ECE 3600 3-Phase Examples

A.Stolp
9/9/09
rev 9/5/20

Ex. 1 A Y-connected load is connected to 208-V, 3-phase.
It draws 1.2kW of power at a power factor of 75%, leading. $P_{3\phi} := 1.2 \cdot kW$ $\quad pf := 0.75$

a) Find the apparent power and the reactive power.

$S_{3\phi} := \dfrac{P_{3\phi}}{pf}$ $\quad S_{3\phi} = 1.6 \cdot kVA$ $\quad Q_{3\phi} := -\sqrt{S_{3\phi}^{\ 2} - P_{3\phi}^{\ 2}}$ $\quad Q_{3\phi} = -1.058 \cdot kVAR$

Negative because the power factor is leading.

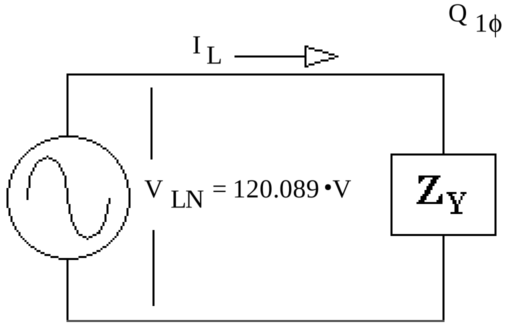
b) Find the line current.

Our Approach 1) Change all Δ-connected loads to equivalent Y-connected loads $\mathbf{Z_Y} = \dfrac{\mathbf{Z_\Delta}}{3}$ NOT NEEDED

2) Find all voltages as $V_{LN} := \dfrac{208 \cdot V}{\sqrt{3}}$ $\quad V_{LN} = 120.089 \cdot V$

3) Change all power numbers to 1ϕ. $P_{1\phi} := \dfrac{P_{3\phi}}{3}$ $\quad P_{1\phi} = 400 \cdot W$ $\quad S_{1\phi} := \dfrac{S_{3\phi}}{3}$ $\quad S_{1\phi} = 533.333 \cdot VA$

$Q_{1\phi} := \dfrac{Q_{3\phi}}{3}$ $\quad Q_{1\phi} = -352.767 \cdot VAR$



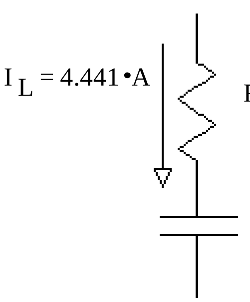
$S_{1\phi} = 533.333 \cdot VA$

$I_L := \dfrac{S_{1\phi}}{V_{LN}}$ $\quad I_L = 4.441 \cdot A$

c) Find the values of the load components, assuming they are connected in series.

The components must be a resistor and a capacitor because there is some real power and the power factor is leading.

assume $\omega = 377 \cdot \dfrac{rad}{sec}$



$R_L := \dfrac{P_{1\phi}}{I_L^{\ 2}}$ $\quad = \dfrac{P_{1\phi}}{I_L^{\ 2}} = 20.28 \cdot \Omega$

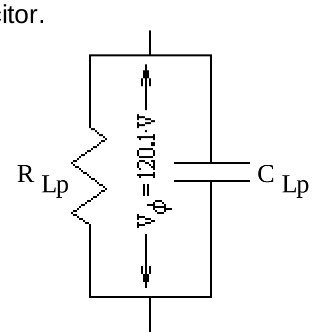
$X_C := \dfrac{Q_{1\phi}}{I_L^{\ 2}}$ $\quad X_C = -17.885 \cdot \Omega$ $\quad = -\dfrac{1}{\omega \cdot C_L}$ $\quad C_L := -\dfrac{1}{\omega \cdot X_C}$ $\quad C_L = 148.3 \cdot \mu F$

d) Find the values of the load components, assuming they are connected in parallel.

Still a resistor and a capacitor.

