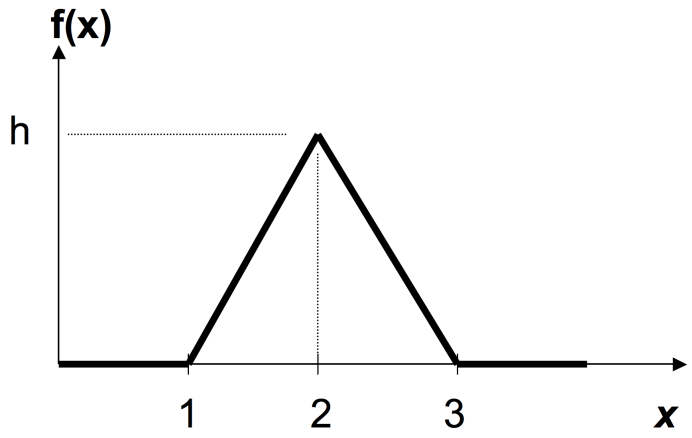
NAME:

ECE 3530 PRACTICE MIDTERM 2
Show your work.
Closed book, limited notes (1 regular size sheet front&back). No laptops.

1. A continuous random variable X has the probability density function

$$f(x) = \begin{cases} 0, & x < 1 \\ hx - h, & 1 \le x \le 2 \\ 3h - hx, & 2 \le x \le 3 \\ 0, & x > 3 \end{cases}$$

which can be graphed as



(a) Find h which makes $f(x)$ a valid probability density function.
(b) Find the cumulative distribution function $F(x)$.

2. Random variable X has a normal probability distribution with mean 10.3 and standard deviation 2.

   (a) Compute the numerical value of $P(7.2 \leq X \leq 13.8)$.

   (b) Find a value d such that X is in the range $10.3 \pm d$ with probability 0.999.

   (c) Let Y be a random variable with variance $\sigma_Y^2 = 6$ and independent of X. Compute the variance of $5X - 3Y$.

3. Let X and Y be two continuous random variables with the joint density function

$$f(x,y) = \begin{cases} x+y, & 0 \le x \le 1 \text{ and } 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Are the random variables X and Y independent? Justify your answer.

(b) Compute the numerical value of $P(Y \ge \frac{1}{2}, X \le \frac{1}{2})$.

4. Let X be the sent bit and Y the received bit in a binary communications channel. The joint probability distribution $f(x, y)$ is given as:

f(x,y)	x=0	x=1
y=0	0.4	0.1
y=1	0.1	0.4

   (a) Compute the numerical value of $P(Y = 1|X = 0)$

   (b) Compute the covariance of random variables X, Y.

   (c) When a single bit is sent and received, we say that an error has occurred if $Y \neq X$. If a 8-bit long message is sent over this communication channel, what is the probability that 1 or less errors will occur?