

NAME:

**ECE 3530 FINAL**

**Show your work.**

**No credit will be given for the correct answer if no work is shown.**

**Four questions each worth 25 points.**

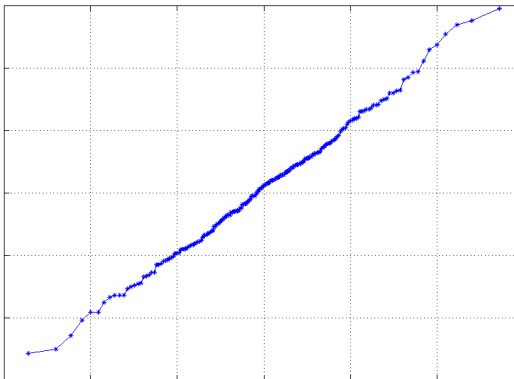
**Closed book, limited notes (3 regular size sheets front&back). No laptops.**

1. A hybrid car manufacturer is studying the miles per gallon (MPG) performance of their new model. Assume that the population distribution for MPG is approximately normal. A sample of 6 cars is tested and the following MPG's are recored:

48, 47, 48, 50, 47, 48

- (a) Compute the sample mean  $\bar{X}$  and the sample variance  $S^2$ .
- (b) Find a 95% lower bound for  $\mu$  the population mean MPG.
- (c) A quality control engineer tests a much larger sample of cars to check if the assumption that *the population distribution is approximately normal* was correct. He plots the following normal-quantile plot:

Normal-quantile plot



Was the assumption correct, why or why not? (1 sentence explanation).

2. A sample of 64 resistors are taken from a production line and are found to have a sample mean of 998 Ohms. The population distribution for the resistances is unknown but the population standard deviation is known to be 10 Ohms.
- (a) Find a 97% confidence interval for the population mean.
- (b) Answer the following two questions by writing either "narrower" or "wider" in the blank space.
- If the sample size were increased to  $n = 1000$  then the confidence interval would be \_\_\_\_\_ than the confidence interval computed from a sample with  $n = 64$ .
  - A 99% confidence interval is \_\_\_\_\_ than a 97% confidence interval.

3. A company manufacturing light bulbs is testing a new model. The company is going to test the hypothesis that the mean life time is 1000 hours vs. the alternative hypothesis that it is less than 1000 hours at the significance level  $\alpha = 0.02$ . Assume that the population distribution for life time is approximately normal.

A sample of 16 light bulbs are found to have sample mean  $\bar{x} = 987.5$  hours and sample variance  $S^2 = 400$ .

- (a) State the critical region and answer whether the null hypothesis  $H_0$  is rejected.  
(b) Find a 90% confidence interval for the population variance  $\sigma^2$ .

4. Companies  $A$  and  $B$  are two competing CPU manufacturers.

The population mean and population standard deviation for the clock speed of company  $A$ 's CPU is known as  $\mu_A = 800$  MHz and  $\sigma_A = 50$  MHz.

The population density function for the clock speed of company  $B$ 's CPU is an uniform distribution given as

$$f(x) = \begin{cases} 0, & x < 770 \text{ MHz} \\ 1/60, & 770 \text{ MHz} \leq x \leq 830 \text{ MHz} \\ 0, & x > 830 \text{ MHz} \end{cases}$$

(a) A sample of  $n_A = 100$  CPUs are taken from company A. Let  $\bar{X}_A$  denote the sample mean of their clock speeds. Compute the probability  $P(790 \leq \bar{X}_A \leq 810)$ .

(b) A sample of  $n_A = 125$  CPUs are taken from company  $A$ . Let  $\bar{X}_A$  denote the sample mean of their clock speeds. A sample of  $n_B = 60$  CPUs are taken from company  $B$ . Let  $\bar{X}_B$  denote the sample mean of their clock speeds.

Find the probability that the sample mean from company  $A$  is at least 8 MHz faster than the sample mean from company  $B$ . In other words, find  $P(\bar{X}_A - \bar{X}_B > 8)$ .

(c) Find a lower bound for the probability that any single CPU from company A will have a clock speed between 700 and 900 MHz.