$R_{Lp} := \dfrac{V_{LN}^{\ 2}}{P_{1\phi}}$

$R_{Lp} = 36.053 \cdot \Omega$



$X_C := \dfrac{V_{LN}^{\ 2}}{Q_{1\phi}}$

$X_C = -40.881 \cdot \Omega$ $\quad = -\dfrac{1}{\omega \cdot C_{Lp}}$

$C_{Lp} := -\dfrac{1}{\omega \cdot X_C}$ $\quad C_{Lp} = 64.9 \cdot \mu F$

e) Correct the power factor with Y-connected components.
Need inductors

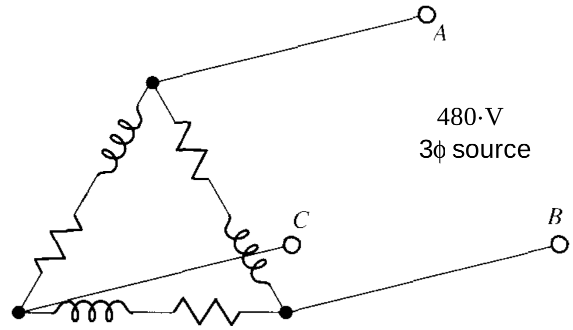
$$Q_{1\phi Ind} := -Q_{1\phi} = \frac{V_\phi^{\,2}}{\omega \cdot L_Y} \qquad L_Y := \frac{V_\phi^{\,2}}{\omega \cdot -Q_{1\phi}} \qquad L_Y = 325.3 \cdot mH$$

f) Correct the power factor with Δ-connected components.

$$L_\Delta := \frac{\left(\sqrt{3} \cdot V_\phi\right)^2}{\omega \cdot -Q_{1\phi}} \qquad L_\Delta = 975.9 \cdot mH$$

OR $\omega \cdot L_\Delta = \mathbf{Z_\Delta} = 3 \cdot \mathbf{Z_y} = 3 \cdot \omega \cdot L_Y \qquad 3 \cdot L_Y = 975.9 \cdot mH$

Ex. 2 From F08, exam 1, Find the following:



All $\mathbf{Z} := (30 + 12 \cdot j) \cdot \Omega$

a) The line current that would be measured by an ammeter.

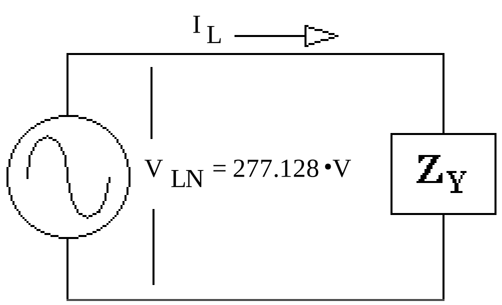
$V_{LL} := 480 \cdot V \qquad \mathbf{Z_\Delta} := (30 + 12 \cdot j) \cdot \Omega$

Our Approach
1) Change all Δ-connected loads to equivalent Y-connected loads

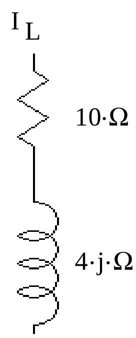
$$\mathbf{Z_Y} := \frac{\mathbf{Z_\Delta}}{3} \qquad \mathbf{Z_Y} = 10 + 4j \ \cdot \Omega$$

2) Find all voltages as V_{LN} $\quad V_{LL} = 480 \cdot V \quad V_{LN} := \frac{V_{LL}}{\sqrt{3}} \quad V_{LN} = 277.128 \cdot V$

3) Change all power numbers to 1φ. NOT NEEDED

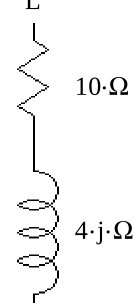


$\mathbf{Z_Y} = 10 + 4j \ \cdot \Omega$ =



$$I_L := \frac{V_{LN}}{\left|\mathbf{Z_Y}\right|} = \frac{277.128 \cdot V}{\sqrt{10^2 + 4^2} \cdot \Omega} = 25.731 \cdot A$$

b) The power consumed by the three-phase load.



$$P_{1\phi} = I_L^{\,2} \cdot 10 \cdot \Omega = 6.62 \cdot kW$$

$$P_{3\phi} = 3 \cdot \left(I_L^{\,2} \cdot 10 \cdot \Omega\right) = 19.86 \cdot kW$$

c) The value of Y-connected impedances that would result in exactly the same line currents and same pf.

$$\mathbf{Z_Y} = 10 + 4j \ \cdot \Omega$$

d) The value of Y-connected capacitors that would correct the pf.

$$Q_{1\phi} := \sqrt{S_{1\phi}^{\,2} - P_{1\phi}^{\,2}} \qquad Q_{1\phi} := \sqrt{\left(V_{LN} \cdot I_L\right)^2 - (6.62 \cdot kW)^2} \qquad Q_{1\phi} = 2.65 \cdot kVAR$$

so we need:

$$Q_C := -Q_{1\phi} \qquad Q_C = -2.65 \cdot kVAR = -\frac{V_{LN}^{\,2}}{\left(\frac{1}{\omega \cdot C}\right)} = -V_{LN}^{\,2} \cdot \omega \cdot C \qquad C := \frac{Q_C}{-V_{LN}^{\,2} \cdot \omega} \qquad C = 91.5 \cdot \mu F$$

