HOMEWORK #4 - DUE: Friday, Feb 22
Write your name on everything you hand in. Show your work.

1. Exercise 3.1 from textbook.

2. (a) The function

$$f(x) = \begin{cases} 1.5 - 2|x|, & -1 \le x \le 1 \\ 0, & otherwise \end{cases}$$

is not a valid probability density function. Why not?

(b) The function $F(x) = 1 + \sin(x)$ is not a valid cumulative distribution function. Why not?

(c) The function

$$F(x) = \begin{cases} 2 - \frac{1}{x}, & x > 1 \\ 0, & otherwise \end{cases}$$

is not a valid cumulative distribution function. Why not?

(d) Is the following valid probability mass function for a discrete random variable? If it is not, state the reason.

$$f(x) = \begin{cases} 0.2, & x = 0 \\ 0.6, & x = 1 \\ 0.2, & x = 2 \\ 0, & otherwise \end{cases}$$

(e) Consider the following probability mass function for a discrete random variable:

$$f(x) = \begin{cases} 2k, & x = -1 \\ 0.5, & x = 0 \\ 3k, & x = 1 \\ 0, & otherwise \end{cases}$$

Find the value k which makes $f(x)$ a valid probability mass function.

3. An electronics company manufactures 3 models of microprocessors. The models sell for the following prices: Model A: $20, Model B: $40 and Model C: $60.

Customers buying microprocessors from this company choose the various models with the following probabilities:

$$P(A) = 0.2 \quad P(B) = 0.7 \quad P(C) = 0.1$$

(a) A market analyst wishes to calculate the average price of chips that customers purchase. Calculate this average or mean price.

(b) The plot below shows the probability density function $f(x)$ for the maximum sustainable clock speed of the model A microprocessor.
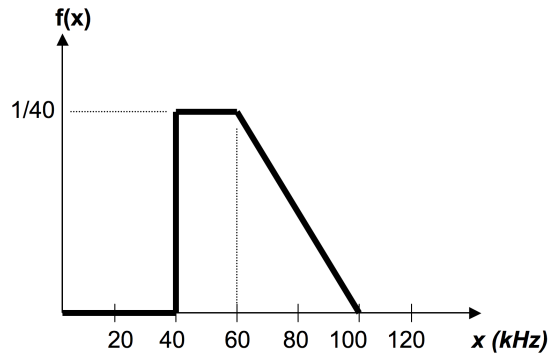
Find the mean of the maximum sustainable clock speed for the model A micropro-
cessor.

(c) Find the cumulative distribution function $F(x)$ for this random variable.

(d) Compute the probability that the maximum sustainable clock speed for a model A
will be less than 60 kHz.

(e) Compute the probability that the maximum sustainable clock speed for a model A
will be between 60 and 80 kHz.

4. Exercise 3.36 from textbook.

5. A cellphone service provider wants to analyze the signal strengths of its network.

(a) Let X be the random variable that is the distance (in miles) that a customer will
be to the nearest cellphone tower. Analyzing their database, the company finds
that the probability density function is

$$f(x) = \begin{cases} 1/4, & 2 \le x \le 6 \\ 0, & otherwise \end{cases}$$

Find the mean and variance of random variable X.

(b) The company is now interested in signal strengths. The company finds that the
signal power (in milliwatts) at a distance x from the tower is $P(x) = 40/x^2$. Given
the probability density function for X in the previous part, what is the expected
signal strength? *Hint: This is asking for $E[P(x)]$*

6. A bag contains 7 red and 3 blue marbles. You draw three marbles from this bag without
replacement. Let X be the number of blue marbles you get.

(a) Is X a discrete or a continuous random variable?

(b) Determine the probability distribution function $f(x)$ and the cumulative distribu-
tion function $F(x)$ for the random variable X.

(c) Using $F(x)$ find $P(X \le 1)$

(d) Compute the mean, variance and standard deviation of the random variable X.

(e) Assume that this is a game in a casino: the payoff is $\$100^X$ and it costs $\$10,000$ to
play the game each time. Compute the average net gain/loss per game.