Midterm 1 solutions

(1) a) $n_1 = 26$, $n_2 = 10$

total outcomes $n_1 \times n_2 = \boxed{260}$

b) $_8P_3 = \dfrac{8!}{(8-3)!} = 8 \times 7 \times 6 = \boxed{336}$

(2) a) red, red, red $\quad \dfrac{6}{8_2} \times \dfrac{5}{7} \times \dfrac{4}{6} = \boxed{5/14}$

b)

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                         6/8   red      36/64   *
              red                                        
        6/8              2/8   blue     12/64   *   *   *
                                                        
        2/8             6/8   red       12/64       *   *   *
              blue                                      
                         2/8   blue     4/64           *
                                         
                                        X   Y   X∩Y
```

$P(X|Y) = \dfrac{P(X \cap Y)}{P(Y)} = \dfrac{12/64 + 12/64}{12/64 + 12/64 + 4/64}$

$= \dfrac{24/64}{28/64} = \dfrac{24}{28} = \boxed{6/7}$

③ a) $X \cup Y = \{2,4,5,6,7\}$

b) $Z \cap Y = \{2\}$

* since we are told $P(Z \cap Y) = 2/10$ this means

$\quad P(2) = 2/10$.

* $Y = \{2,4,6\}$. since all even outcomes have equal probability and we just found $P(2) = 2/10$, we find $P(Y) = 6/10$
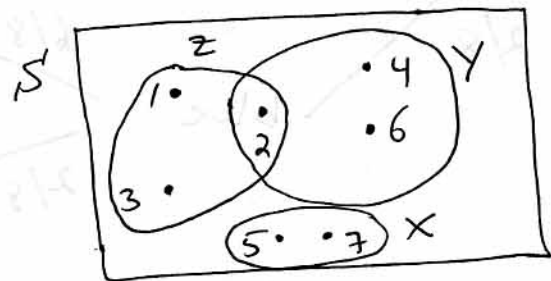
* $P(Y \cup Z) = P(Y) + P(Z) - P(Y \cap Z)$

$$= \frac{6}{10} + \frac{3}{10} - \frac{2}{10}$$

$$= \boxed{7/10}$$

* $Y \cup Z = \{1,2,3,4,6\}$

$\quad X = \{5,7\}$

Notice $X = (Y \cup Z)'$

(can also see from Venn Diagram)

Therefore $P(X) = 1 - P(Y \cup Z)$

$$= 1 - \frac{7}{10} = \boxed{\frac{3}{10}}$$

(4) a) $P(C_2 | X) = \dfrac{P(X|C_2) P(C_2)}{P(X|C_1)P(C_1) + P(X|C_2)P(C_2) + P(X|C_3)P(C_3) + P(X|C_4)P(C_4)}$

Bayes rle

$$= \frac{1 \times 1/8}{1/2 \times 1/8 + 1 \times 1/8 + 3/4 \times 1/4 + 3/4 \times 1/2}$$

$$= \frac{1/8}{1/16 + 2/16 + 3/16 + 6/16} = \frac{2/16}{12/16} = \boxed{\frac{1}{6}}$$

The denominator is $P(X)$ which ya need for part (b). So $P(X) = 12/16$

b) The question asks for $P(Y' \cap X')$

$P(Y' \cap X') = P((Y \cup X)') = 1 - P(Y \cup X)$

$$= 1 - [P(Y) + P(X) - \underbrace{P(X)P(Y)}_{\text{using independence}}]$$

$$= 1 - \left[\frac{4}{5} + \frac{12}{16} - \frac{4}{5} \wedge \frac{12}{16} \right]$$

$$= 1 - \frac{4 \times 16 + 12 \times 5 - 4 \times 12}{5 \times 16}$$

$$= 1 - 76/80 = 4/80 = \boxed{1/20}$$