HOMEWORK #3 - DUE: Friday, Feb 1
Write your name on everything you hand in. Show your work.

1. There is a deck of 12 cards numbered 1 through 12. You draw a card from the deck, look at it and then put it back in the deck. You draw a total of two cards in this way. Lets define the following events:

   - A: You get an even numbered card and an odd numbered card regardless of their order.
   - B: The first card you draw is even numbered.
   - C: The sum of the cards is even.
   - D: The sum of the cards is 18.

   (a) Are events A and B independent?
   (b) Are events A and C independent?
   (c) Compute $P(D)$.
   (d) Compute $P(D|C)$.

2. Textbook exercise 2.94: The probability that Tom will be alive in 20 years is 0.7, and the probability that Nancy will be alive in 20 years is 0.9. If we assume independence for both, what is the probability that neither will be alive in 20 years?

3. We have a bag with 5 red and 15 green marbles. You draw 3 marbles out of this bag without replacement.

   (a) What is the probability of getting a red, red and green marble in that order?
   (b) What is the probability of not getting any green marbles?
   (c) What is the probability that you got a red, red and green in that order given that you got 2 red and 1 green marble?
   (d) What is the probability that you got 2 red and 1 green marble given that you got a red, red and green in that order?

   Hint: You will need material from Monday, Jan 28 lecture for the following problems.

4. A company which manufactures pacemakers is testing new electrodes made of Gold or Platinum or Rhodium. Electrodes are always made from one and only one of these metals. Define the following events:

   - Au: The electrode is made of Gold.
   - Pt: The electrode is made of Platinum.
   - Rh: The electrode is made of Rhodium.

The electrodes must adhere to a silicone substrate for many years. Let X be the event that the electrode adheres. Conditional probabilities of adherence are given:

$$P(X|Au) = 0.9 \; , \;\; P(X|Pt) = 0.8 \; , \;\; P(X|Rh) = 0.6$$

Probabilities of different metals being used to manufacture the electrodes are given:

$$P(Au) = 0.2 \; , \;\; P(Pt) = 0.7, \;\; P(Rh) = 0.1$$

(a) Find $P(Au|X)$.

(b) Find $P(Au'|X)$.

(c) Find $P(X)$.

5. Textbook exercise 2.128: A rare disease exists in which only 1 in 500 are affected. A test for the disease exists but of course is not infallible. A correct positive result (patient actually has the disease) occurs 95% of the time while a false positive result (patient does not have the disease) occurs 1% of the time. If a randomly selected individual is tested and the result is positive, what is the probability that the individual has the disease?