NAME:

UID:

ECE 3530 Final

Show your work (except for Question 1)
Four questions each worth 25 points.
All necessary tables provided at the back of the exam.
Closed book, limited notes (3 regular size sheets front&back). No laptops.

1. Answer the following multiple choice questions by circling the correct answer:

   (a) Which of the following confidence levels will result in a narrower confidence interval for the same sample size?

   (a) 99% (b) 95%

   (b) Which of the following sample sizes will result in the widest confidence interval for the same confidence level?

   (a) n=100 (b) n=1000 (c) n=200 (d) All n give the same size interval

   (c) If a hypothesis is rejected at significance level $\alpha = 0.001$, is it possible that the hypothesis is not rejected if the test was done at significance level $\alpha = 0.01$ (with everything else staying the same)?

   (a) Yes (b) No

   (d) An experiment consists of taking the average of 100 rolls of a fair dice. Let $\bar{X}$ be the average of the 100 rolls. Does $\bar{X}$ have a normal distribution?

   (a) Yes (b) No

   (e) A linear model is fitted to two different samples using least squares regression and the coefficient of determination (R^2) is computed for each sample. The first sample yields $R^2 = 0.37$ while the second sample yields $R^2 = 0.81$. For which sample is the linear model a better fit?

   (a) First sample (b) Second sample

2. An engineer wants to investigate the relationship between the electrical resistivity of a certain metal and temperature. Let x be the temperature in degrees Fahrenheit. Let Y be the electrical resistivity in $n\Omega m$. The engineer collects the following data:

i	1	2	3
x_i	50	150	250
y_i	100	115	130

(a) Find the equation of the fitted regression line.

(b) Estimate the electrical resistivity of this metal at a temperature of 300 degrees Fahrenheit.

a) $\bar{X} = \frac{50+150+250}{3} = 150$

$\bar{y} = \frac{100+115+130}{3} = 115$

$b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

or $b = \frac{S_{xy}}{S_{xx}}$

$S_{xx} = (50-150)^2 + 0 + (250-150)^2$

$= 20000$

$S_{xy} = (50-150)(100-115) + 0 + (130-115)(250-150)$

$= 3000$

$b = 3/20$

$a = \bar{y} - b\bar{x} = 115 - \frac{3}{20} \times 150 = 92.5$

$\boxed{\hat{y} = \frac{3}{20} x + 92.5}$ $\quad$ 0.15 $\quad \frac{185}{2}$

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b) $x = 300 \rightarrow \hat{y} = \frac{3}{20} * 200 + 92.5$

$= \boxed{137.5}$ $\quad \frac{275}{2}$

3. A DC power supply manufacturer wants to test the hypothesis that the mean output voltage is 100 V (for a variety of different loads). Assume that the output voltage has a normal probability distribution.

   A quality control engineer measures the output voltage when the supply is connected to 16 different loads and computes a sample mean $\bar{x} = 103.6$ and sample variance $S^2 = 36$.

   (a) Perform a hypothesis test at significance level $\alpha = 0.05$ for the null hypothesis that the mean output voltage is 100 V.

   (b) Find a 99% confidence interval for the population variance σ^2.

(15) a) $H_0 : \mu = 100 \qquad H_1 : \mu \neq 100$

Reject H_0 if

$$\bar{X} > \mu_0 + t_{\alpha/2} \frac{S}{\sqrt{n}}$$

or $\bar{X} < \mu_0 - t_{\alpha/2} \frac{S}{\sqrt{n}}$

$v = 15$

$t_{0.025} = 2.131$

$S = \sqrt{36} = 6$

$$\left.\begin{array}{l} \bar{x} > 100 + 2.31 \dfrac{6}{\sqrt{16}} = 103.1965 \\ \bar{x} < 100 - 2.31 \dfrac{6}{\sqrt{16}} = 96.8035 \end{array}\right] \text{Critical region}$$

$\bar{X} = 103.6 > 103.1965$ so Reject H_0

(10) b) $\dfrac{(n-1)S^2}{\chi^2_{\alpha/2}} < \sigma^2 < \dfrac{(n-1)S^2}{\chi^2_{1-\alpha/2}}$, $\alpha = 0.01$

$v = 15 \qquad \chi^2_{0.005} = 32.801$

$\chi^2_{0.995} = 4.601$

$$\frac{15 \times 36}{32.801} < \sigma^2 < \frac{15 \times 36}{4.601}$$

$\boxed{16.46 < \sigma^2 < 117.36}$

~~Reject H_0 if $X < 3$ or $X > 3$~~

(4) a) σ known $n > 30$ $\rightarrow$ z-distribution

$H_0: \mu = 100$ $\qquad \alpha = 0.05$

$H_1: \mu \neq 100$ $\qquad \mu_0 = 100$

Reject H_0 if $\bar{X} < \mu_0 - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

or $\bar{X} > \mu_0 + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$z_{\alpha/2} = z_{0.025} = 1.96$

Reject if $\bar{X} < 100 - 1.96 \frac{6}{\sqrt{36}} = 98.04$

or $\bar{X} > 100 + 1.96 \frac{6}{\sqrt{36}} = 101.96$

$\bar{X} = 102 > 101.96$ so Reject.

b) From linear combinations we know that $\mu_{X+Y} = \mu_X + \mu_Y = 100 + 100 = 200$

also $\sigma^2_{X+Y} = \sigma_X^2 + \sigma_Y^2 = 6^2 + 6^2 = 72$

so $\sigma_W = \sigma_{X+Y} = \sqrt{72} = 8.49$

$\alpha = 0.02$ z-distribution $z_{\alpha/2} = z_{0.01} \cong 2.33$

$$\bar{w} - z_{\alpha/2} \frac{\sigma_w}{\sqrt{n}} < \mu_w < \bar{w} + z_{\alpha/2} \frac{\sigma_w}{\sqrt{n}}$$

$$201 - 2.33 \times \frac{8.49}{\sqrt{100}} < \mu_w < 201 + 2.33 \times \frac{8.49}{\sqrt{100}}$$

$$\boxed{198.022 < \mu_w < 202.978}$$