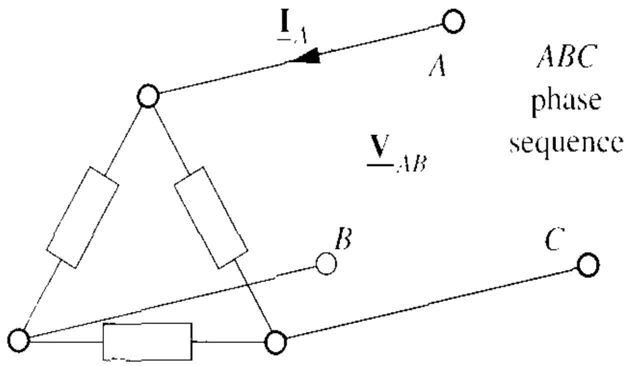


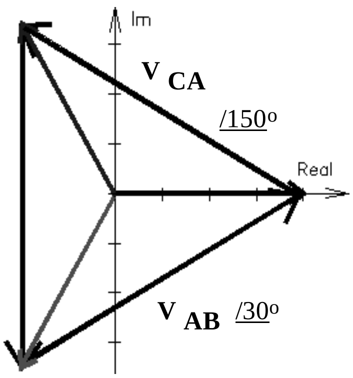
Figure P1.7

Ex. 3 For the three-phase delta-connected load in fig P1 .7,
The line-to-line voltage and line current are:

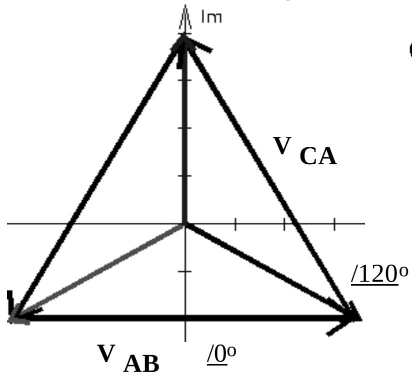
$\mathbf{V}_{AB} := 480 \cdot V \ \underline{/0^o}$ $\qquad \mathbf{I_A} = 10A\underline{/-40^o}$

a) What is $\mathbf{V_{CA}}$?

Normal phase angles



Rotate CW 30 deg



$\mathbf{V}_{CA} := 480 \cdot V \ \underline{/120^o}$

$= 480 \cdot V \ \underline{/-240^o}$

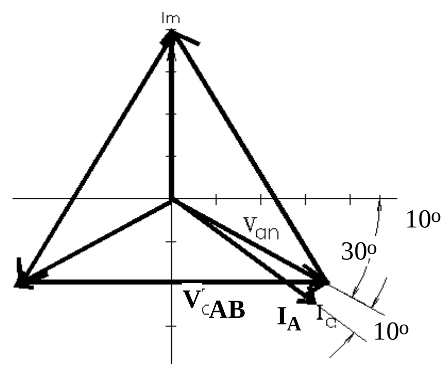
b) What is the phase current in the load?

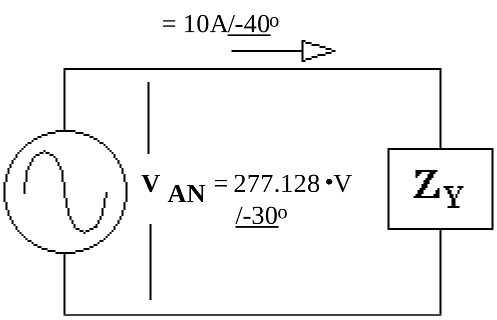
$$I_{LL} = \frac{I_L}{\sqrt{3}} \qquad \frac{10 \cdot A}{\sqrt{3}} = 5.774 \bullet A$$

c) What is the time-average power into the load?

$\mathbf{V}_{AN} := \frac{480 \cdot V}{\sqrt{3}} \ \underline{/-30^o}$ Since $\mathbf{I_A} = 10A\underline{/-40^o}$ $\qquad$ **I** lags **V** by 10^o

$\theta := 10 \cdot deg$





$$P_{1\phi} = (277.128 \cdot V \cdot 10 \cdot A) \cdot \cos(\theta) = 2.729 \bullet kW$$

$$P_{3\phi} = 3 \cdot (277.128 \cdot V \cdot 10 \cdot A) \cdot \cos(\theta) = 8.188 \bullet kW$$

d) What is the phase impedance?

