HW 7 Solutions

(1) a) $\bar{X} = 98.44$

Sample median using average of $\frac{n}{2}$'th and $\frac{n}{2}+1$'th observations after sorting $= 97.85$

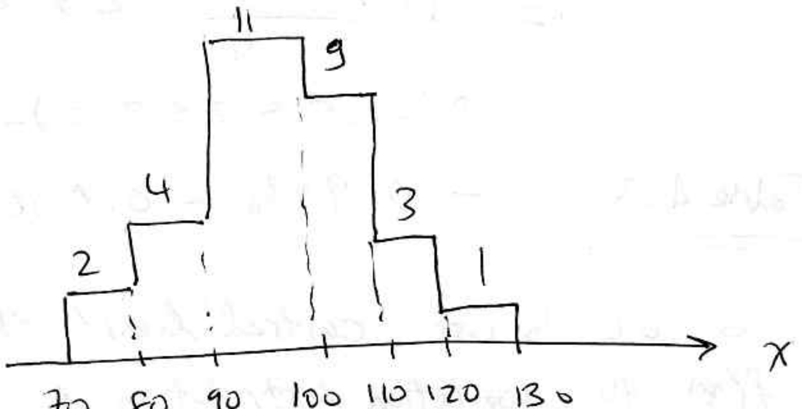
Sample median using $q(0.5)$ definition

$q(0.5) = 0.5 \times 30 = 15$'th element after sorting $= 97.5$

both acceptable

b) $s^2 = 104.621$ so $S = \sqrt{104.621} = 10.23$

c)



d) The normal-quantile plot looks approximately linear so we can say the sample came from a normal distribution.

(2) a)

b)

(3) a) $n < 30$ and the underlying population distribution is unknown, so we can't use the central limit theorem

b) $n < 30$, but underlying population distribution is approximately normal so ok. to use central limit theorem

$$\mu_{\bar{X}} = \mu = 200 \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{3} = 4$$

$$P(190 \le \bar{X} \le 210) = P\left(\frac{190 - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \le z \le \frac{210 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right)$$

$$= P\left(\frac{190 - 200}{4} \le z \le \frac{210 - 200}{4}\right)$$

$$= P(-2.5 \le z \le 2.5) = P(z \le 2.5) - P(z \le -2.5)$$

$\underline{\text{Table A.3}}$ $= 0.9938 - 0.0062 = \boxed{0.9876}$

c) $n \geq 30$ so ok to use central limit theorem even though $f(x)$ the population distribution is not normal

We first need the population μ and σ

$$\mu = \sum x f(x) = 0 \times \frac{3}{8} + 1 \times \frac{3}{8} + 2 \times \frac{1}{8} + 3 \times \frac{1}{8} = \underline{1}$$

$$\sigma^2 = E[x^2] - \mu^2 = \left(0 \times \frac{3}{8} + 1 \times \frac{3}{8} + 4 \times \frac{1}{8} + 9 \times \frac{1}{8}\right) - 1^2 = \underline{1}$$

$$\sigma = \sqrt{1} = 1$$

$$\mu_{\bar{x}} = \mu = 1 \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{100}} = 0.1$$

$$P(\bar{X} > 1.2) = P\left(z > \frac{1.2 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right) = P\left(z > \frac{1.2 - 1}{0.1}\right) = P(z > 2)$$

$$= 1 - P(z \le 2)$$

$$= 1 - \underbrace{0.9772}_{\text{Table A.3}} = \boxed{0.0228}$$

(4) a) $P(1.6 \le X_A \le 2.4)$

$= P(2 - 2\times 0.2 \le X_A \le 2 + 2\times 0.2)$

$= P(\mu_A - 2\sigma_A \le X_A \le \mu_A + 2\sigma_A)$

$\geq 1 - \frac{1}{2^2} = 0.75$ Using Chebyshev's theorem with $k=2$

Notice there is no sample in this part of the question.

b) $n = 64 \geq 30$ can use central limit theorem

$\sigma_A = \sqrt{0.04} = 0.2$

$\mu_{\bar{X}_A} = \mu_A = 2 \qquad \sigma_{\bar{X}_A} = \frac{\sigma_A}{\sqrt{64}} = \frac{0.2}{8} = 0.025$

$P(\bar{X}_A \le 1.9575) = P\left(Z \le \frac{1.9575 - 2}{0.025}\right)$

$= P(Z \le -1.7) = 0.0446$

c) Since both samples greater than 30 observations can use central limit theorem

$$Z = \frac{(\bar{X}_A - \bar{X}_B) - (\mu_{\bar{X}_A} - \mu_{\bar{X}_B})}{\sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}} = \frac{(\bar{X}_A - \bar{X}_B) - (2 - 1.9)}{\sqrt{\frac{0.04}{64} + \frac{0.0975}{100}}}$$

$$Z = \frac{(\bar{X}_A - \bar{X}_B) - 0.1}{0.04}$$

so $P(0.094 \le \bar{X}_A - \bar{X}_B \le 0.162) = P\left(\frac{0.094 - 0.1}{0.04} \le Z \le \frac{0.162 - 0.1}{0.04}\right)$

$= P(-0.15 \le Z \le 1.55) = P(Z \le 1.55) - P(Z \le -0.15)$

$= 0.9394 - 0.4404 = \boxed{0.499}$

(2)

a) $P(X^2 > \chi_\alpha^2) = 0.01 \quad v = 21$

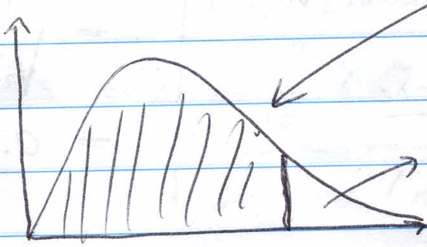
$\alpha = 0.01$

Row $v=21$ column $\alpha = 0.01$ Table A.5

$\chi_\alpha^2 = \chi^2_{0.01} = 38.932$

b) $P(X^2 < \chi_\alpha^2) = 0.95 \quad v = 6$

because $X^2 < \chi_\alpha^2$



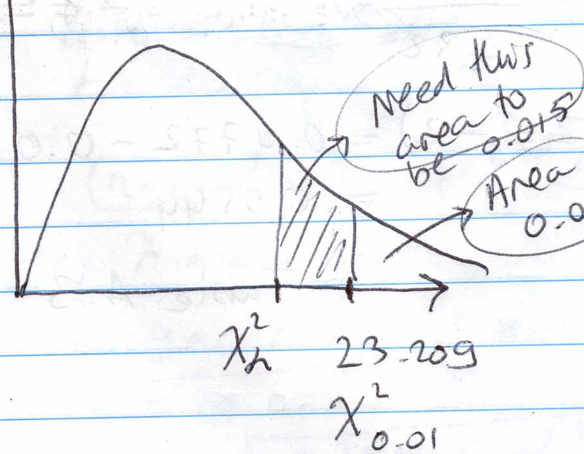
$\alpha = 1 - 0.95 = 0.05$

Row $v = 6$ column $\alpha = 0.05$ Table A.5

$\chi_\alpha^2 = 12.592$

c) $P(\chi_\alpha^2 < X^2 < \underbrace{23.209}) = 0.015 \quad v=10$

From row $v=10$ of table A.5 notice that this is $\alpha = 0.01$



$\Rightarrow$ So we need to find $\chi^2_{0.015+0.01}$

$\chi^2_{0.025} = 20.483$

Row $v=10$ Table A.5