NAME:

ECE 3530 PRACTICE MIDTERM
Show your work.
Closed book, limited notes (1 regular size sheet front&back). No laptops.

1. There is a deck of 15 cards numbered 1 through 15.

   (a) If you draw 5 cards from the deck without replacement, what is the probability that your hand will contain the cards 10 and 13?

   (b) If you draw 2 cards from the deck with replacement, what is the probability that the sum of the values of the 2 cards you draw will be an even number?

   (c) If you draw one card from the deck, then roll one fair dice (6-sided) and finally flip one fair coin (heads or tails). What is the probability that you get an even numbered card OR heads on the coin toss OR both?

   Hint: Remember choosing without replacement means that we do not put the first card we chose back in the deck before we choose the second, and so forth. Choosing with replacement means we put the card back in the deck before drawing the next one.

2. An online bookseller uses one of four shipping companies to send packages to its customers. Any package can be sent with one and only one of these companies. Define the following events:

   - C_1: the package is shipped with company 1
   - C_2: the package is shipped with company 2
   - C_3: the package is shipped with company 3
   - C_4: the package is shipped with company 4

   The bookseller uses the shipping companies with the following probabilities:

   $$P(C_1) = 0.5 \quad P(C_2) = 0.25 \quad P(C_3) = 0.125 \quad P(C_4) = 0.125.$$

   Let X be the event that the package arrives on time at its destination. Depending on the shipping company used, the probability of X varies:

   $$P(X|C_1) = 0.85 \quad P(X|C_2) = 0.9 \quad P(X|C_3) = 0.8 \quad P(X|C_4) = 0.8.$$

   (a) Compute the numerical value of $P(C_2 \cup C_3)$.

   (b) Given that a package has arrived on time what is the probability that it was shipped with company C_1? In other words, compute the numerical value of $P(C_1|X)$.

   (c) Are the events C_1' and X independent? Justify your answer.

3. The sample space of an experiment consists of the following outcomes:

$$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

The following events are defined:

- $X =$ The outcome of the experiment is an even number
- $Y = \{6, 7, 8\}$
- $Z = \{3, 7\}$

We are also given the following information:

- $P(X \cap Y) = 1/3$
- X and Y are independent events
- Probabilities of all even numbered outcomes are equal. In other words:

$$P(2) = P(4) = P(6) = P(8)$$

Answer the following questions:

(a) Draw the Venn diagram for events X, Y, Z, and all the outcomes in S.

(b) Define a new event W which together with events X and Z forms a partition of the sample space S. Define W by listing the outcomes included in it.

(c) Compute the numerical value of $P(X \cup Y)$.

4. Two bags A and B each contain a mixture of red balls and black balls. Bag A contains a total of 10 balls of which 5 are red and 5 are black. Bag B contains a total of 16 balls of which 8 are red and 8 are black. In step 1 of a game, a blind folded person chooses one of the two bags with equal probability. Still blind folded, in step 2 of the game he chooses 3 balls without replacement from the bag he chose in the step 1 of the game.

   (a) What is the probability that he will get 3 red balls in step 2, if he chose bag A in step 1?

   (b) If he got 3 red balls in step 2, what is the probability that he chose bag B in step 1?