$\mathbf{Z}_Y := \frac{277.128 \cdot V}{10 \cdot A} \ \underline{/ -30 - (-40)^o}$ $\qquad \mathbf{Z}_Y = 27.71 \bullet \Omega \ \underline{/10^o}$

$\mathbf{Z}_\Delta = 3 \cdot \mathbf{Z}_Y = 83.14 \bullet \Omega \ \underline{/10^o}$

Ex. 4 In the three-phase circuit shown in Fig. P1.9. find the following:

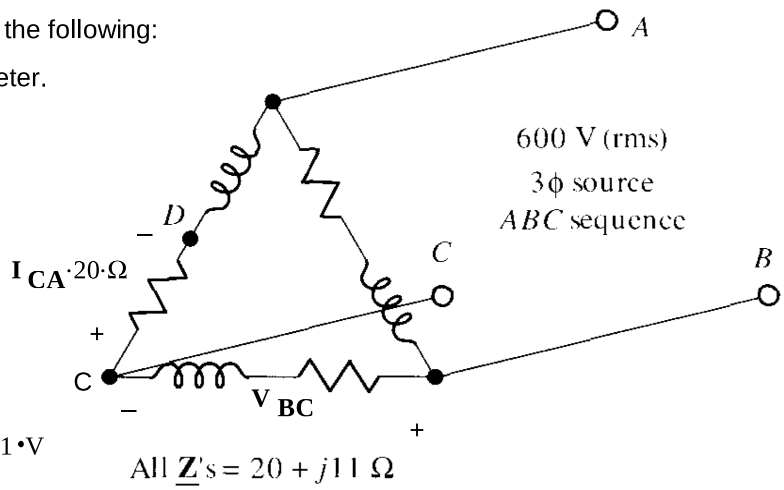


Figure P1.9

a) The line current that would be measured by an ammeter.

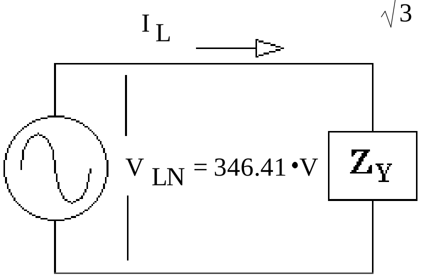
Direct way

$V_{LL} := 600 \cdot V$ $\qquad$ $\mathbf{Z}_{\Delta} := (20 + 11 \cdot j) \cdot \Omega$

$I_{AB} := \left| \frac{V_{LL}}{\mathbf{Z}_{\Delta}} \right|$ $\qquad$ $I_{AB} = 26.286 \cdot A$

$I_A := \sqrt{3} \cdot I_{AB}$ $\qquad$ $I_A = 45.53 \cdot A$

Our Approach $\qquad$ $V_{LN} := \frac{600 \cdot V}{\sqrt{3}}$ $\qquad$ $V_{LN} = 346.41 \cdot V$

