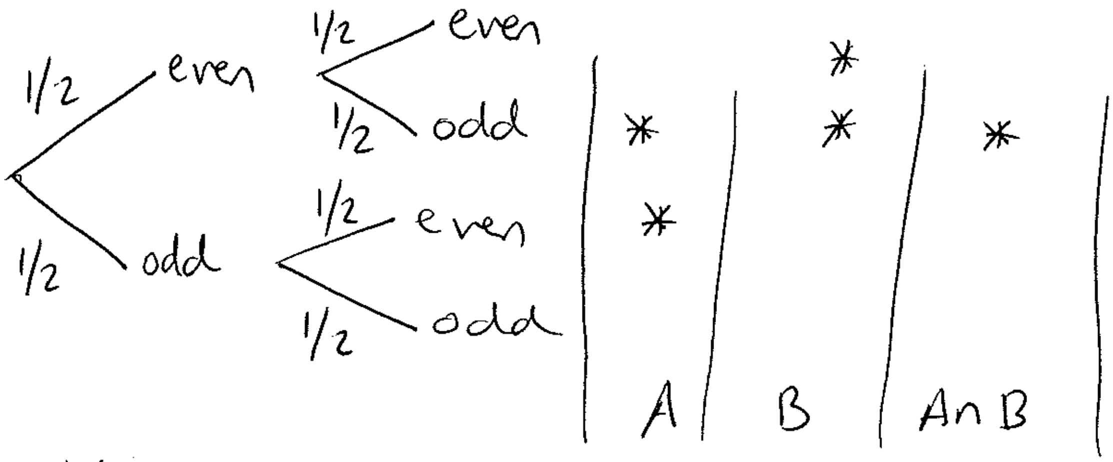
HW #3

① a) To check lets see if $P(A)P(B) \stackrel{?}{=} P(A\cap B)$



$P(A) = \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2}$

$P(B) = \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2}$

$P(A\cap B) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$ which is $P(A)P(B)$

so independent

b) $C = \{(\text{even},\text{even}), (\text{odd},\text{odd})\}$

$A\cap C = \emptyset \qquad P(A\cap C) = 0 \neq \underbrace{P(A)}_{1/2}\ \underbrace{P(C)}_{\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2}}$

Not independent

c) $D = \{(6,12), (7,11), (8,10), (9,9), (10,8), (11,7), (12,6)\}$

$P(D) = \underbrace{\frac{1}{12}\times\frac{1}{12}}_{\text{prob. of each pair (outcome)}} \times \underbrace{7}_{\#\text{ outcomes in } D} = 7/144$

d) $P(D|C) = \dfrac{P(D\cap C)}{P(C)} = \dfrac{P(D)}{P(C)} = \dfrac{7/144}{1/2} = 7/72$

$D\cap C = D$ since if the sum is 18 it is also even

(2) A: Tom alive in 20 years

B: Nancy " " " "

Question asks $P(A' \cap B')$

$$P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B)$$

De Morgan

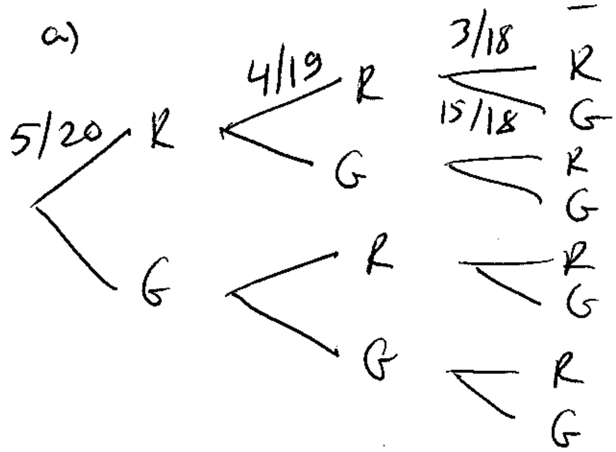
$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - [P(A) + P(B) - P(A)P(B)]$$ independence

$$= 1 - [0.7 + 0.9 - 0.7 \times 0.9]$$

$$= 0.03$$

(3) a)



* part a.

$$P(RRG) = \frac{\cancel{5}}{\cancel{20}} \times \frac{4}{19} \times \frac{\cancel{15}^{5}}{\cancel{18}_{6}} = 5/114$$

event Y.

b) X: ~~not~~ not getting any green marbles

~~X~~ $= \{RRR\}$

$$P(X) = \frac{\cancel{5}}{\cancel{20}} \times \frac{4}{19} \times \frac{3}{\cancel{18}_{6}} = 1/114$$

c) Z: 2 red 1 green Y: RRG

$$P(Y|Z) = \frac{P(Y \cap Z)}{P(Z)} = \frac{P(Y)}{P(Z)} = \frac{5/114}{3 \times 5/114} = 1/3$$

$Z \cap Y = Y$

$Z = \{RRG, RGR, GRR\}$

$P(Z) = 3 \times 5/114$

d) $P(Z|Y) = \frac{P(Z \cap Y)}{P(Y)} = \frac{P(Y)}{P(Y)} = 1$

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4) a) Using Bayes Rule

$$P(Av|X) = \frac{P(X|Av)\,P(Av)}{P(X|Av)\,P(Av) + P(X|Pt)\,P(Pt) + P(X|Rh)\,P(Rh)}$$

$$= \frac{0.9 \times 0.2}{0.5 \times 0.2 + 0.8 \times 0.7 + 0.6 \times 0.1} = 0.225$$

b) $P(Av|X) + P(Av'|X) = 1$

$P(Av'|X) = 1 - 0.225 = 0.775$

c) $P(X) =$ this denominator $= 0.8$

5) D: has disease   X: test positive

$P(D) = 1/500$ $P(X|D) = 0.95$ $P(X|D') = 0.01$

$$P(D|X) = \frac{P(X|D)\,P(D)}{P(X|D)\,P(D) + P(X|D')\,P(D')}$$

$$= \frac{0.95 \times 1/500}{0.95 \times 1/500 + 0.01 \times \frac{499}{500}} = 0.1599$$