HW 1

1) a) $S = \{HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT\}$

b) $A = \{TTTT\}$ $P(A) = 1/16$

$B = \{HTTT, THTT, TTHT, TTTH\}$, $P(B) = \frac{4}{16} = \frac{1}{4}$

$C = \{HHHH, HHHT, HHTH, HHTT\}$, $P(C) = \frac{4}{16} = \frac{1}{4}$

$D = \{HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, THHH, THHT, THTH, TTHH\}$, $P(D) = 11/16$

c) $A \cap B = \phi \rightarrow$ Yes, $B \cap C = \phi \rightarrow$ Yes, $C \cap D \neq \phi \rightarrow$ No

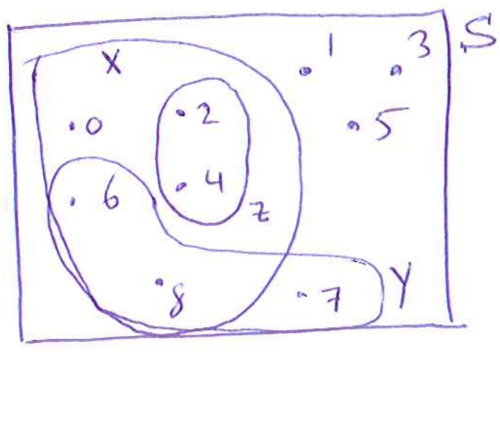
d) $P(A \cup D) = P(A) + P(D) - P(A \cap D)$ $A \cap D = \phi$

$= \frac{1}{16} + \frac{11}{16} - 0 = \frac{12}{16} = \frac{3}{4}$ $\leftarrow P(A \cap D) = 0$

e) $P(C \cup D) = P(C) + P(D) - P(C \cap D)$ $C \cap D = C$

$= \frac{1}{4} + \frac{11}{16} - \frac{1}{4} = \frac{11}{16}$ $\leftarrow P(C \cap D) = P(C)$

2) a)



b) Let x be the prob. of the even outcomes. Then the prob. for the odd outcomes is $2x$. There are 5 even and 4 odd outcomes so

$5x + 4 \cdot 2x = 1$

$13x = 1$

$x = 1/13$

$\longrightarrow$

so $P(0) = P(2) = P(4) = P(6) = P(8) = 1/13$

$P(1) = P(3) = P(5) = P(7) = 2/13$

then $P(Z \cup Y) = P(\{2,4,6,8,7\}) = 4/13 + 2/13 = 6/13$

$P(X \cup Y) = P(\{0,2,4,6,7,8\}) = \frac{5}{13} + \frac{2}{13} = \frac{7}{13}$

$P(X' \cap Z) = P(\{1,3,5,7\} \cap \{2,4\}) = P(\phi) = 0$

$P((X \cap Z)') = 1 - P(X \cap Z) = 1 - P(\{2,4\}) = 1 - \frac{2}{13} = \frac{11}{13}$

$P((X \cup Y) \cap Z) = P((X \cap Z) \cup (Y \cap Z))$
$= P(\{2,4\} \cup \phi) = P(\{2,4\}) = 2/13$

$P((X \cap Z) \cup Y) = P(\{2,4\} \cup \{6,7,8\})$
$= P(\{2,4,6,7,8\}) = \frac{4}{13} + \frac{2}{13} = \frac{6}{13}$

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(3) All outcomes equally likely. 500 outcomes in S.
Lets give the events names:
S_m = smokes, D: drinks, E: eats between meals
# of elements in S_m is 210, D is 258, E is 216
$S_m \cap D$ is 122, $E \cap D$ is 83, $S_m \cap E$ is 97
$S_m \cap D \cap E$ is 52

a) $P(S_m \cap D') = ?$ D, D' form a partition so

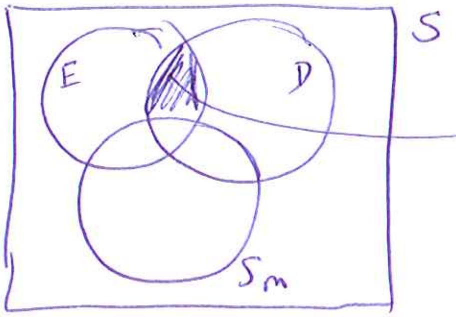
Law of total prob

$P(S) = P(S_m \cap D) + P(S_m \cap D')$

$\frac{210}{500} = \frac{122}{500} + P(S_m \cap D')$

$P(S_m \cap D') = \frac{210 - 122}{500} = \frac{88}{500}$

b) $P(E \cap D \cap S_m') = ?$ Lets use the venn diagram



→ Asking for this area.
Notice that this area + $P(E \cap D \cap S_m)$ is equal to $P(E \cap D)$

so $P(E \cap D \cap S_m') = P(E \cap D) - P(E \cap D \cap S_m)$

$$= \frac{83}{500} - \frac{52}{500} = \frac{31}{500}$$

Note: this is also using the law of total prob.

c) $P(S_m' \cap E') = ?$

$S_m' \cap E' = (S_m \cup E)'$ De Morgan

$P(S_m' \cap E') = 1 - P(S_m \cup E) = 1 - \frac{329}{500} = \frac{171}{500}$

$P(S_m \cup E) = P(S_m) + P(E) - P(S_m \cap E)$

$$= \frac{210 + 216 - 97}{500} = \frac{329}{500}$$

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(4) a) $A \cap B \cap C = (A \cap C) \cap B$, but we are told that $P(A \cap C)$ which means $A \cap C = \emptyset$ So $(A \cap C) \cap B = \emptyset \cap B) = \emptyset \rightarrow P(A \cap B \cap C) = 0$

b) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C)$
$+ P(A \cap B \cap C) = 0.1 + 0.6 + 0.6 - 0.1 - 0$
$- 0.4 + 0 = 0.8$

c) A house built after 1990 that doesn't have a 2 car garage $\rightarrow A \cap B'$. Want to show $P(A \cap B') = 0$

Total probability $\rightarrow$ $P(A) = P(A \cap B) + P(A \cap B')$

$$0.1 = 0.1 + P(A \cap B')$$

$$P(A \cap B') = 0.$$

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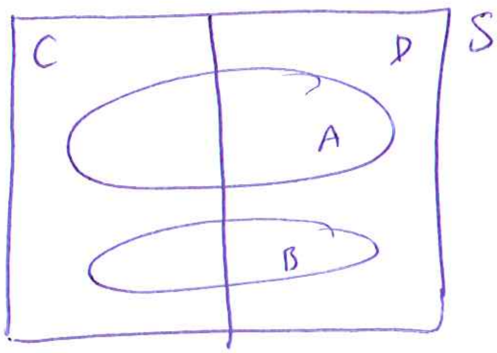
(5) a) 

b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= 0.65 + 0.25 - 0 = 0.9$$

c) $P((A \cup B)') = 1 - P(A \cup B)$

$$= 1 - 0.9 = 0.1$$

d) C, D form a partition

so using law of total prob. $P(B) = P(B \cap C) + P(B \cap D)$

$$0.25 = 0.05 + P(B \cap D)$$

$$P(B \cap D) = 0.25 - 0.09 = 0.16$$

e) $D = C' \rightarrow P(D) = 1 - P(C) = 1 - 0.7 = 0.3$