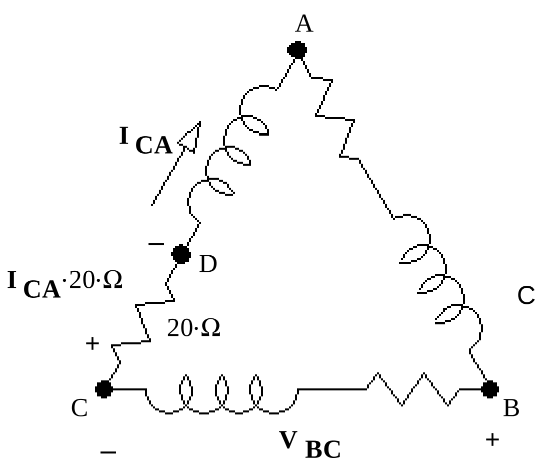


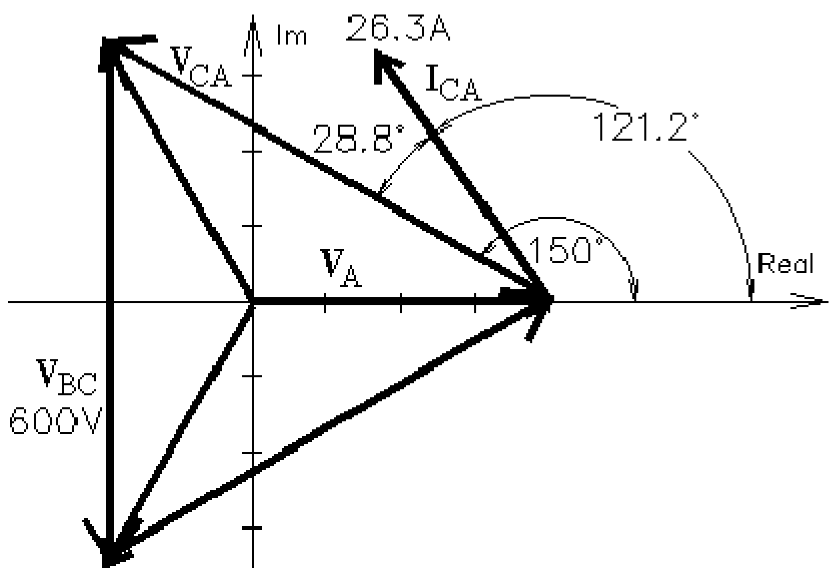
$\mathbf{Z}_Y := \frac{\mathbf{Z}_{\Delta}}{3}$ $\qquad$ $\mathbf{Z}_Y = 6.667 + 3.667j \cdot \Omega$

$I_L := \frac{V_{LN}}{|\mathbf{Z}_Y|} = \frac{346.41 \cdot V}{\sqrt{6.667^2 + 3.667^2}}$ $\qquad$ $I_L = 45.53 \cdot A$

b) The power factor of the three-phase load. $\qquad$ $\theta := \text{atan}\left(\frac{11}{20}\right)$ $\qquad$ $\theta = 28.811 \cdot \text{deg}$ $\qquad$ pf $\quad \cos(\theta) = 0.876$

c) The voltage that would be measured between B and D by a voltmeter.





Using $\mathbf{V}_A$ as reference (0º):

$\mathbf{V}_{BC} := 600 \cdot V \cdot e^{-j \cdot 90 \cdot \text{deg}}$

$\mathbf{I}_{CA} := 26.286 \cdot A \cdot e^{j \cdot (150 - 28.811) \cdot \text{deg}}$

$\mathbf{V}_{CD} := \mathbf{I}_{CA} \cdot 20 \cdot \Omega$

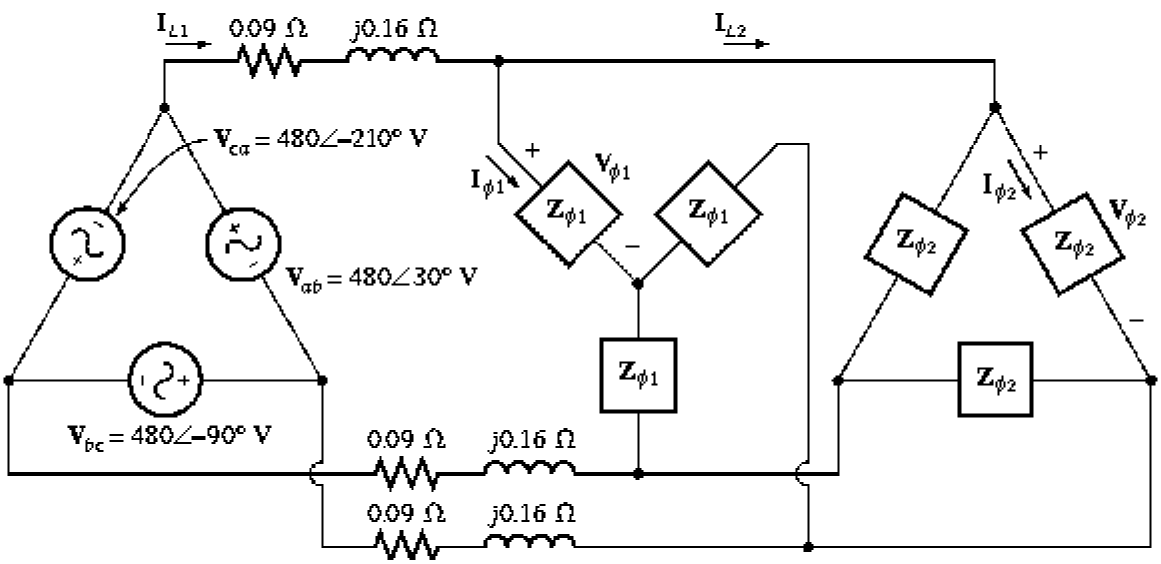
$\mathbf{V}_{CD} = -272.251 + 449.734j \cdot V$

$\mathbf{V}_{BD} := \mathbf{V}_{BC} + \mathbf{V}_{CD}$ $\qquad$ $\mathbf{V}_{BD} = -272.251 - 150.266j \cdot V$ $\qquad$ $|\mathbf{V}_{BD}| = 311 \cdot V$

(must be the sum, NOT the difference, see the + and - signs on the drawing.)

