HOMEWORK #5 - DUE: Friday, March 1

Write your name on everything you hand in. Show your work.

1. The following MATLAB function computes and returns $P(X = x)$ according to the binomial distribution $b(x; n, p)$ where n is the number of trials in the Bernoulli process, p is the probability of success in each trial and x is the number of successes we are interested computing the probability of.

```
function b = BinomialDist (x, n, p)
b = (factorial(n)/(factorial(x)*factorial(n-x)))*(p^x)*((1-p)^(n-x));
```

   (a) Save the above function in to a file named BinomialDist.m
   Use the function to compute the numerical value of $b(x = 89; n = 100, p = 0.8)$. You call the function from MATLAB command line with $BinomialDist(89, 100, 0.8)$ in this case.

   If one wants to generate and graph the entire Binomial distribution for a given n and p, the following function can be used:

```
function b = BinomialDistGraph (n, p)
for x = 0:n
    b(x+1) = (factorial(n)/(factorial(x)*factorial(n-x)))*p^x*(1-p)^(n-x);
end;
bar(0:n,b);
axis([0 n 0 max(b)]);
```

   Save this function into a file called BinomialDistGraph.m

   This function computes all $P(X = x)$ for the given n and p values in an array named b and uses the *bar* command to graph it.

   (b) Use the function given above to graph the Binomial distribution for $n = 25$ and $p = 0.67$. What is the mean, variance and standard distribution?

   (c) Use the function given above to graph the Binomial distribution for $n = 25$ and $p = 0.03$. What is the mean, variance and standard distribution?

   Note: *Do NOT attach printouts of the graphs to your HW.*

   Note2: *MATLAB is available on the computers in the CADE lab. Please see the TA if you have problems with MATLAB.*

2. (a) Textbook exercise 5.11
   (b) Textbook exercise 5.12

3. The internet connection speed at any time from your home can depend on the amount of overall internet traffic at that time. Let the random variable X denote the speed of connection in megabits per second (MBPS). Assuming X is uniformly distributed on the interval 0.75 to 1.25 MBPS, answer the following questions:
   (a) Find the mean connection speed and standard deviation.
   (b) What is the probability that the connection speed will be less than 0.8 MBPS at any given time?
   (c) What is the probability that the connection speed will be between 0.875 MBPS and 1.125 MBPS at any given time?

4. Let the random variable X denote the annual snow fall amount at a well known Utah ski resort. X has a normal distribution with mean 500 inches and standard deviation 50 inches.
   (a) What is the probability that in a given year the snow fall will be between 432 and 568 inches?
   (b) Find a value d such that X is in the range $500 \pm d$ with probability 0.999.
   (c) What is the probability that at least 8 out of 10 consecutive years will have annual snowfall amount greater than 522 inches?

5. Let T be a random variable that is the time to failure (in years) of a certain type of electrical component. T has an exponential probability density function

$$f(x;\beta) = \begin{cases} \frac{1}{\beta}e^{-\frac{x}{\beta}}, & x > 0 \\ 0, & otherwise \end{cases}$$

   with $\beta = 2$ years.
   (a) Compute the probability that a given component will fail in 5 years or less.
   (b) A laboratory uses 10 of these components. Let X be the number of components out of the 10 that have failed in 5 years or less. Compute the probability that 6 components have failed in 5 years or less.
   (c) For the same laboratory, compute the probability that all 10 components have failed in 5 years or less.