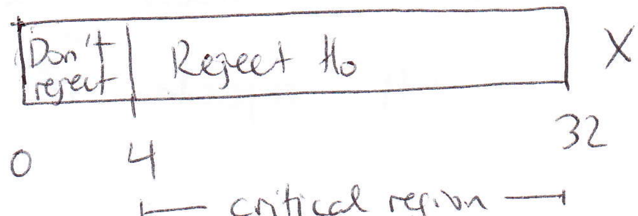
HW 9 Solutions

1. a) $H_0 : p = 0.05$

$H_1 : p > 0.05$



b) $\alpha = P(X \geq 4 \text{ when } p = 0.05)$

$$= \sum_{x=4}^{32} b(x; n=32, p=0.05) = 0.0738$$

c) Testing against $p = 0.15$

$\beta = P(X < 4 \text{ when } p = 0.15)$

$$= \sum_{x=0}^{3} b(x; n=32, p=0.15) = 0.2721$$

Testing against $p = 0.2$

$\beta = P(X < 4 \text{ when } p = 0.2)$

$$= \sum_{x=0}^{3} b(x; n=32, p=0.2) = 0.0931$$

d) $\alpha = P(X \geq 30 \text{ when } p = 0.05)$

$$\approx P\left(Z \geq \frac{30 - np}{\sqrt{np(1-p)}}\right) = P\left(Z \geq \frac{30 - 400 \times 0.05}{\sqrt{400 \times 0.05 \times 0.95}}\right)$$

$$= P(Z \geq 2.29) = 1 - P(Z < 2.29)$$

$$= 1 - 0.989 = 0.011$$

Table A3

(2) a) $H_0: \mu = 352$ $H_1: \mu > 352$

b) σ known $\longrightarrow$ use standard normal distribution.

H_0 rejected if $\bar{x} > 352 + z_\alpha \frac{\sigma}{\sqrt{n}}$ [1-tailed test]

$\alpha = 0.01$ closest value to $1-\alpha = 0.99$ is for

$z_\alpha = 2.33$

Reject H_0 if $\bar{x} > 352 + 2.33 \frac{28}{\sqrt{49}}$

Critical region: $\bar{x} > 361.32$

Since $\bar{x} = 360$ Do not reject H_0.

c) $\alpha = 0.001$ will result in a smaller critical region (larger critical value in this case).

So if we can't reject H_0 at $\alpha = 0.01$, we won't reject it at $\alpha = 0.001$.

d) $\beta = P(\bar{X} < 361.32 \text{ when } \mu = 368)$

$= P\left(z < \frac{361.32 - \mu}{\sigma/\sqrt{n}}\right) = P\left(z < \frac{361.32 - 368}{28/\sqrt{49}}\right)$

$= P(z < -1.67) = 0.0475$

③ a) $H_0 = \mu = 50 \quad H_1 = \mu \neq 50$

b) Since σ unknown we will use the t-distribution.

2 tailed test so H_0 rejected if

$$\bar{x} < \mu_0 - t_{\alpha/2} \frac{s}{\sqrt{n}} \quad \text{or} \quad \bar{x} > \mu_0 + t_{\alpha/2} \frac{s}{\sqrt{n}}$$

$\alpha = 0.01 \qquad v = 16-1 = 15 \qquad t_{0.005} = 2.947$

$$t_{\alpha/2} \frac{s}{\sqrt{n}} = 2.947 \cdot \frac{5}{\sqrt{9}} = 4.91$$

Reject H_0 if $\bar{x} < 50 - 4.91 = 45.09$

or $\bar{x} > 50 + 4.91 = 54.91$

Reject	Don't reject	Reject
	45.09 ... 54.91	

Since $\bar{x} = 45.4$, it is not rejected.

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④ a) $H_0 = \mu = 20 \quad H_1 = \mu < 20$

Since the pop. standard dev is unknown, we will use t-values. Also, this is a 1-tailed test

Reject H_0 if $\bar{x} < \mu_0 - t_{\alpha} \frac{s}{\sqrt{n}}$

$\alpha = 0.05 \quad v = n-1 = 24 \quad t_{0.05} = 1.711$ (Table A4)

Reject H_0 if $\quad \bar{x} < 20 - 1.711 \dfrac{3}{\sqrt{25}} = 18.97$

since $\bar{x} = 18.8$ Reject H_0.

b) Decrease the critical value. P(Type I error) is the prob. of rejecting H_0 when H_0 is true. It can be decreased by decreasing the size of the critical region. Decreasing the size of the critical region corresponds to decreasing the critical value in this problem since we reject H_0 when $\bar{x}$ is less than the critical value.

c) $$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

$s = 3 \longrightarrow s^2 = 9 \qquad v = n-1 = 24$

$\alpha = \cancel{0.05}\ 0.1\ \cancel{\alpha/2 = 0.025}$

$\chi^2_{0.05} = 36.415 \qquad \chi^2_{0.95} = 13.848$ Table A5

$$\frac{24 \times 9}{36.415} < \sigma^2 < \frac{24 \times 9}{13.848}$$

$$5.93 < \sigma^2 < 15.6$$

(5) a) $\bar{X} = \frac{1}{7}\sum_{i=1}^{7} X_i = 380$ $\bar{Y} = \frac{1}{7}\sum_{i=1}^{7} y_i = 49.54$

$$S_{xx} = \sum_{i=1}^{7} (X_i - 380)^2 = 2800$$

$$S_{xy} = \sum_{i=1}^{7} (X_i - 380)(y_i - 49.54) = 1774$$

$$b = \frac{S_{xy}}{S_{xx}} = \frac{1774}{2800} = 0.6336$$

$$a = \bar{y} - b\bar{x} = -191.2143$$

so $\boxed{\hat{y} = -191.2143 + 0.6336\,x}$

b) $x = 430 \Rightarrow$ $\hat{y} = -191.2143 + 0.6336 \times 430$
$= \boxed{81.2337}$

c) $b - t_{\alpha/2} \frac{s}{\sqrt{S_{xx}}} < \beta < b + t_{\alpha/2} \frac{s}{\sqrt{S_{xx}}}$

$v = n - 2 = 5$, $\alpha = 0.05$ $\Rightarrow t_{0.025} = 2.571$

next we need s

$$s^2 = \frac{S_{yy} - b S_{xy}}{n-2} \quad (*)$$

$$S_{yy} = \sum_{i=1}^{n} (y_i - \bar{y})^2 = \sum_{i=1}^{7} (y_i - 49.54)^2 = 1132.9$$

substitute into $(*)$ $s^2 = \frac{1132.9 - 0.6336 \times 1774}{5} = 1.79$

$$s = \sqrt{1.7963} = 1.3403$$

$$0.6336 - 2.571 \times \frac{1.3403}{\sqrt{2800}} < \beta < 0.6336 + 2.571 \times \frac{1.3403}{\sqrt{2800}}$$

$$\boxed{0.5684 < \beta < 0.6987}$$

d) $$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{S_{yy} - bS_{xy}}{S_{yy}}$$

$$= 1 - \frac{1132.9 - 0.6336 \times 1774}{1132.9}$$

$$= \boxed{0.9921}$$ very high quality fit.