Final exam solutions

① a) $\bar{X} = \frac{1}{6}(48+47+48+50+47+48) = \boxed{48}$

$$s^2 = \frac{1}{5}\left[(48-48)^2+(47-48)^2+(48-48)^2 + (50-48)^2+(47-48)^2+(48-48)^2\right]$$

$$= \frac{1}{5}\left[0+1+0+4+1+0\right] = \frac{6}{5} \boxed{=1.2}$$

b) $\alpha = 0.05$ t-distribution with $v = 6-1 = 5$

$t_{0.05} = 2.015$

$$\mu > \bar{X} - t_{0.05}\frac{s}{\sqrt{n}} = 48 - 2.015 \times \frac{\sqrt{1.2}}{\sqrt{6}}$$

$$\cong 47.1$$

$$\boxed{\mu > 47.1}$$

c) Yes, it was correct because the normal-quantile plot shows a line.

② a) $n = 64$, $\bar{X} = 998$, $\sigma = 10$

Since σ known, $n \geq 30$ $\Rightarrow$ z-distribution.

$$\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$\alpha = 0.03 \qquad \alpha/2 = 0.015$

$z_{0.015} = 2.17$

$$998 - 2.17 \times \frac{10}{\sqrt{64}} < \mu < 998 + 2.17 \times \frac{10}{\sqrt{64}}$$

$$998 - 2.7125 < \mu < 998 + 2.7125$$

$$995.2875 < \mu < 1000.7125$$

b) • narrower ($n = 1000$ narrower than $n = 64$)

• wider (99% wider than 97%)

(3) a) $H_0 : \mu = 1000$, $H_1 : \mu < 1000$

$s^2 = 400$, $\bar{x} = 987.5$, $\alpha = 0.02$

This is a one-sided test so

reject H_0 if $\bar{x} < \mu - t_\alpha \dfrac{s}{\sqrt{n}}$

$t_{0.02} = 2.249$ from $v = 16 - 1 = 15$
Table A.4

$s = \sqrt{400} = 20$

so reject H_0 if $\bar{x} < 1000 - 2.249 \times \dfrac{20}{\sqrt{16}}$

$$\boxed{\bar{x} < 988.755}$$

Since $\bar{x} = 987.5$ $\boxed{H_0 \text{ is rejected}}$

b) χ^2-distribution (Chi-square)

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

90% confidence interval $\Rightarrow \alpha = 0.1$

$V = 16 - 1 = 15$

so From Table A.5 $\chi^2_{\alpha/2} = \chi^2_{0.05} = 24.99$

$\chi^2_{1-\alpha/2} = \chi^2_{0.95} = 7.261$

$$\frac{15 \times 400}{24.996} < \sigma^2 < \frac{15 \times 400}{7.261}$$

$$\boxed{240.04 < \sigma^2 < 826.33}$$

4 a) $\mu_A = 800$, $\sigma_A = 50$, $n_A = 100$

Z-distribution since σ_A known.

$P(790 \le \bar{X}_A \le 810) = P\left(\frac{790-800}{5} \le Z \le \frac{810-800}{5}\right)$

$$Z = \frac{\bar{X}_A - \mu_A}{\sigma_A / \sqrt{n_A}}$$

$= P(-2 \le Z \le 2)$

$$Z = \frac{\bar{X}_A - 800}{50/\sqrt{100}}$$

$= P(Z \le 2) - P(Z \le -2)$

$$= \frac{\bar{X}_A - 800}{5}$$

$= 0.9772 - 0.0228$

$= \boxed{0.9544}$

b) We first need to compute μ_B and σ_B^2.

Since $f(x)$ is a uniform distribution on the interval 770 to 830, we know that

$$\mu_B = \frac{770 + 830}{2} = 800$$

and $$\sigma_B^2 = \frac{(830-770)^2}{12} = \frac{60^2}{12} = 300$$

Then $$z = \frac{(\bar{X}_A - \bar{X}_B) - (\mu_A - \mu_B)}{\sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}}$$

$\boxed{\begin{array}{l} n_A = 125 \\ n_B = 60 \end{array}}$

$$z = \frac{(\bar{X}_A - \bar{X}_B) - (800 - 800)}{\sqrt{\frac{2500}{125} + \frac{300}{60}}} = \frac{\bar{X}_A - \bar{X}_B}{\sqrt{25}} = \frac{(\bar{X}_A - \bar{X}_B) -}{5}$$

So $$P(\bar{X}_A - \bar{X}_B > 8) = P\left(z > \frac{8-0}{5}\right)$$

$$= P(z > 1.6) = 1 - P(z < 1.6)$$
$$= 1 - 0.9452$$
$$= \boxed{0.0548}$$

c) Use Chebyshov's Theorem

$$P(700 < X_A < 900) =$$
$$P(800 - 100 < X_A < 800 + 100) =$$
$$P(\mu_A - 2\sigma_A < X_A < \mu_A + 2\sigma_A) \geq 1 - \frac{1}{2^2} = 0.75$$