HOMEWORK #9 - DUE: Friday, April 19

Write your name on everything you hand in. Show your work.

1. In communications let p denote the probability that a bit sent by the transmitter will be received erroneously on the other end of the channel. A telecom company has determined that for a communication channel an error rate of $p > 0.05$ is unacceptable. They want to test the hypothesis that $p = 0.05$ vs the alternative hypothesis that $p > 0.05$.

   To perform the test, a predetermined 32 bit long message is sent (this is a sample with $n = 32$) and the number of erroneous bits are counted at the receiving end. Let X denote the number of erroneous bits. If $X \geq 4$ the channel will be deemed unacceptable.

   (a) Specify the null and alternative hypothesis. Draw a diagram illustrating the critical region.

   (b) Compute the probability of committing a type I error.

   Hint: Use the Binomial distribution MATLAB function we had used for a previous homework.

   (c) Compute the probability of committing a type II error when testing against the alternatives $p = 0.15$ and $p = 0.2$.

   (d) To decrease the probability of both types of errors simultaneously, we can send a longer message instead. A 400-bit long message is sent. H_o will be rejected if the number of erroneous bits are greater than or equal to 30. Using the normal approximation to the Binomial distribution, compute α the probability of type I error.

2. To meet the *ISO 4* standard a clean room for semiconductor manufacturing must not have more than 352 particles (size 0.5 microns or larger) per cubic meter. Assume that the number of particles per cubic meter has an approximately normal probability distribution with population standard deviation $\sigma = 28$.

   An engineering working at a CPU manufacturing plant wants to test the hypothesis that μ the mean for number of particles per cubic meter is equal to 352 vs the alternative hypothesis that it is greater than 352. He takes air samples on 49 different occasions and finds $\bar{x} = 360$.

   (a) State the null and alternative hypothesis.

   (b) What is the critical region if the hypothesis test is to be conducted at significance level $\alpha = 0.01$? Is the null hypothesis rejected?

   (c) Based on your answer for part (b), can you determine if the null hypothesis would be rejected if the test was performed at a level of significance $\alpha = 0.001$?

   (d) What is the probability of type II error for the critical region computed in the previous part when testing against the specific alternative $\mu = 368$?

3. A DC power supply manufacturer wants to test the hypothesis that the mean output voltage is 50 V (for a variety of different loads). Assume that the output voltage has a normal probability distribution.

   A quality control engineer measures the output voltage when the supply is connected to 9 different loads and computes a sample mean $\bar{x} = 45.4$ and sample standard deviation $S = 5$.

   (a) State the null and alternative hypothesis.

   (b) What is the critical region if the hypothesis test is to be conducted at significance level $\alpha = 0.01$? Is the null hypothesis rejected?

4. A company manufacturing pacemakers is testing a new electrode. The electrodes must adhere to a silicone substrate for at least 20 years. The company is going to test the hypothesis that the mean adherence time is 20 years vs. the alternative that it is less than 20 years at the significance level $\alpha = 0.05$. The experiment will be conducted with a sample of 25 volunteers. Assume that the population distribution for the adherence time is approximately normally distributed.

   The average adherence time for the pacemakers in the 25 volunteers is found to be 18.8 years and the standard deviation of the sample is found to be 3 years.

   (a) Is the null hypothesis rejected?

   (b) If the company wants to decrease the probability of making a type I error without increasing the sample size, should the critical value be increased or decreased?

   (c) Find the 90% confidence interval for the population variance σ^2.

5. A CPU manufacturer is interested in studying the relationship between clock speed and the operating temperature that results at that clock speed for a particular CPU model. Let x be the clock speed in MHz and let Y be the temperature in ^oC. The following data was collected:

i	x_i	y_i
1	350	31.4
2	360	35.6
3	370	41.8
4	380	51.0
5	390	56.8
6	400	62.8
7	410	67.4

   (a) Find the equation of the regression line.

   (b) Estimate the temperature for clock speed $x = 430MHz$.

   (c) Find the 95% confidence interval for β.

   (d) Compute the coefficient of determination R^2. Is this a high quality fit?