Ex. 5 When all you have is impedances and an input voltage, it gets messy & luckily, it's not a common problem.

Textbook problem 2-2. Figure P2-1 shows a three-phase power system with two loads. The Δ-connected generator is producing a line voltage of 480 V, and the line impedance is 0.09 + j0.16 Ω. Load 1 is Y-connected, with a phase impedance of 2.5Ω /36.87° and load 2 is Δ-connected, with a phase impedance of 5Ω /-20° .



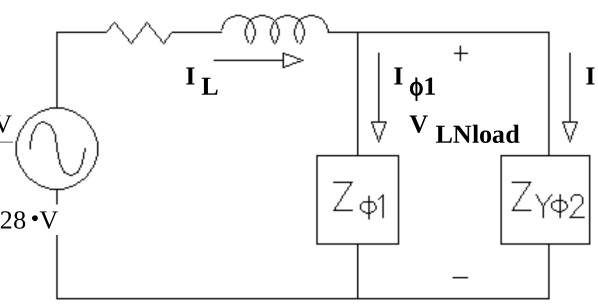
$$Z_{\phi1} := 2.5\cdot e^{j\cdot 36.87\cdot deg}\cdot\Omega \qquad Z_{\phi2} := 5\cdot e^{-j\cdot 20\cdot deg}\cdot\Omega$$

a) What is the line voltage at the two loads?

Find an equivalent Y-only circuit: $Z_{line} := (0.09 + 0.16\cdot j)\cdot\Omega$

$V_Y := \dfrac{480\cdot V}{\sqrt{3}}$

$V_Y = 277.128\cdot V$



$Z_{Y\phi2} := \dfrac{Z_{\phi2}}{3}$

$Z_{Y\phi2} = 1.566 - 0.57j\ \cdot\Omega$

$$Z_{Yloads} := \frac{1}{\dfrac{1}{Z_{\phi1}} + \dfrac{1}{Z_{Y\phi2}}} \qquad Z_{Yloads} = 1.13 + 0.044j\ \cdot\Omega \qquad |Z_{Yloads}| = 1.131\cdot\Omega \quad \arg(Z_{Yloads}) = 2.254\cdot deg$$

$$Z_{Ytot} := Z_{line} + Z_{Yloads} \qquad Z_{Ytot} = 1.22 + 0.204j\ \cdot\Omega \qquad |Z_{Ytot}| = 1.237\cdot\Omega \quad \arg(Z_{Ytot}) = 9.516\cdot deg$$

$$I_L := \frac{V_Y}{Z_{Ytot}} \qquad I_L = 220.998 - 37.047j\ \cdot A \qquad |I_L| = 224.082\cdot A \quad \arg(I_L) = -9.516\cdot deg$$

$$V_{LNload} := I_L\cdot Z_{Yloads} \qquad V_{LNload} = 251.311 - 32.025j\ \cdot V \qquad |V_{LNload}| = 253.343\cdot V \quad \arg(V_{LNload}) = -7.262\cdot deg$$

$$V_{Lload} := V_{LNload}\cdot\sqrt{3} \qquad V_{Lload} = 435.283 - 55.47j\ \cdot V \qquad |V_{Lload}| = 438.803\cdot V$$

b) What is the voltage drop on the transmission lines?

$\mathbf{V}_{\mathbf{linedrop}} := \mathbf{I}_{\mathbf{L}} \cdot \mathbf{Z}_{\mathbf{line}}$ $\mathbf{V}_{\mathbf{linedrop}} = 25.817 + 32.025j \cdot V$ $|\mathbf{V}_{\mathbf{linedrop}}| = 41.136 \cdot V$

$\arg(\mathbf{V}_{\mathbf{linedrop}}) = 51.126 \cdot deg$

Check: $\mathbf{V}_{\mathbf{Y}} - \mathbf{V}_{\mathbf{LNload}} = 25.817 + 32.025j \cdot V$

c) Find the real and reactive powers supplied to each load.

