HW2

1) a) multiplication rule $n_1 = 3$, $n_2 = 3$, $n_3 = 2$ $n_1 n_2 n_3 = 18$

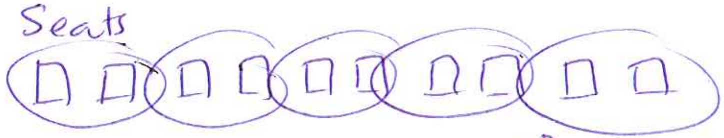
company choices, form choices, strength choices

b) multiplication rule $n_1 = n_2 = n_3 = n_4 = n_5 = 4$ 4 choices each

$n_1 n_2 n_3 n_4 n_5 = 4^5 = 1024$

c) permutation: $_{10}P_{10} = \frac{10!}{(10-10)!} = \frac{10!}{0!} = 10! = 3,628,800$

d) Seats



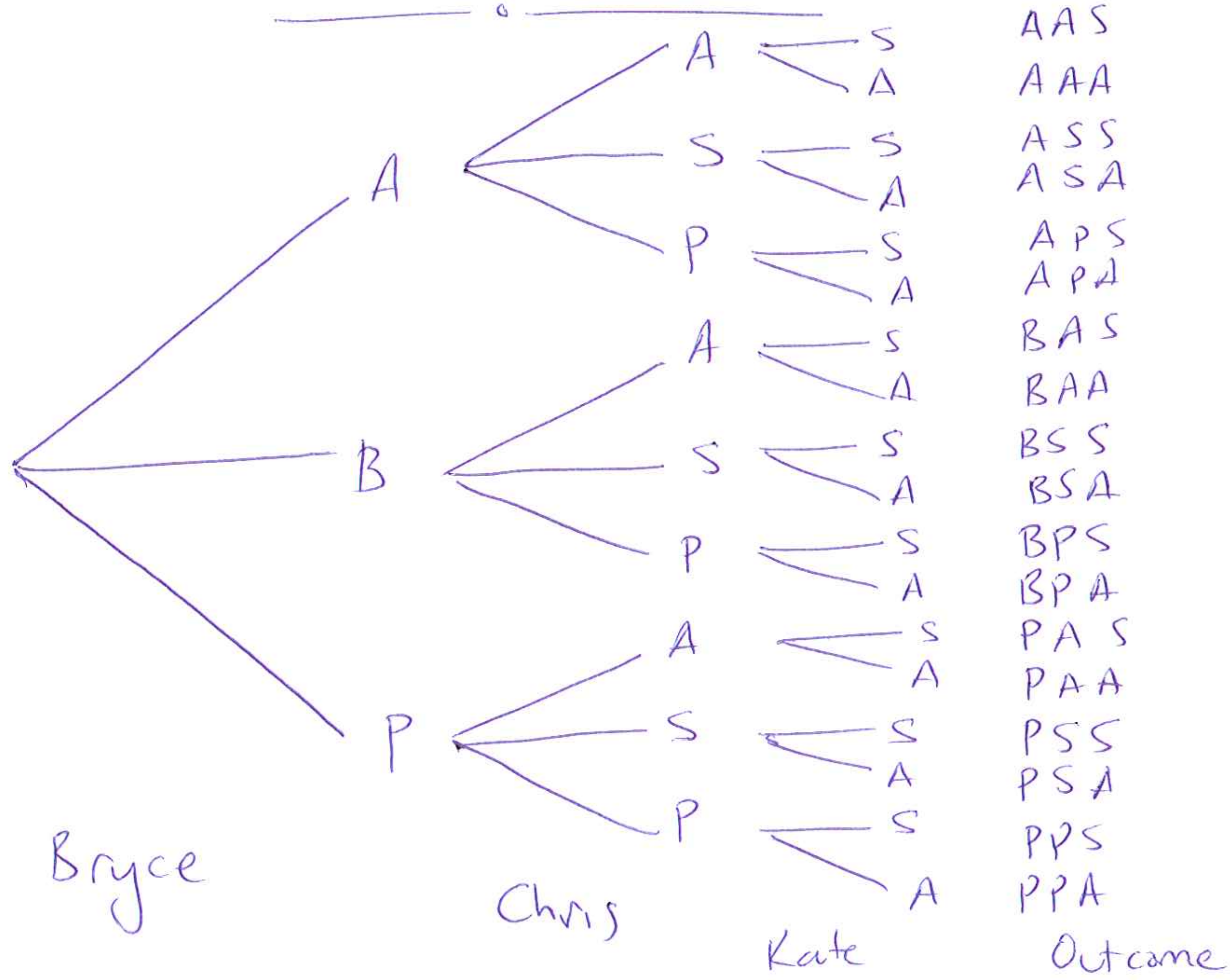
5 objects 5 couples $_5P_5 = 5!$

each couple has 2 choices husband-wife or wife-husband

$n_1 = n_2 = n_3 = n_4 = n_5 = 2$ $n_1 n_2 n_3 n_4 n_5 = 2^5 = 32$

total ways $5! \cdot 2^5 = 3840$

2) a)



