Homework 8 Solutions

1. a) $P(T < 2.365)$ $v = 7$

Look for 2.365 along row $v=7$ for t-distribution
When you find the correct column, look at α value
at the top. $\alpha = 0.025$

$2.365 = t_{0.025}$

$P(T > 2.365) = 0.025$

$P(T < 2.365) = 1 - 0.025 = 0.975$

b) $P(T > 1.318)$ $v = 24$

$\alpha = 0.1$ $P(T > 1.318) = 0.1$

c) $P(-1.356 < T < 2.179)$ $v = 12$

$P(T > 2.179) = 0.025$ (row $v=12$)

$P(T < 2.179) = 1 - 0.025 = 0.975$

$P(T < -1.356)$ can't find directly in the table, so use symmetry

$P(T < -1.356) = P(T > 1.356)$
$= 0.1$

$P(-1.356 < T < 2.179) =$

$P(T < 2.179) - P(T < -1.356) = 0.975 - 0.1 = 0.875$

d) $P(T > -2.567)$ $\quad v = 17$

$P(T < -2.567) = P(T > 2.567)$

$= 0.01$

$P(T > -2.567) = 1 - P(T < -2.567)$

$= 1 - 0.01 = 0.99$

---

2 a) Known pop. variance σ^2 so use standard normal dist (z)

$\sigma = 40$, $n = 64$, $\bar{x} = 450$

$1 - \alpha = 0.97 \Rightarrow \alpha = 0.03$

2-sided confidence interval $\frac{\alpha}{2} = 0.015$

$z_{0.015}$ = the critical vale leaving an area of 0.015 to the right

But Table A.3 lists areas to the left of critical values

Way 1

Look for $1 - 0.015 = 0.985$

$P(z < 2.17) = 0.985$

$z_{0.015} = 2.17$

Way 2

Look for 0.015

$P(z < -2.17) = 0.985$

use symmetry

$z_{\alpha/2} = -z_{\alpha/2}$

$= -(-2.17) = 2.17$

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$450 - 2.17\frac{40}{\sqrt{64}} < \mu < \bar{x} + 2.17\frac{40}{\sqrt{64}}$$

$450 - 10.85 < \mu < 450 + 10.85$

$\boxed{439.15 < \mu < 460.85}$ 97% confidence interval

b) Increase sample size n

c) $1 - \alpha = 0.99 \quad \alpha = 0.01$

$P(Z < 2.33) = 0.9901$ (closest entry to 0.99)

$z_{0.01} = 2.33$

$\mu > \bar{x} - z_{0.01} \frac{\sigma}{\sqrt{n}} = 450 - 2.33 \frac{40}{\sqrt{64}}$

$\boxed{\mu > 438.35}$ 99% lower bound

---

(3) Pop. variance σ^2 unknown $\rightarrow$ use t-distribution (ok since pop. distribution is approximately normal)

a) $S = 35, \quad \bar{x} = 500, \; n = 25 \rightarrow v = 24$

$1 - \alpha = 0.95 \quad \alpha = 0.05$

$t_{\alpha/2} = t_{0.025} = 2.064$ row $v = 24$

$\bar{x} - t_{\alpha/2} \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2} \frac{s}{\sqrt{n}}$

$500 - 2.064 \frac{35}{\sqrt{25}} < \mu < 500 + 2.064 \frac{35}{\sqrt{25}}$

$\boxed{485.552 < \mu < 514.448}$ 95% confidence interval

b) Wider

c) $1-\alpha = 0.9$ $\alpha = 0.1$

$t_\alpha = t_{0.1} = 1.318$ row $v = 24$

$$\mu > \bar{x} - t_\alpha \frac{s}{\sqrt{n}} = 500 - 1.318 \frac{35}{\sqrt{25}}$$

$\boxed{\mu > 490.774}$ 90% lower bound

4 $n = \left(\frac{z_{\alpha/2}\,\sigma}{e}\right)^2$ rounded up

$\alpha = 0.05$ $z_{0.025} = 1.96$

$$n = \left(\frac{1.96 \times 50}{2.5}\right)^2 = 1536.64$$

$n = 1537$

5 Pop. variances known, both sample sizes > 30

$\bar{X}_1 - \bar{X}_2$ is a normal distribution

$$(\bar{X}_1 - \bar{X}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$1-\alpha = 0.99 \quad \alpha = 0.01 \quad z_{\alpha/2} = z_{0.005} = 2.575$

Table A.3 $\quad P(z > 2.57) = P(z < -2.57) = 0.0051$

$P(z > 2.58) = P(z < -2.58) = 0.0049$

both equally close so I averaged them

$$z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} = 2.575\sqrt{\frac{10^2}{40}+\frac{20^2}{150}} = 5.85$$

$$\bar{X}_1 - \bar{X}_2 = 540 - 525 = 15$$

$$15 - 5.85 < \mu_1 - \mu_2 < 15 + 5.85$$

$$\boxed{9.15 < \mu_1 - \mu_2 < 20.85}$$

99% confidence interval

b) Pop variances unknown but pop distributions approx normal so use t-distribution

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{40\times 400 + 20\times 100}{60}$$

$$= 300$$

$1-\alpha = 0.99 \quad \alpha = 0.01 \quad v = 62-2 = 60$

$t_{0.005} = 2.66$ from row $v = 60$

$$(\bar{X}_1-\bar{X}_2) - t_{\alpha/2} s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1-\bar{X}_2) + t_{\alpha/2} s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$t_{\alpha/2}\, s_p \sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 2.66 \times \sqrt{300} \times \sqrt{\frac{1}{41}+\frac{1}{21}}$$

$$= 12.36$$

$$20 - 12.36 < \mu_1 - \mu_2 < 20 + 12.36$$

$$\boxed{7.64 < \mu_1 - \mu_2 < 32.36}$$ 99% confidence interval

⑥ Chi-squared distribution

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

$\bar{X} = 3 \quad s^2 = 0.815$

$1-\alpha = 0.95 \quad \alpha = 0.05$

$\chi^2_{\alpha/2} = \chi^2_{0.025} = 11.143$

$\chi^2_{1-\alpha/2} = \chi^2_{0.975} = 0.484$

row $r = 4$

$$\frac{4 \times 0.815}{11.143} < \sigma^2 < \frac{4 \times 0.815}{0.484}$$

$$\boxed{0.293 < \sigma^2 < 6.736}$$

95% confidence interval