$I_{\phi 1} := \dfrac{|\mathbf{V}_{\mathbf{LNload}}|}{|\mathbf{Z}_{\boldsymbol{\phi}\mathbf{1}}|}$ $I_{\phi 1} = 101.337 \cdot A$ $I_{L2} := \dfrac{|\mathbf{V}_{\mathbf{LNload}}|}{|\mathbf{Z}_{\mathbf{Y}\boldsymbol{\phi}\mathbf{2}}|}$ $I_{L2} = 152.006 \cdot A$

$P_{3\phi 1} := 3 \cdot I_{\phi 1}^{2} \cdot \mathrm{Re}(\mathbf{Z}_{\boldsymbol{\phi}\mathbf{1}})$ $P_{3\phi 1} = 61.615 \cdot kW$ $P_{3\phi 2} := 3 \cdot I_{L2}^{2} \cdot \mathrm{Re}(\mathbf{Z}_{\mathbf{Y}\boldsymbol{\phi}\mathbf{2}})$ $P_{3\phi 2} = 108.562 \cdot kW$

$Q_{3\phi 1} := 3 \cdot I_{\phi 1}^{2} \cdot \mathrm{Im}(\mathbf{Z}_{\boldsymbol{\phi}\mathbf{1}})$ $Q_{3\phi 1} = 46.212 \cdot kVAR$ $Q_{3\phi 2} := 3 \cdot I_{L2}^{2} \cdot \mathrm{Im}(\mathbf{Z}_{\mathbf{Y}\boldsymbol{\phi}\mathbf{2}})$ $Q_{3\phi 2} = -39.513 \cdot kVAR$

d) Find the real and reactive power losses in the transmission line.

$P_{3\phi L} := 3 \cdot (|\mathbf{I}_{\mathbf{L}}|)^{2} \cdot \mathrm{Re}(\mathbf{Z}_{\mathbf{line}})$ $P_{3\phi L} = 13.557 \cdot kW$

$Q_{3\phi L} := 3 \cdot (|\mathbf{I}_{\mathbf{L}}|)^{2} \cdot \mathrm{Im}(\mathbf{Z}_{\mathbf{line}})$ $Q_{3\phi L} = 24.102 \cdot kVAR$

e) Find the real power, reactive power, and power factor supplied by the generator.

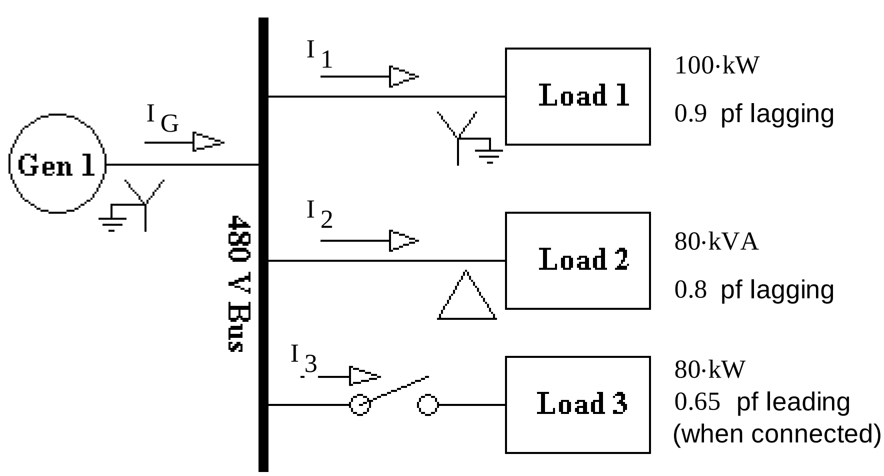
$P_{3\phi gen} := P_{3\phi L} + P_{3\phi 1} + P_{3\phi 2}$ $P_{3\phi gen} = 183.734 \cdot kW$

$Q_{3\phi gen} := Q_{3\phi L} + Q_{3\phi 1} + Q_{3\phi 2}$ $Q_{3\phi gen} = 30.801 \cdot kVAR$ $pf = \dfrac{P_{3\phi gen}}{3 \cdot |\mathbf{V}_{\mathbf{Y}}| \cdot |\mathbf{I}_{\mathbf{L}}|} = 0.986$ lagging

f) What is the efficiency of this system? $\eta = \dfrac{P_{3\phi 1} + P_{3\phi 2}}{P_{3\phi gen}} = 92.621 \cdot \%$

The next example uses a "one-line diagram" to show how a generator is connected to 3 loads. In these diagrams, one line represents all 3 phases and neutral. Because the individual lines are not shown, there may be notes or symbols to indicate Y or Δ connections. All powers given will be 3-phase values, all voltages will be line voltages (that is line-to-line) and all currents will line currents. The term "bus" refers common connection area.

