**HOMEWORK #6 - DUE: Friday, March 8**

Write your name on everything you hand in. Show your work.

1. Textbook exercise 3.49. Additionally answer the following question:

   (d) Are X and Y independent?

2. Textbook exercise 3.42. Additionally answer the following question:

   (b) Are X and Y independent?

3. Let $X$ and $Y$ be two jointly distributed continuous random variables representing voltages measured in Volts at two different nodes of a circuit. $X$ has the marginal density function

$$g(x) = \begin{cases} \frac{2x+1}{2}, & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$

   You are also given the conditional density function

$$f_Y(y|x) = \begin{cases} \frac{2(x+y)}{2x+1}, & 0 \le x \le 1 \text{ and } 0 \le y \le 1 \\ 0, & otherwise \end{cases}$$

   (a) Compute $P(Y \le 0.5|X = 0.5)$ the probability that $Y$ is less than or equal to 0.5 given that we know $X = 0.5$.

   (b) Compute the probability that the voltage $X$ is in the range $1/4 \le X \le 1/2$ and the voltage $Y$ is less than $1/2$ at the same time. In other words compute the probability $P(\frac{1}{4} \le X \le \frac{1}{2}, Y \le \frac{1}{2})$.

   (c) Compute the covariance of $X$ and $Y$.

4. A game consists of rolling a pair of dice, one red and one blue. Let $X$ denote the outcome of the red dice and let $Y$ denote the outcome of the blue dice. $X$ and $Y$ are independent random variables. Both dice are rigged and they have the following marginal distrubtions:

$$g(x) = \begin{cases} 1/4, & x = 1, 2 \\ 1/8, & x = 3, 4, 5, 6 \\ 0, & otherwise \end{cases} \quad h(y) = \begin{cases} 1/3, & y = 1, 2 \\ 1/12, & y = 3, 4, 5, 6 \\ 0, & otherwise \end{cases}$$

   (a) Make a table showing the joint distribution $f(x, y)$.

   (b) Find the probability that the sum of the two dice is greater than 8.

5. Discrete random variables $X$ and $Y$ have the following joint probability distribution

| f(x,y) | x=0 | x=1 |
|---|---|---|
| y=0 | 0.1 | 0 |
| y=1 | 0.1 | 0.1 |
| y=2 | 0.1 | 0.2 |
| y=3 | 0 | 0.4 |

(a) Compute the correlation coefficient $\rho_{XY}$.

(b) Compute the probability $P(Y \geq 2, X = 0)$.

(c) Compute the probability $P(Y \geq 2|X = 0)$.

(d) If this is a game and you win $\$(100X + 10Y^2)$ each time you play this game. What is the expected amount you win per game?

(e) There is a third random variable $Z$ which is independent from $Y$. The mean and variance for $Z$ are given as $\mu_Z = 1$ and $\sigma_Z^2 = 0.5$. Compute the mean and variance of the linear combination $2Y - 4Z$.