

Homework 5 solutions

① a) 0.0069 b) $\mu = 16.75$ $\sigma^2 = 5.5275$ $\sigma = 2.3511$
 c) $\mu = 0.75$ $\sigma^2 = 0.7275$ $\sigma = 0.8529$

② a) ~~0.0069~~ $b(5; n=7, p=0.9) = 0.124$
 b) $\sum_{x=0}^3 b(x; n=9, p=0.25) = 0.8343$

③ a) $\mu = \frac{0.75 + 1.25}{2} = 1$
 $\sigma^2 = \frac{(1.25 - 0.75)^2}{12} = 0.0208$
 $\sigma = \sqrt{0.0208} = 0.1443$

b) $P(X < 0.8) = \int_{-\infty}^{0.8} f(x) dx = \int_{0.75}^{0.8} 2 dx$
 $= 2x \Big|_{0.75}^{0.8} = 0.1$

$$f(x) = \begin{cases} 2 & ; \quad 0.75 \leq x \leq 1.25 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

c) $P(0.875 < X < 1.125) = \int_{0.875}^{1.125} 2 dx = 0.5$

④ $P(432 < X < 568) =$

$$P\left(\frac{432 - 500}{50} < Z < \frac{568 - 500}{50}\right) =$$

$$P(-1.36 < Z < 1.36) = P(Z < 1.36) - P(Z < -1.36)$$

$$= 0.9131 - 0.0869 = 0.8262$$

$$b) P(500 - d < X < 500 + d) = 0.999$$

$$P\left(\frac{500 - d - 500}{50} < z < \frac{500 + d - 500}{50}\right) = 0.999$$

$$P\left(-\frac{d}{50} < z < \frac{d}{50}\right) = 0.999$$

$$\& P\left(z < -\frac{d}{50}\right) = 1 - 0.999$$

$$P(z < -d/50) = 0.0005$$

$$-d/50 = -3.31$$

$$d = 165.5$$

$$c) P(X > 522) = P\left(z > \frac{522 - 500}{50}\right)$$

$$= P(z > 0.44) = 1 - P(z < 0.44)$$

$$= 1 - 0.67 = 0.33$$

prob at least 8 out of 10 years > 522

$$= \sum_{x=8}^{10} b(x; n=10, p=0.33)$$

$$= 0.0032$$

$$\textcircled{5} \text{ a) } P(T \leq 5) = \int_{-\infty}^5 f(x) dx$$

$$= \int_0^5 \frac{1}{2} e^{-x/2} dx = -e^{-x/2} \Big|_0^5$$

$$= -e^{-2.5} - (-e^{-0}) = -0.082 + 1 = 0.918$$

b) X has a binomial distribution with $n=10$, $p=0.918$

$$P(X=6) = b(x=6; n=10, p=0.918)$$

$$= 0.0057$$

$$\text{c) } P(X=10) = b(x=10; n=10, p=0.918)$$

$$= 0.425$$