Ex. 6 The one-line diagram below shows a single, Y-connected generator and 3 loads. Assume all lines are lossless.



Find:

a) The phase voltage and currents in Load 1.

$V_{LL} := 480 \cdot V$ $\quad V_{LN} := \frac{V_{LL}}{\sqrt{3}}$ $\quad V_{LN} = 277.128 \cdot V = V_{L1\phi}$

$pf_{L1} := 0.9$ $\quad S_{L1.1\phi} := \frac{100 \cdot kW}{3 \cdot pf_{L1}}$ $\quad I_1 := \frac{S_{L1.1\phi}}{V_{LN}}$ $\quad I_1 = 133.646 \cdot A = I_{L1\phi}$

b) The phase voltage and currents in Load 2.

$V_{LL} := 480 \cdot V = V_{L2\phi}$ $\quad pf_{L2} := 0.8$ $\quad S_{L2.1\phi} := \frac{80 \cdot kVA}{3}$

$I_2 := \frac{S_{L2.1\phi}}{V_{LN}}$ $\quad I_2 = 96.225 \cdot A = \sqrt{3} \cdot I_{L2\phi}$ $\quad I_{L2\phi} = \frac{I_2}{\sqrt{3}} = 55.556 \cdot A$

c) The real, reactive and apparent power supplied by the generator with the switch to load 3 open.

$P_1 := 100 \cdot kW$ $\quad P_2 := 80 \cdot kVA \cdot pf_{L2}$ $\quad P_2 = 64 \cdot kW$ $\quad P_G := P_1 + P_2$ $\quad P_G = 164 \cdot kW$

$Q_1 := \sqrt{\left(\frac{100 \cdot kW}{pf_{L1}}\right)^2 - (100 \cdot kW)^2}$ $\quad Q_1 = 48.432 \cdot kVAR$ $\quad Q_2 := \sqrt{(80 \cdot kVA)^2 - (64 \cdot kW)^2}$ $\quad Q_1 = 48.432 \cdot kVAR$

$Q_G := Q_1 + Q_2$ $\quad Q_G = 96.432 \cdot kVAR$

$S_G := \sqrt{P_G^{\,2} + Q_G^{\,2}}$ $\quad S_G = 190.25 \cdot kVAR$

d) The total line current from the generator, I_G, with the switch to load 3 open. $\quad I_G = \frac{\left(\frac{S_G}{3}\right)}{V_{LN}} = 228.836 \cdot A$

e) The real, reactive and apparent power supplied by the generator with the switch to load 3 closed.

$pf_{L3} := 0.65$ $\quad S_{L3.1\phi} := \frac{80 \cdot kW}{3 \cdot pf_{L3}}$ $\quad Q_3 := -\sqrt{\left(\frac{80 \cdot kW}{pf_{L3}}\right)^2 - (80 \cdot kW)^2}$ $\quad Q_3 = -93.53 \cdot kVAR$

$P_G := P_1 + P_2 + 80 \cdot kW$ $\quad P_G = 244 \cdot kW$ $\quad Q_G := Q_1 + Q_2 + Q_3$ $\quad Q_G = 2.902 \cdot kVAR$

$S_G := \sqrt{P_G^{\,2} + Q_G^{\,2}}$ $\quad S_G = 244.017 \cdot kVAR$

f) How does the total line apparent power from the generator, S_G, compare to the sum of the three individual apparent powers, $S_1 + S_2 + S_3$? If they aren't equal, why not? (Switch closed)

$3 \cdot S_{L1.1\phi} + 80 \cdot kVAR + 3 \cdot S_{L3.1\phi} = 314.188 \cdot kVAR \neq S_G = 244.017 \cdot kVAR$

Can't Add Magnitudes

g) The total line current from the generator, I_G, with the switch to load 3 closed.

$I_G := \frac{\left(\frac{S_G}{3}\right)}{V_{LN}}$ $\quad I_G = 293.507 \cdot A$

h) How does the total line current from the generator, I_G, compare to the sum of the three individual currents, $I_1 + I_2 + I_3$? If they aren't equal, why not? (Switch closed)

$I_3 := \frac{S_{L3.1\phi}}{V_{LN}}$ $\quad I_3 = 148.039 \cdot A$

$I_1 + I_2 + I_3 = 377.909 \cdot A \neq I_G = 293.507 \cdot A$

Can't Add Magnitudes