HOMEWORK #1 - DUE: Friday, Jan 18

Write your name on everything you hand in. Show your work.

1. A coin is tossed 4 times. We use H to denote heads and T to denote tails. For instance, HTHH means we got the sequence heads, tails, heads, heads.

   (a) Write down the elements of the sample space.
   Hint: Using a tree diagram can be helpful.

   (b) List the outcomes included in each of the following events. Also compute the probabilities for each event. Assume that the coin is fair.

   - Event A: Number of times the coin came up heads is 0.
   - Event B: Number of times the coin came up heads is 1.
   - Event C: The first two tosses came up heads.
   - Event D: Number of times the coin came up heads is greater than 1.

   (c) Answer YES or NO to the following questions:

   - Are the events A and B mutually exclusive?
   - Are the events B and C mutually exclusive?
   - Are the events C and D mutually exclusive?

   (d) Compute the probability $P(A \cup D)$.

   (e) Compute the probability $P(C \cup D)$.

2. You are given the sample space $\mathcal{S} = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and the events X=even numbers, Y=numbers greater than or equal to 6, $Z = \{2, 4\}$. You are told that $P(0) = P(2) = P(4) = P(6) = P(8)$ and $P(1) = P(3) = P(5) = P(7)$. Furthermore, you are told that odd numbered outcomes are twice as likely as the even numbered outcomes.

   (a) Draw the Venn diagram showing the sample space $\mathcal{S}$ and events X,Y,Z. *Note: The areas of the events in a Venn diagram don't have to be proportional to their probabilities, but you should correctly show the relationship of the events. For instance, if two events are mutually exclusive, they should not intersect in the Venn diagram.*

   (b) List the elements of the each of the following and compute their probability.

   - $Z \cup Y$
   - $X \cup Y$
   - $X' \cap Z$
   - $(X \cap Z)'$
   - $(X \cup Y) \cap Z$
   - $(X \cap Z) \cup Y$

3. (Exercise 2.54 from textbook) Suppose that in a senior college class of 500 students it is found that 210 smoke, 258 drink alcoholic beverages, 216 eat between meals, 122 smoke and drink alcoholic beverages, 83 eat between meals and drink alcoholic beverages, 97

smoke and eat between meals and 52 engage in all three of these bad health practices. If a member of this senior class is selected at random, find the probability that the student

(a) smokes but does not drink alcoholic beverages

(b) eats between meals and drinks alcoholic beverages but does not smoke

(c) neither smokes nor eats between meals

4. Let $\mathcal{S}$ represent the sample space of all houses in zip code 84106. You are also given the events:

   - A = Houses built after 1990
   - B = Houses with a 2-car garage
   - C = Houses with lead-based paint

   $P(A) = 0.1$, $P(B) = 0.6$, $P(C) = 0.6$, $P(A \cap C) = 0$, $P(A \cap B) = 0.1$ and $P(B \cap C) = 0.4$.

   (a) Compute the probability $P(A \cap B \cap C)$. *Hint:* $A \cap B \cap C = (A \cap C) \cap B$

   (b) Compute the probability $P(A \cup B \cup C)$.

   (c) Using rules of probability and the probabilities you've been given above, show that it is not possible to find a house in this zipcode built after 1990 that doesn't have a 2-car garage.

   Hint: First write this event as an intersection of two other events or their complements.

5. Let $\mathcal{S}$ represent the sample space of all automobiles produced in company X's manufacturing plant. Lets also define the following events:

   - A = Exterior color black, B = Exterior color silver, C = Automatic transmission, D = Manual transmission

   We are give the following probabilities: $P(A) = 0.65$ and $P(B) = 0.25$, $P(C) = 0.7$ and $P(B \cap C) = 0.09$.

   You are told that events A and B are mutually exclusive. Similarly, a car can't have automatic and manual transmission at the same time so C and D are mutually exclusive as well. Furthermore, you are told that events C and D form a partition of the sample space.

   (a) Draw a Venn diagram to show all the events.

   (b) Compute the probability $P(A \cup B)$?

   (c) Compute the probability $P((A \cup B)')$.

   (d) Compute the probability $P(B \cap D)$.

   (e) Compute the probability $P(D)$.