SOLUTIONS

NAME and UID:

ECE 3530 Midterm 1
Show your work.
No credit will be given for the correct answer if no work is shown.
Four questions each worth 25 points.
Closed book, limited notes (1 regular size sheet front&back). No laptops.

1. (a) User IDs on an old computer system consist of one letter from the English alphabet followed by a single digit between 0 and 9. The English alphabet contains 26 letters. How many IDs are possible?

   (b) There are 4 users in total of this computer system. Their names are Mike, Bryce, Chris and Kate. What is the probability that Mike has the ID *X1* and Kate has the ID *W3*? Note that two people can not have the same ID. Also all IDs are equally likely to be used.

a) $n_1 = 26$ $n_2 = 10$ $n_1 n_2 = 260$

b) Sample space: ${}_{260}P_4$

event: ${}_{258}P_2$ (2 to pick IDs from 258)

$$\text{probability} = \frac{{}_{258}P_2}{{}_{260}P_4} = \frac{258!}{256!} \cdot \frac{256!}{260!}$$

$$= \frac{1}{260 \times 259} = \frac{1}{67340}$$

2. Consider all households in Utah. Lets define the following events:

- A: the household has an internet connection
- B: the house is a 2-story building
- C: the house has greater than or equal to 3 bedrooms

We are also told the following: The probability that a household in Utah does NOT have an internet connection is 0.1. The probability that a house in Utah is a 2-story building is 0.3. The probability that a house in Utah is a 2-story building and has greater than or equal to 3 bedrooms at the same time is 0.26. Events A,B are independent.

Based on this answer the following questions:

(a) Compute $P(A \cup B)$.

(b) What is the probability that a house in Utah is a 2-story building but has less than 3 bedrooms?

given: $P(A') = 0.1$ $\quad P(B) = 0.3$ $\quad P(B \cap C) = 0.26$

A, B independent

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A) = 1 - P(A') = 1 - 0.1 = 0.9$

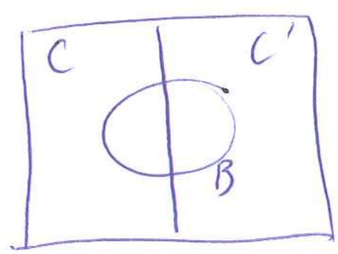
$P(A \cap B) = P(A)P(B)$ using independence

So $P(A \cup B) = 0.9 + 0.3 - 0.9 \times 0.3$

$= 0.93$

b) Asking for $P(B \cap C')$

using law of total probability



$P(B) = P(B \cap C) + P(B \cap C')$

$P(B) - P(B \cap C) = P(B \cap C')$

$P(B \cap C') = 0.3 - 0.26 = 0.04$

3. We have a deck of 13 cards: One of each of the numbers 1 through 10 and Jack, Queen and King.

   (a) If I draw 4 cards from this deck with replacement, what is the probability that I will get the cards 1,2,1,2 in that order?

   (b) I draw 4 cards from this deck without replacement. What is the probability that my hand contains the King and the Queen or my hand contains the King and the Jack. Note that if my hand has the King, the Queen and the Jack, this is also included in this event defined in the previous statement.

a) $\left(\frac{1}{13}\right)^4 = \frac{1}{28561}$

b) Let X = hand has K, Q

Y = " " K, J

Asking $P(X \cup Y)$

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

$$P(X) = \frac{{}_{11}C_2}{{}_{13}C_4} = \frac{11!}{9!\,2!} \cdot \frac{9!\,4!}{13!} = \frac{4 \times 3 \times 2 \times 1}{13 \times 12 \times 2} = 1/13$$

$$P(Y) = \frac{{}_{11}C_2}{{}_{13}C_4} = 1/13$$

$X \cap Y$ = hand has K, Q, J

$$P(X \cap Y) = \frac{{}_{10}C_1}{{}_{13}C_4} = \frac{10!}{9!\,1!} \cdot \frac{9!\,4!}{13!} = \frac{4 \times 3 \times 2 \times 1}{13 \times 12 \times 11} = 2/13 \times 11$$

$$P(X \cup Y) = \frac{1}{13} + \frac{1}{13} - \frac{2}{13 \times 11}$$

$$= \frac{2 \times 11 - 2}{13 \times 11} = 20/13 \times 11 = 20/143$$

4. A team of explorers climbing Mount Everest have 2 identical looking satellite radios. We will call them radio A and radio B. Lets define the events:

- X: The temperature on Mountain Everest is below 0 degrees Fahrenheit,
- Y: The temperature on Mountain Everest is between 0 degrees Fahrenheit and 32 degrees Fahrenheit,
- Z: The temperature on Mountain Everest is above 32 degrees Fahrenheit.

We are given the probabilities:

$$P(X) = 0.4 \quad P(Y) = 0.4 \quad P(Z) = 0.2$$

Radio A is temperature sensitive and works:

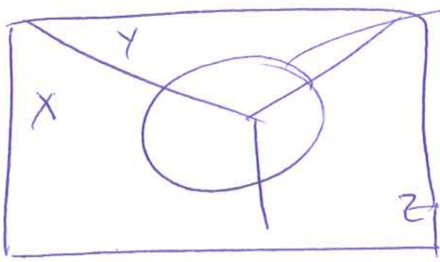
- %50 of the time given that the temperature is below 0 degrees Fahrenheit.
- %80 of the time given that the temperature is between 0 degrees Fahrenheit and 32 degrees Fahrenheit.
- %100 of the time given that the temperature is above 32 degrees Fahrenheit.

Radio B is insensitive to the temperature and works %90 of the time regardless of the temperature.

(a) What is the probability that radio A works at a given moment?
Hint: This part of the question has nothing to do with Radio B.

(b) On a really cold day with the temperature at negative 25 degrees Fahrenheit, I pick one of the two radios with equal probability and find that it doesn't work. What is the probability that I picked radio A?

a)



→ Radio A works call this event W

$$P(W) = P(X \cap W) + P(Y \cap W) + P(Z \cap W)$$

$$P(W) = P(W|X)P(X) + P(W|Y)P(Y) + P(W|Z)P(Z)$$

$$= 0.5 \times 0.4 + 0.8 \times 0.4 + 1 \times 0.2$$

$$= 0.2 + 0.32 + 0.2 = 0.72$$

b)

1/2 → A: 0.5 works, 0.5 does not
1/2 → B: 0.9 works, 0.1 does not

$$P(A | \text{does not work}) = \frac{P(\text{does not work} | A)\, P(A)}{P(\text{doesn't work} | A)\, P(A) + P(\text{doesn't work} | B)\, P(B)}$$

$$P(A \mid \text{does not work}) = \frac{0.5 \times 0.5}{0.5 \times 0.5 + 0.1 \times 0.5} = \frac{0.5}{0.6}$$

$$= 5/6$$