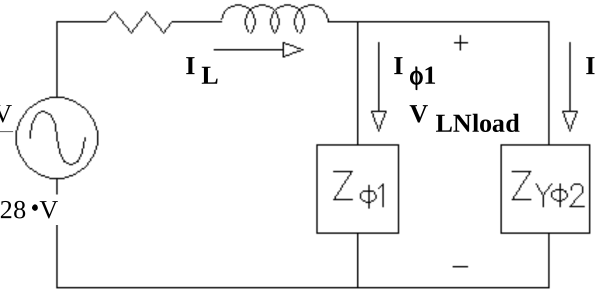
$\mathbf{Z}_{\phi1} := 2.5 \cdot e^{j \cdot 36.87 \cdot \deg} \cdot \Omega$ $\quad$ $\mathbf{Z}_{\phi2} := 5 \cdot e^{-j \cdot 20 \cdot \deg} \cdot \Omega$

a) What is the line voltage at the two loads?

Find an equivalent Y-only circuit:

$Z_{line} := (0.09 + 0.16 \cdot j) \cdot \Omega$

$V_Y := \frac{480 \cdot V}{\sqrt{3}}$ $\quad V_Y = 277.128 \cdot V$



$Z_{Y\phi 2} := \frac{Z_{\phi 2}}{3}$ $\quad Z_{Y\phi 2} = 1.566 - 0.57j \cdot \Omega$

$Z_{Yloads} := \frac{1}{\frac{1}{Z_{\phi 1}} + \frac{1}{Z_{Y\phi 2}}}$ $\quad Z_{Yloads} = 1.13 + 0.044j \cdot \Omega$ $\quad |Z_{Yloads}| = 1.131 \cdot \Omega$ $\quad \arg(Z_{Yloads}) = 2.254 \cdot deg$

$Z_{Ytot} := Z_{line} + Z_{Yloads}$ $\quad Z_{Ytot} = 1.22 + 0.204j \cdot \Omega$ $\quad |Z_{Ytot}| = 1.237 \cdot \Omega$ $\quad \arg(Z_{Ytot}) = 9.516 \cdot deg$

$I_L := \frac{V_Y}{Z_{Ytot}}$ $\quad I_L = 220.998 - 37.047j \cdot A$ $\quad |I_L| = 224.082 \cdot A$ $\quad \arg(I_L) = -9.516 \cdot deg$

$V_{line} := I_L \cdot Z_{line}$ $\quad V_{line} = 25.817 + 32.025j \cdot V$ $\quad |V_{line}| = 41.136 \cdot V$ $\quad \arg(V_{line}) = 51.126 \cdot deg$

$V_{LNload} := I_L \cdot Z_{Yloads}$ $\quad V_{LNload} = 251.311 - 32.025j \cdot V$ $\quad |V_{LNload}| = 253.343 \cdot V$ $\quad \arg(V_{LNload}) = -7.262 \cdot deg$

$V_{Lload} := V_{LNload} \cdot \sqrt{3}$ $\quad V_{Lload} = 435.283 - 55.47j \cdot V$ $\quad |V_{Lload}| = 438.803 \cdot V$ $\quad \arg(V_{Lload}) = -7.262 \cdot deg$

b) What is the voltage drop on the transmission lines?

$V_{line} := I_L \cdot Z_{line}$ $\quad V_{line} = 25.817 + 32.025j \cdot V$ $\quad |V_{line}| = 41.136 \cdot V$ $\quad \arg(V_{line}) = 51.126 \cdot deg$

c) Find the real and reactive powers supplied to each load.

$I_{\phi 1} := \frac{|V_{LNload}|}{|Z_{\phi 1}|}$ $\quad I_{\phi 1} = 101.337 \cdot A$ $\qquad I_{\phi 2} := \frac{|V_{LNload}|}{|Z_{Y\phi 2}|}$ $\quad I_{\phi 2} = 152.006 \cdot A$

$P_{3\phi 1} := 3 \cdot I_{\phi 1}^2 \cdot Re(Z_{\phi 1})$ $\quad P_{3\phi 1} = 61.615 \cdot kW$ $\qquad P_{3\phi 2} := 3 \cdot I_{\phi 2}^2 \cdot Re(Z_{Y\phi 2})$ $\quad P_{3\phi 2} = 108.562 \cdot kW$

$Q_{3\phi 1} := 3 \cdot I_{\phi 1}^2 \cdot Im(Z_{\phi 1})$ $\quad Q_{3\phi 1} = 46.212 \cdot kVAR$ $\qquad Q_{3\phi 2} := 3 \cdot I_{\phi 2}^2 \cdot Im(Z_{Y\phi 2})$ $\quad Q_{3\phi 2} = -39.513 \cdot kVAR$

d) Find the real and reactive power losses in the transmission line.

$P_{3\phi L} := 3 \cdot (|I_L|)^2 \cdot Re(Z_{line})$ $\quad P_{3\phi L} = 13.557 \cdot kW$ $\qquad Q_{3\phi L} := 3 \cdot (|I_L|)^2 \cdot Im(Z_{line})$ $\quad Q_{3\phi L} = 24.102 \cdot kVAR$

e) Find the real power, reactive power, and power factor supplied by the generator.

$P_{3\phi gen} := P_{3\phi L} + P_{3\phi 1} + P_{3\phi 2}$ $\quad P_{3\phi gen} = 183.734 \cdot kW$

$Q_{3\phi gen} := Q_{3\phi L} + Q_{3\phi 1} + Q_{3\phi 2}$ $\quad Q_{3\phi gen} = 30.801 \cdot kVAR$ $\qquad pf = \frac{P_{3\phi gen}}{3 \cdot |V_Y| \cdot |I_L|} = 0.986$ lagging

f) What is the efficiency of this system? $\quad \eta = \frac{P_{3\phi 1} + P_{3\phi 2}}{P_{3\phi gen}} = 92.621 \cdot \%$