b) From the tree, this event has the following outcomes

$X = \{AAS, AAA, ASS, ASA, APA, BAA, BSS, PAA, PSS, PPS, PPA\}$

Since all outcomes equally likely

$P(X) =$ Prob. of event $= \dfrac{\#\text{ outcomes in event}}{\#\ \text{''}\ \ \text{''}\ \ S} = \dfrac{11}{18}$

c) Note that this is the complement of X

so $P(X') = 1 - P(X) = 1 - 11/18 = 7/18$

d) use multiplication rule $n_1 = 3, n_2 = 3, n_3 = 2, n_4 = 4$

$3 \cdot 3 \cdot 2 \cdot 4 = 72$

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3 a) $_{25}P_5 = \dfrac{25!}{(25-5)!} = 25 \times 24 \times 23 \times 22 \times 21 = 6{,}375{,}600$

b) How many outcomes in this event

choices $\boxed{1} \times \boxed{23} \times \boxed{22} \times \boxed{21}\ \boxed{1} = {}_{23}P_3 = \dfrac{23!}{(23-3)!} = 23 \times 22 \times 21$

23 cards
3 spots
order matters

Prob $= \dfrac{\cancel{23 \times 22 \times 21}}{25 \times 24 \times \cancel{23 \times 22 \times 21}} = \dfrac{1}{600}$

c) Order doesn't matter here.

$S: {}_{25}C_5$ event: ${}_{23}C_3$

prob $= \dfrac{{}_{23}C_3}{{}_{25}C_5} = \dfrac{23!}{\cancel{20!}\,3!} \cdot \dfrac{5!\,\cancel{20!}}{25!} = \dfrac{\cancel{5}^{1} \times \cancel{4}^{1}}{\cancel{25}_{5} \times \cancel{24}_{6}} = \dfrac{1}{30}$

4) a) order matters, sampling without replacement

$$_{10}P_6 = \frac{10!}{(10-6)!} = 151,200$$

b) 1 outcome in event so prob $= \dfrac{1}{151,200}$

c) Dave sits in A, 9 students 5 seats left

$$\frac{\text{event} \rightarrow\ _9P_5}{S \rightarrow\ _{10}P_6} = \frac{9!}{\cancel{4!}} \frac{\cancel{4!}}{10!} = \frac{1}{10}$$

d) Order doesn't matter in this part

$$\frac{\text{event} \rightarrow 1}{S \rightarrow\ _{10}C_6} = \frac{6!\,4!}{10!} = \frac{1}{210}$$

e) Seat A occupied, 9 students 5 seats left
order does not matter

$$\frac{\text{event} \rightarrow\ _9C_5}{S \rightarrow\ _{10}C_6} = \frac{\cancel{9!}}{\cancel{5!}\,\cancel{4!}} \frac{\overset{6}{\cancel{6!}}\,\cancel{4!}}{\underset{10}{\cancel{10!}}} = 6/10$$

f) Let X be the event Dave sits in A, Michael in C
Let Y " " " " " " ", Bill sits in F

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

$$P(X) = \frac{_8P_4}{_{10}P_6} = 1/90 \qquad P(Y) = \frac{_8P_4}{_{10}P_6} = 1/90$$

$X \cap Y$: Dave – A
Michael – C
Bill – F

$$P(X \cap Y) = \frac{_7P_3}{_{10}P_6} = \frac{1}{720}$$

$$P(X \cup Y) = \frac{1}{90} + \frac{1}{90} - \frac{1}{720} = \frac{8+8-1}{720} = \frac{15}{720}$$

5

a) $8^4 = 4096$

b) ~~A~~ event : $\boxed{1} \times \boxed{8} \times \boxed{8} \quad \boxed{1} = 64$

$$\text{Prob} = \frac{64}{4096} = 1/64$$

c) $4096 - 1 = 4095$

d) ${}_{4096}P_3 = 4096 \times 4095 \times 4094 = 68{,}669{,}153{,}280$