HOMEWORK #7 - DUE: Friday, April 5

Write your name on everything you hand in. Show your work.

1. In trying to assess the accuracy of a GPS receiver, 30 observations of position errors (in centimeters) have been made:

89.9	104.0	93.7	94.4	86.0	115.8	95.3	90.7	88.0	121.8
97.4	95.2	101.4	98.2	97.5	111.4	101.6	103.3	97.1	78.7
85.7	75.0	107.5	106.6	103.0	111.5	99.8	104.4	97.3	100.9

Note: You may enter the data given above into MATLAB manually or download the data file (Q1data.mat) from the class web page.

(a) Compute the sample mean and median. Both definitions of the median are acceptable. **Hint:** Use MATLAB's **sort** command.

(b) Compute the sample standard deviation.

(c) Sketch the histogram plot of the data with the following bins:

Bin 1: $70 \leq X < 80$	Bin 2: $80 \leq X < 90$	Bin 3: $90 \leq X < 100$
Bin 4: $100 \leq X < 110$	Bin 5: $110 \leq X < 120$	Bin 6: $120 \leq X < 130$

(d) Using a normal-quantile-plot determine whether you think the position errors for this GPS receiver are normally distributed. Give a one sentence explanation.

Use the following MATLAB function to make a normal-quantile-plot of the sample:

```
function NormalQuantilePlot (x)
y = sort(x);
n = length(x);
f = ((1:n)-3/8)/(n+1/4);
q = 4.91*(f.^0.14 - (1-f).^0.14);
figure(1);clf;plot(q,y,'*-');grid;
```

Save the above function in to a file named NormalQuantilePlot.m and use it on the given sample. You do not need to attach the plot or sketch it for this part.

2. (a) In MATLAB use the command **x=rand(1,500)** to generate your own sample (n=500) drawn from a uniform distribution. Use **NormalQuantilePlot (x)** to see how it differs from linearity. Sketch the plot (or attach a printout).

(b) Use the command **x=randn(1,500)** to generate your own sample (n=500) drawn from a standard normal distribution. Use **NormalQuantilePlot (x)** to see how well it fits a line. Sketch the plot (or attach a printout).

3. For each of the scenarios described below, answer whether the central limit theorem can be used reliably to compute the probability that is asked. If the answer is yes, use Table A.3 to determine the numerical value of the probability that is asked.

(a) A manufacturing process for resistors has unknown population distribution $f(x)$ but we know that the mean $\mu = 100$ Ohms and the standard deviation $\sigma = 5$ Ohms. If a random sample of 10 resistors are picked, what is the probability that the sample mean will be larger than 105 Ohms?

(b) A manufacturing process for resistors is known to have an approximately normal distribution $f(x)$ with mean $\mu = 200$ Ohms and standard deviation $\sigma = 12$ Ohms. If a random sample of 9 are resistors is picked, what is the probability that the sample mean will be between 190 and 210 Ohms?

(c) Let X be the number of computers sold at the U. bookstore on any given day. X has the following population distribution

$$f(x) = \begin{cases} 3/8 & x = 0 \\ 3/8 & x = 1 \\ 1/8 & x = 2 \\ 1/8 & x = 3 \\ 0, & otherwise \end{cases}$$

If a random sample of 100 days are recorded, what is the probability that the sample mean will be greater than 1.2?

4. Jack and Mark are two neighbors who have home internet connections through different providers. Let X_A be the random variable denoting the connection speed at Jack's house and Let X_B be the random variable denoting the connection speed at Mark's house (units in MBPS). We don't know the probability density functions for these random variables, but we know that the mean and variance for X_A are $\mu_A = 2$, $\sigma_A^2 = 0.04$ and the mean and variance for X_B are $\mu_B = 1.9$, $\sigma_B^2 = 0.0975$.

   (a) Find a lower bound for the probability that the internet connection speed at Jack's house is between 1.6 and 2.4 MBPS. *Hint: Use Chebyshev's theorem.*

   (b) Jack measures the connection speed at his house on 64 different occasions. Let $\bar{X}_A$ denote the sample mean. Find the probability $P(\bar{X}_A \leq 1.9575)$.

   (c) Jack measures the connection speed at his house on 64 different occasions while Mark measures it on 100 different occasions. Let $\bar{X}_A$ and $\bar{X}_B$ denote the sample mean for Jack and Mark, respectively. Find $P(0.094 \leq \bar{X}_A - \bar{X}_B \leq 0.162)$.

5. Exercise 8.42 from textbook: For a chi-squared distribution find χ^2_α such that

   (a) $P(X^2 > \chi^2_\alpha) = 0.01$ when $v = 21$.

   (b) $P(X^2 < \chi^2_\alpha) = 0.95$ when $v = 6$.

   (c) $P(\chi^2_\alpha < X^2 < 23.209) = 0.015$ when $v = 10$.