NAME:

ECE 3530 MIDTERM 1
Show your work.
Four questions each worth 25 points.
Closed book, limited notes (1 regular size sheet front&back). No laptops.

1. (a) User IDs on an old computer system consist one letter from the English alphabet followed by a single digit between 0 and 9. The English alphabet contains 26 letters. How many different IDs are possible?

   (b) There is a deck of eight cards numbered 1-8. You draw 3 cards from this deck without replacement and lay them down on a table from left to right in the order they are drawn. How many possible outcomes are there?

2. A bag contains a total of 8 marbles of which 6 are red and 2 are blue.

   (a) You draw 3 marbles without replacement. Find the probability that all three marbles are red.

   (b) We start from scratch with the bag containing a total of 8 marbles of which 6 are red and 2 are blue. You draw 2 marbles with replacement. Define the following events:

   - X: you get at least one red marble
   - Y: you get at least one blue marble

   Find the conditional probability $P(X|Y)$.

3. An experiment has the sample space $\mathcal{S} = \{1, 2, 3, 4, 5, 6, 7\}$. The following events are also defined

   - $X = \{5, 7\}$
   - $Y =$ the outcome is even
   - $Z =$ the outcome is less than or equal to 3

   (a) List the elements in $X \cup Y$

   (b) In addition to the information given above, we are further told that:

   - All even outcomes have equal probability. In other words, $P(2) = P(4) = P(6)$
   - $P(Z \cap Y) = 2/10$
   - $P(Z) = 3/10$

   Find the probabilities $P(Y \cup Z)$ and $P(X)$.

4. A traveling salesman can be found at one of four cities (C_1,C_2,C_3,C_4) at any given time. His probability of being at these cities are

$$P(C_1) = 1/8,\ P(C_2) = 1/8,\ P(C_3) = 1/4,\ P(C_4) = 1/2.$$

He always carries his cellphone with him. Let X be the event that his cellphone has coverage. The conditional probabilities that his cell phone has coverage given what city he is at are as following:

$$P(X|C_1) = 1/2,\ P(X|C_2) = 1,\ P(X|C_3) = 3/4,\ P(X|C_4) = 3/4.$$

(a) Given that today his cellphone has coverage, what is the probability that he is at city C_2?

(b) Let Y be the event that his cellphone is at least 70% charged. He usually remembers to recharge his cell phone every evening so the probability that his cell phone is at least 70% charged at any given time is $4/5$. Events X and Y are independent. What is the probability that his cellphone is less than 70% charged AND out of coverage at the same time?